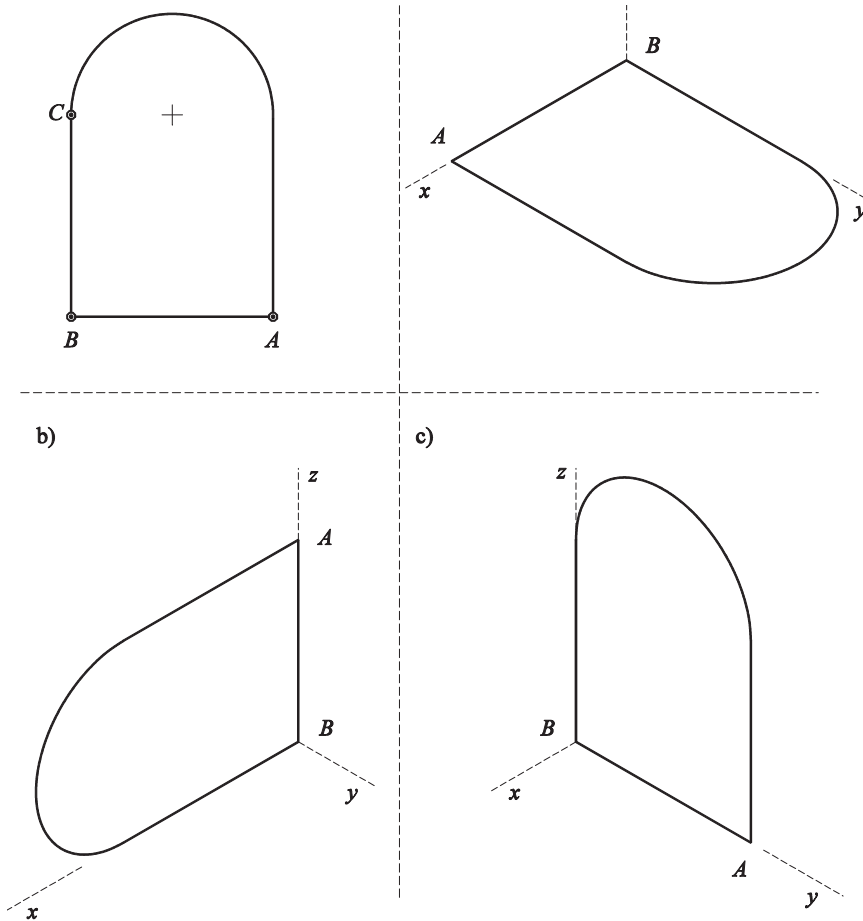
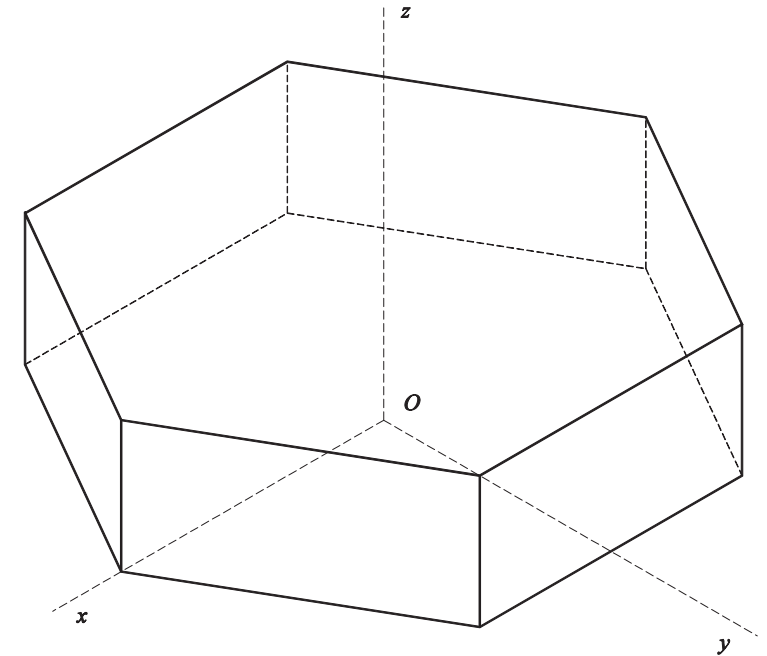


## 2 Technical isometry

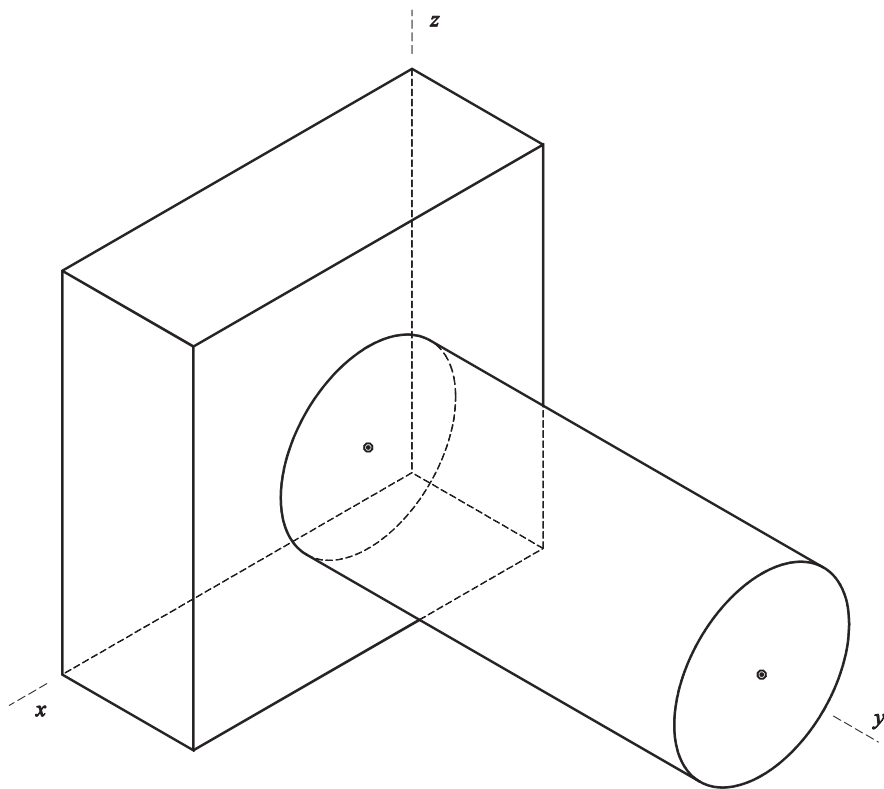
- **Exercise 2.1.** In technical isometry, construct the planar figure given by its orthogonal projection. Point  $B$  lies at origin  $O$ . The figure lies
  - a) in the plane  $(x, y)$ ,  $A \in x$ , choose solution for  $x_A > 0$  and  $y_C > 0$ ,
  - b) in the plane  $(x, z)$ ,  $A \in z$ , choose solution for  $z_A > 0$  and  $x_C > 0$ ,
  - c) in the plane  $(y, z)$ ,  $A \in y$ , choose solution for  $y_A > 0$  and  $z_C > 0$ .



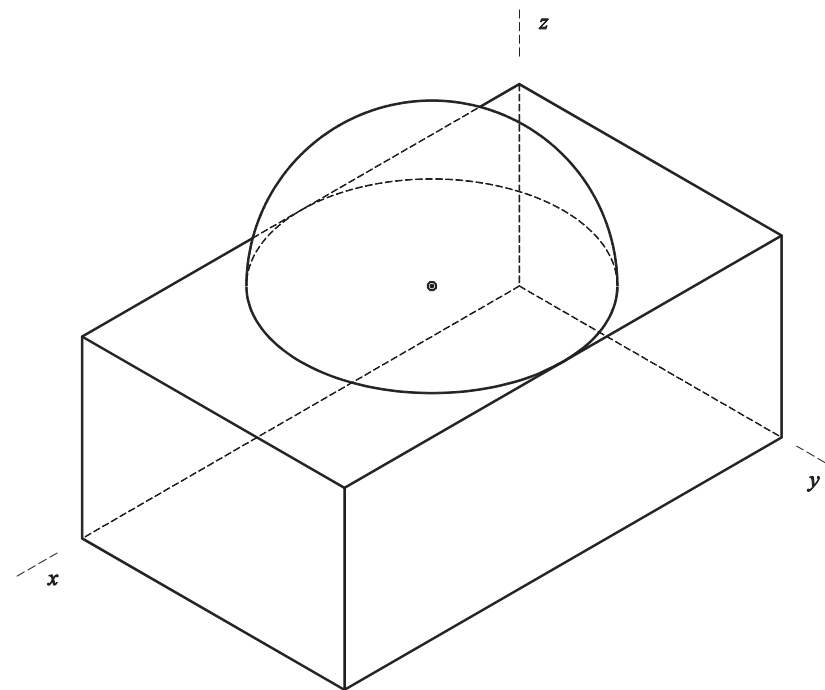
- **Exercise 2.2.** In technical isometry, construct the regular hexagonal prism given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.



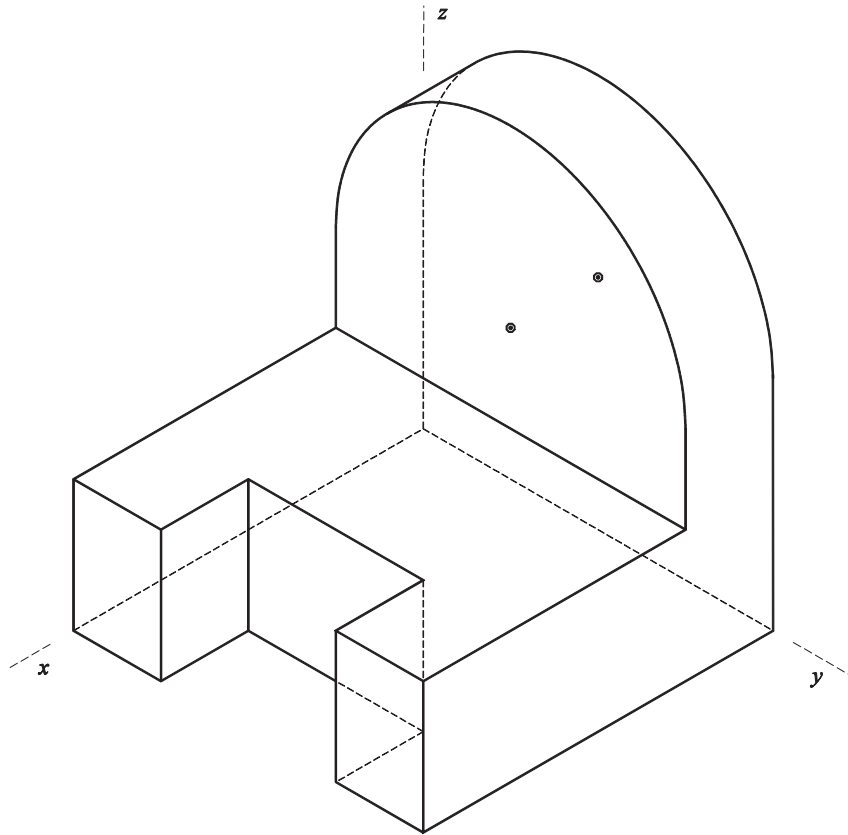
- **Exercise 2.3.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.



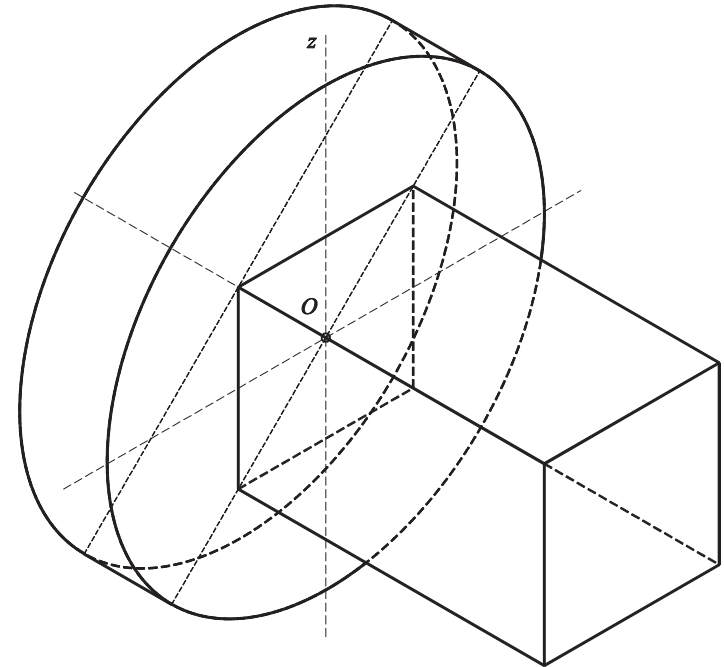
- **Exercise 2.4.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.



- **Exercise 2.5.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.

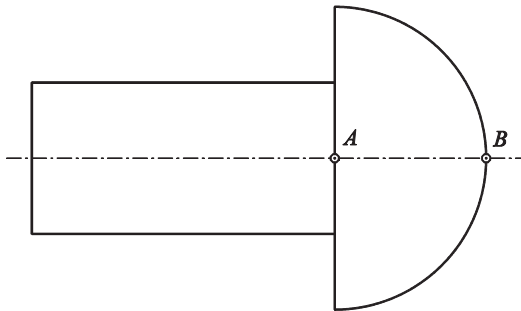


- **Exercise 2.6.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.

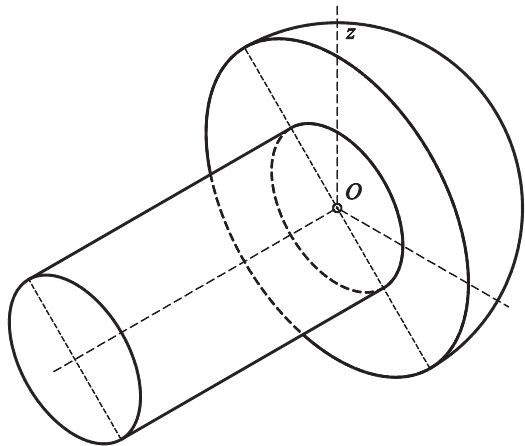


■ **Exercise 2.7.** In technical isometry, construct the rotary object (hemisphere and cylinder of revolution) given by its orthogonal projection in scale 1:1. Measure all dimensions you need. Indicate the visibility. Centre  $A$  of the hemisphere lies at origin  $O$ , the axis of revolution is identical with

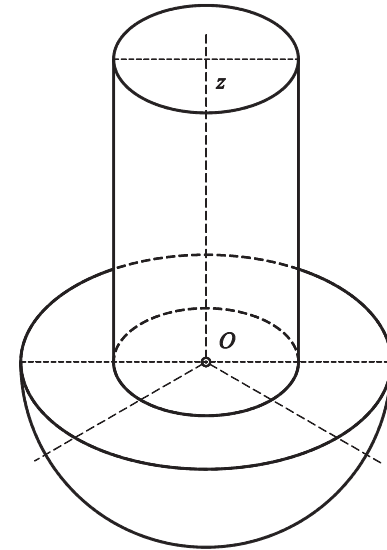
- a)  $x$ -axis, choose solution for  $x_B < 0$ ,
- b)  $z$ -axis, choose solution for  $z_B < 0$ ,
- c)  $y$ -axis, choose solution for  $y_B > 0$ .



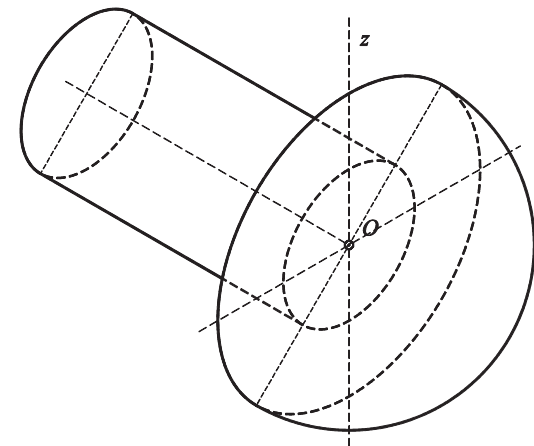
a)



b)



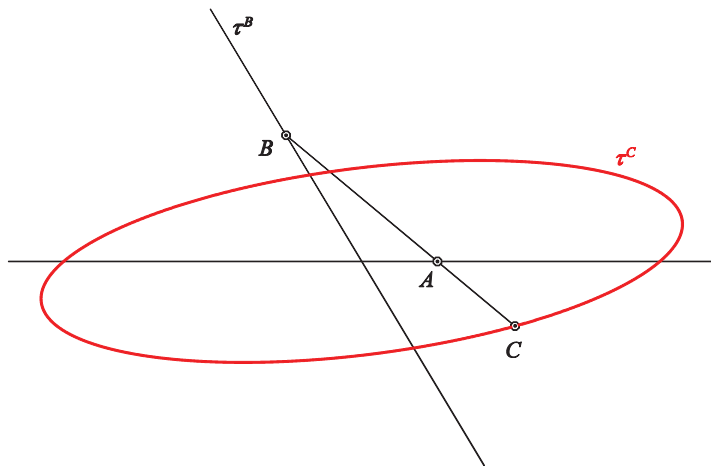
c)



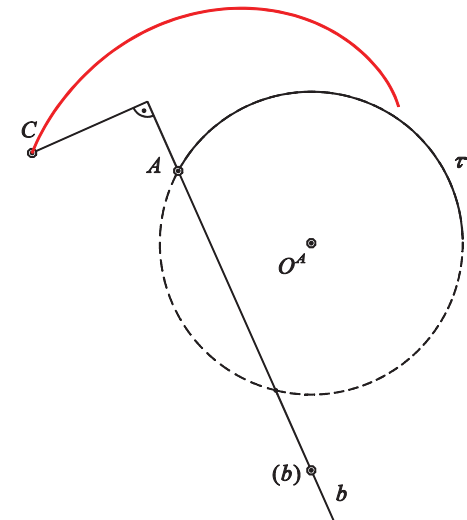
### 3 Planar kinematic geometry

#### 3.1 Motion given by trajectories and envelopes

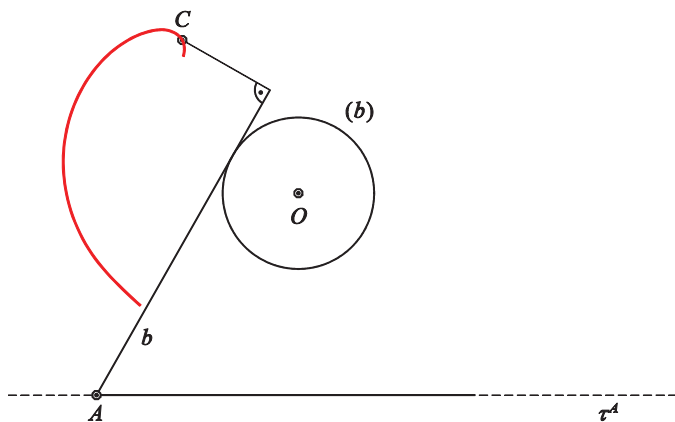
■ **Exercise 3.1.** Motion is given by trajectory  $\tau^A$  of point  $A$  and trajectory  $\tau^B$  of point  $B$ . Construct at least three new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines.



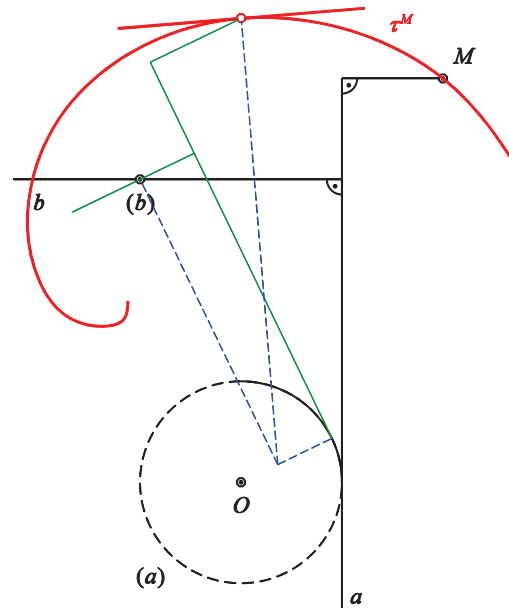
■ **Exercise 3.2.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope ( $b$ ) of straight line  $b$ . Construct at least three new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines. Consider the continuous part of trajectory  $\tau^A$  only.



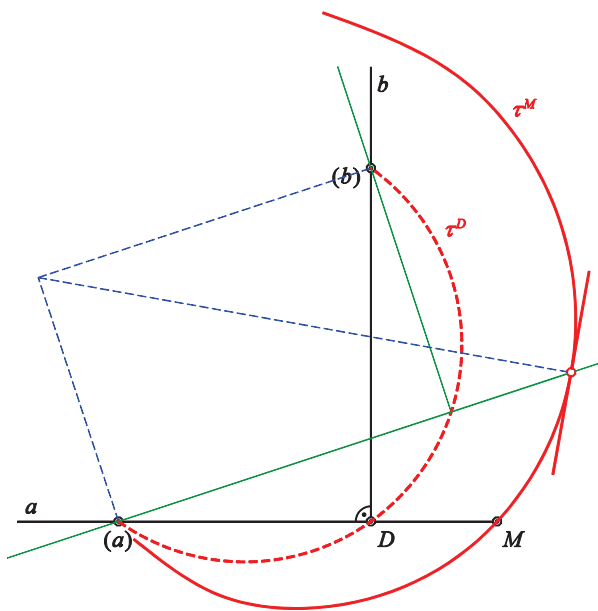
- **Exercise 3.3.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines. Consider the continuous part of trajectory  $\tau^A$  only.



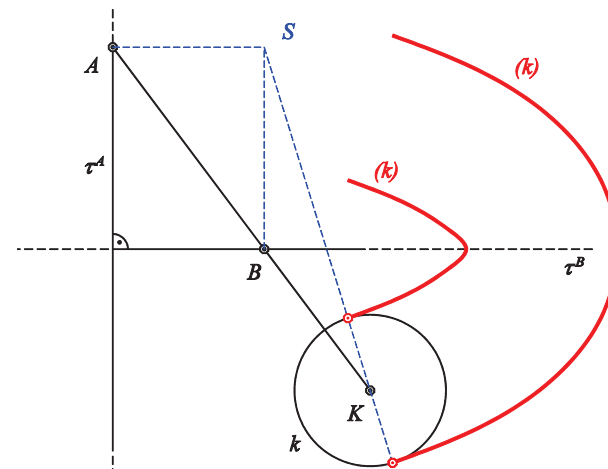
- **Exercise 3.4.** Motion is given by envelope  $(a)$  of straight line  $a$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of point  $M$ . Construct tangent lines to the trajectory  $\tau^M$  at each position. Sketch the part of trajectory  $\tau^M$  determined by all positions of point  $M$  and the corresponding tangent lines. Consider the continuous part of envelope  $(a)$  only.



- **Exercise 3.5.** Motion is given by envelope (a) of straight line  $a$  and envelope (b) of straight line  $b$ . Construct at least three new positions of point  $M$ . Construct tangent lines to the trajectory  $\tau^M$  at each position. Sketch the part of trajectory  $\tau^M$  determined by all positions of point  $M$  and the corresponding tangent lines.

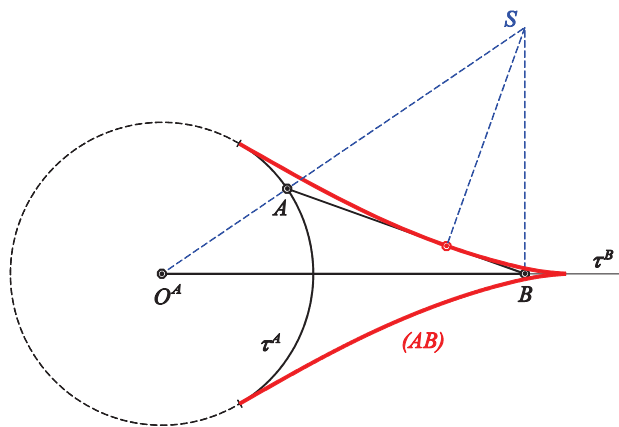


- **Exercise 3.6.** Motion is given by trajectory  $\tau^A$  of point  $A$  and trajectory  $\tau^B$  of point  $B$ . Construct at least three new positions of circle  $k$ . Construct point of contact between the circle and its envelope  $(k)$  at each position. Sketch the part of envelope  $(k)$  determined by all positions of circle  $k$  and the corresponding points of contact. Consider the continuous parts of both trajectories only.

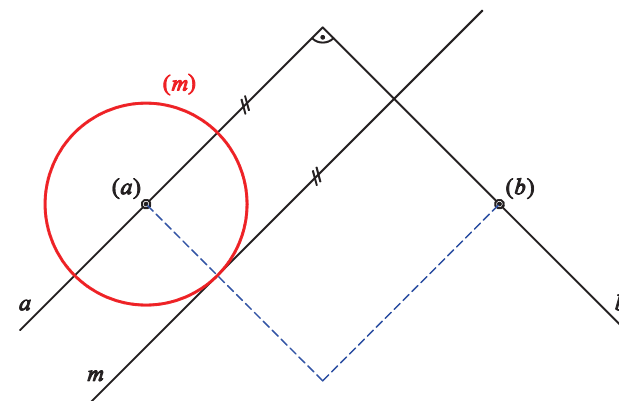


- **Exercise 3.7.** Motion is given by trajectory  $\tau^A$  of point  $A$  and trajectory  $\tau^B$  of point  $B$ . Construct at least three new positions of straight line  $AB$ . Construct point of contact between straight line  $AB$  and its envelope  $(AB)$  at each position. Sketch the part of envelope  $(AB)$  determined by all positions of straight line  $AB$  and the corresponding points of contact.

Consider the continuous parts of both trajectories only.

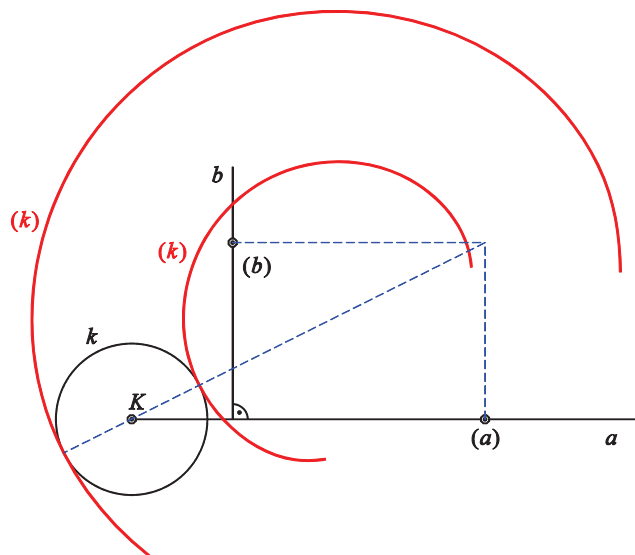


- **Exercise 3.8.** Motion is given by envelope  $(a)$  of straight line  $a$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of straight line  $m$ . Construct point of contact between straight line  $m$  and its envelope  $(m)$  at each position. Sketch the part of envelope  $(m)$  determined by all positions of straight line  $m$  and the corresponding points of contact.

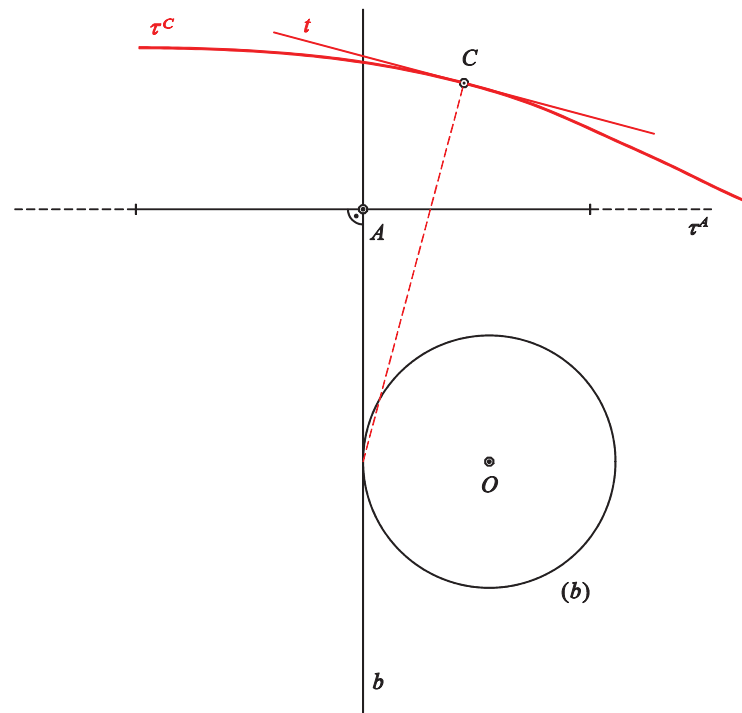




- **Exercise 3.9.** Motion is given by envelope  $(a)$  of straight line  $a$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of circle  $k$ . Construct point of contact between circle  $k$  and its envelope  $(k)$  at each position. Sketch the part of envelope  $(k)$  determined by all positions of circle  $k$  and the corresponding points of contact.



- **Exercise 3.10.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines. Consider the continuous parts of trajectory  $\tau^A$  only.

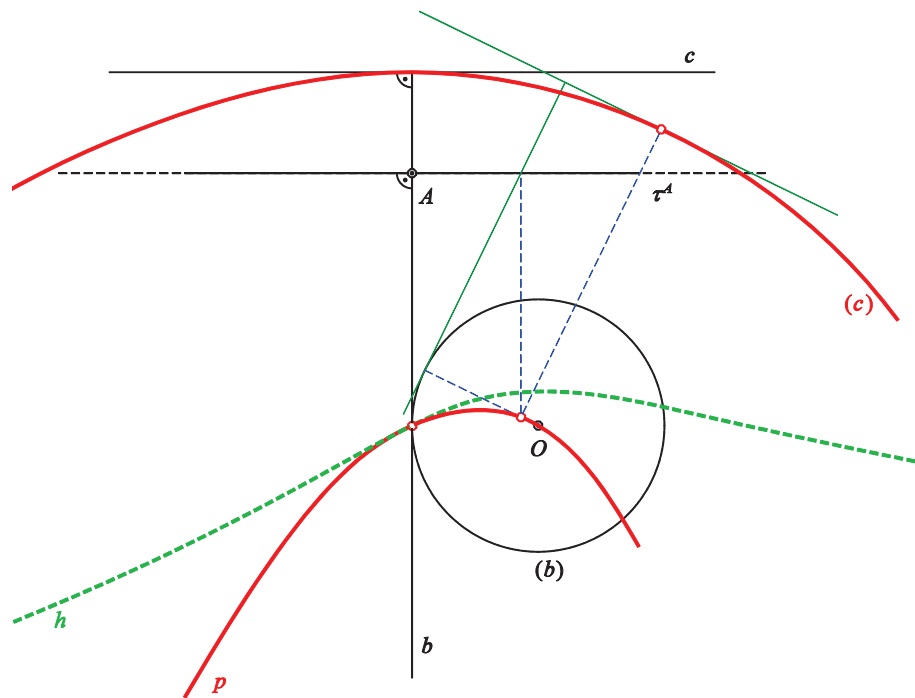


■ **Exercise 3.11.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of straight line  $c$ . Construct point of contact between the line and its envelope  $(c)$  at each position. Sketch the part of envelope  $(c)$  determined by all positions of straight line  $c$  and the corresponding points of contact.

Construct the corresponding part of fixed centrode  $p$ .

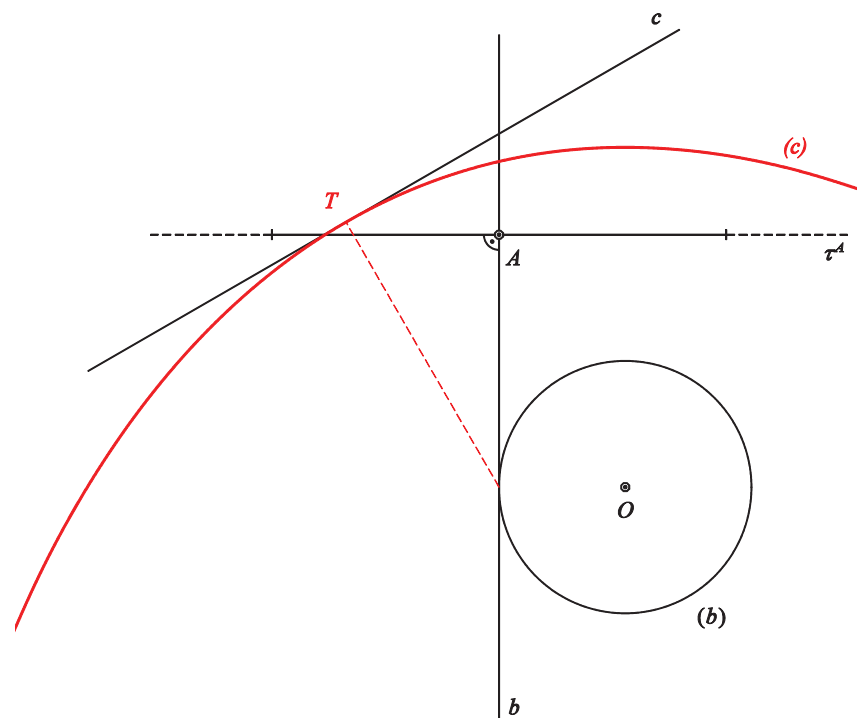
Construct the corresponding part of moving centrode  $h^0$  at the given instant.

Consider the continuous parts of trajectory  $\tau^A$  only.

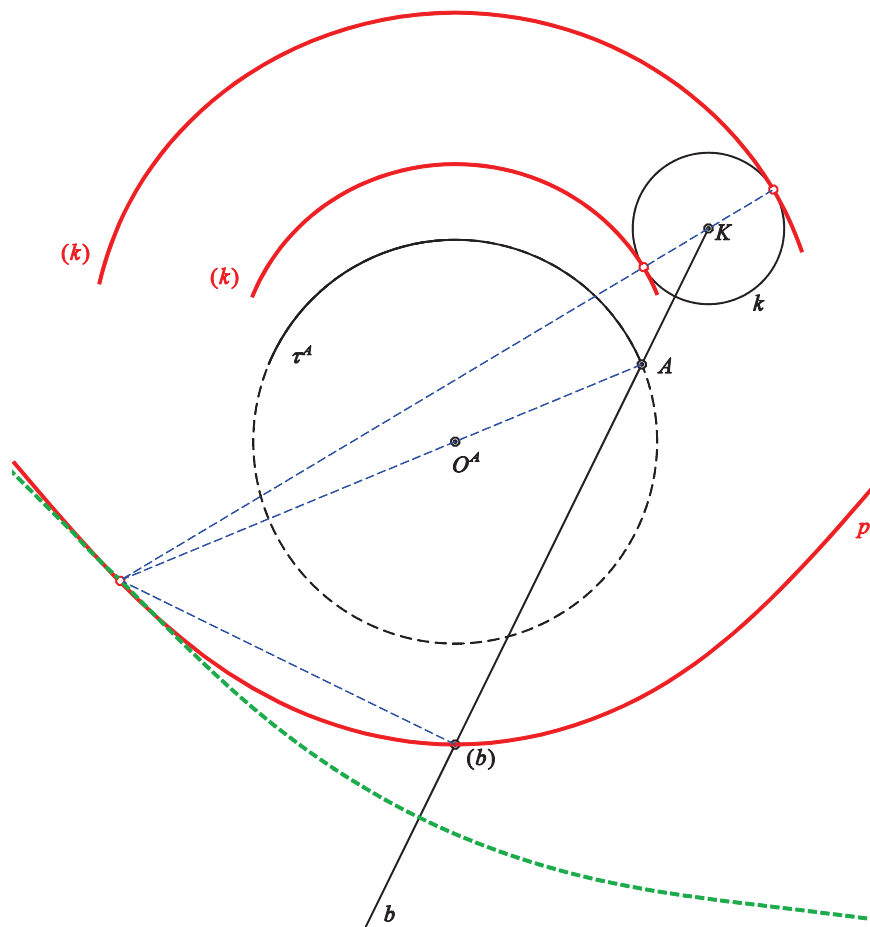


■ **Exercise 3.12.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of straight line  $c$ . Construct point of contact between the line and its envelope  $(c)$  at each position. Sketch the part of envelope  $(c)$  determined by all positions of straight line  $c$  and the corresponding points of contact.

Consider the continuous parts of trajectory  $\tau^A$  only.



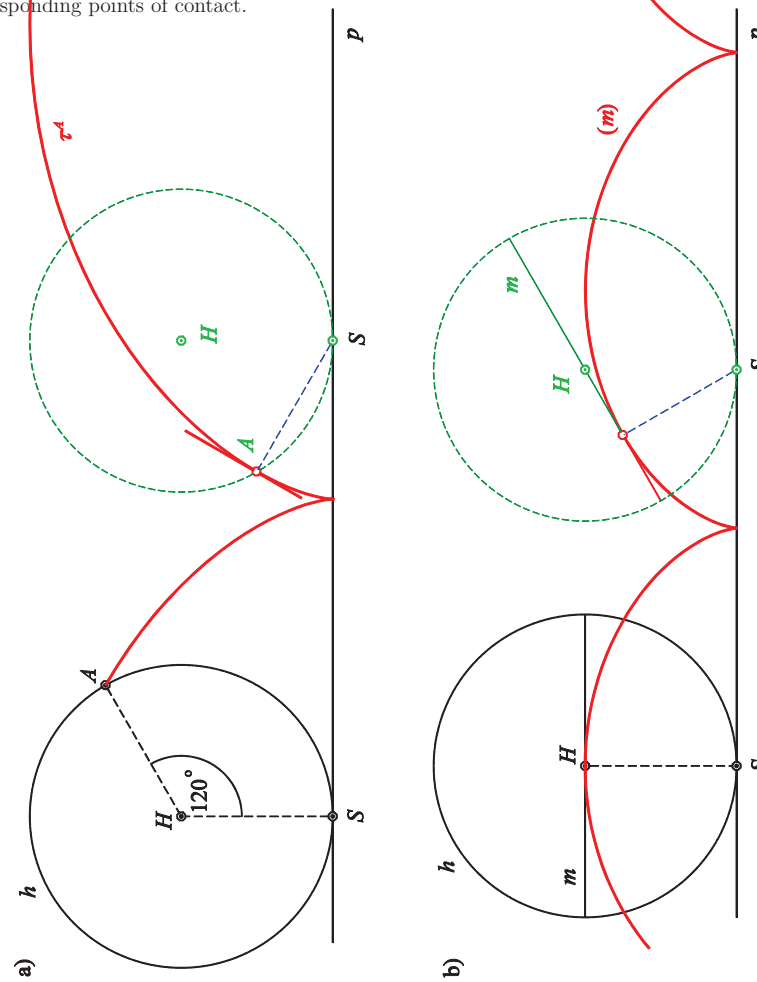
- **Exercise 3.13.** Motion is given by trajectory  $\tau^A$  of point  $A$  and envelope  $(b)$  of straight line  $b$ . Construct at least three new positions of circle  $k$ . Construct point of contact between circle  $k$  and its envelope  $(k)$  at each position. Sketch the part of envelope  $(k)$  determined by all positions of circle  $k$  and the corresponding points of contact. Construct the corresponding part of fixed centrode  $p$ . Construct the corresponding part of moving centrode  $h^0$  at the given instant. Consider the continuous part of trajectory  $\tau^A$  only.



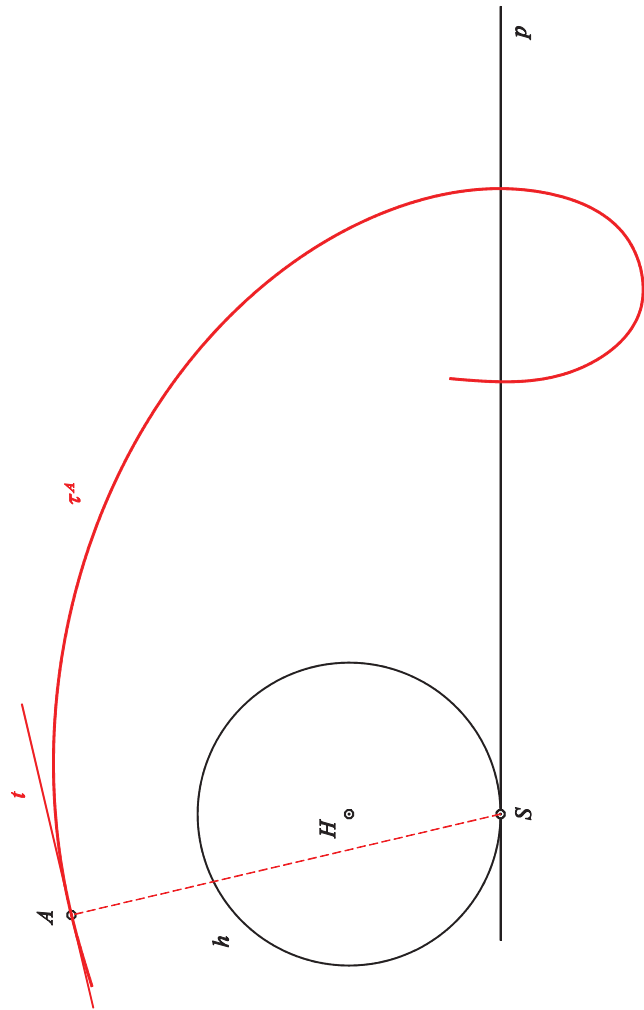
### 3.2 Cyclic motion

- **Exercise 3.14.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ .

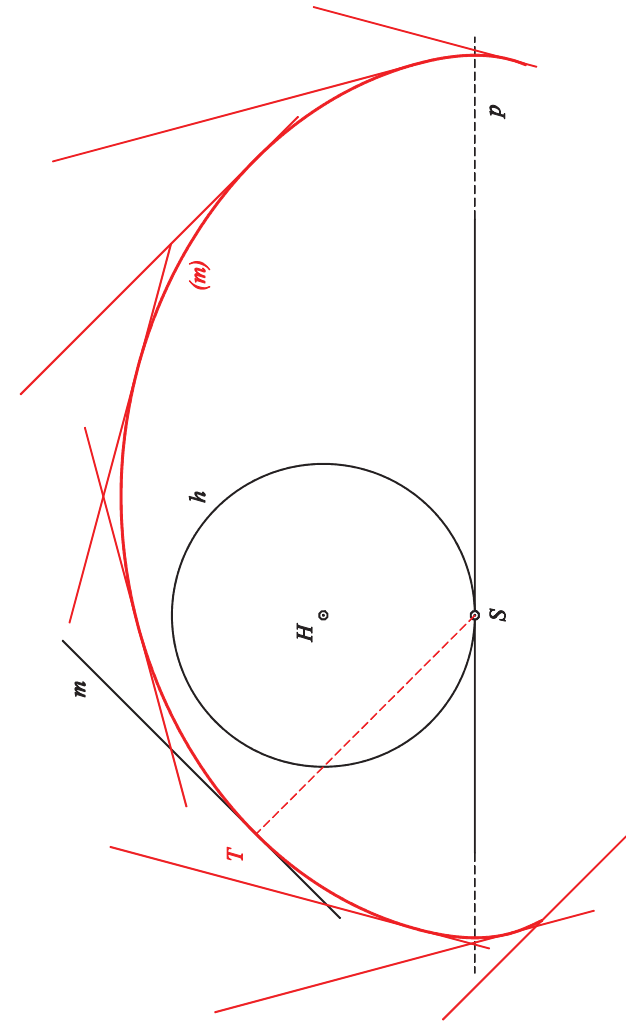
  - Construct a sufficient number of new positions of point  $A$ . Construct tangent lines to the trajectory  $\tau^A$  at each position. Sketch the part of trajectory  $\tau^A$  determined by all positions of point  $A$  and the corresponding tangent lines.
  - Construct point of contact between line  $m$  and its envelope  $(m)$  at each position. Sketch the part of envelope  $(m)$  determined by all positions of straight line  $m$  and the corresponding points of contact.



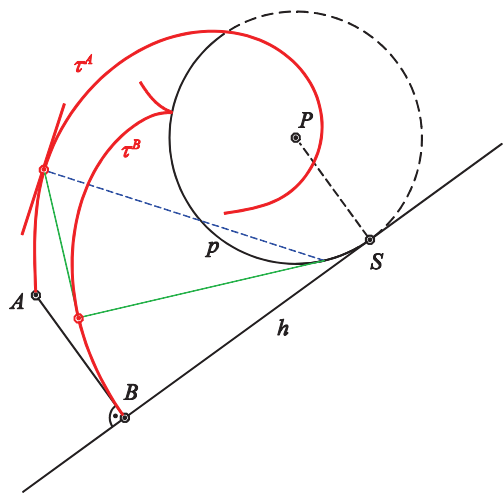
- **Exercise 3.15.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $A$ . Construct tangent lines to the trajectory  $\tau^A$  at each position. Sketch the part of trajectory  $\tau^A$  determined by all positions of point  $A$  and the corresponding tangent lines.



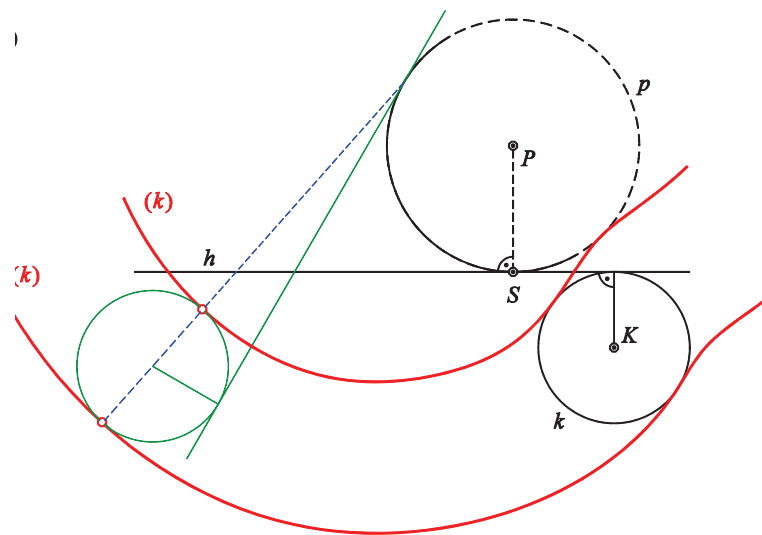
- **Exercise 3.16.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of straight line  $m$ . Construct point of contact between line  $m$  and its envelope  $(m)$  at each position. Sketch the part of envelope  $(m)$  determined by all positions of straight line  $m$  and the corresponding points of contact. Consider the continuous part of fixed centrode  $p$  only.



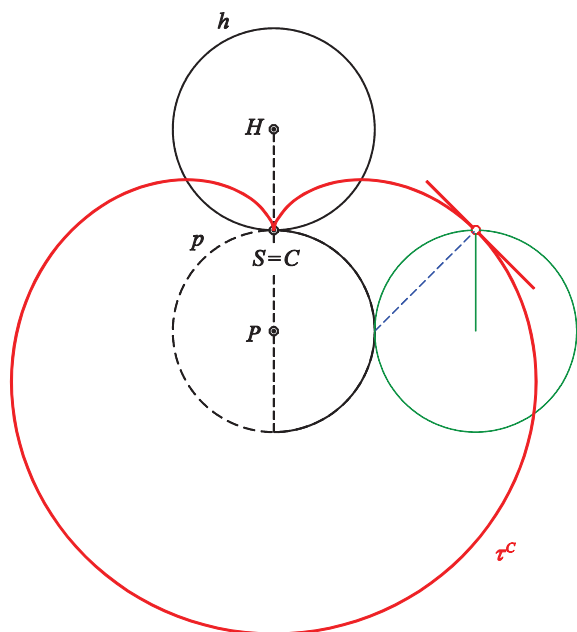
- **Exercise 3.17.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of points  $A$  and  $B$ . Construct tangent lines to the trajectories  $\tau^A$  and  $\tau^B$  at each position. Sketch the part of trajectories  $\tau^A$  and  $\tau^B$  determined by all positions of points  $A$  and  $B$  and the corresponding tangent lines. Consider the continuous part of fixed centrode  $p$  only.



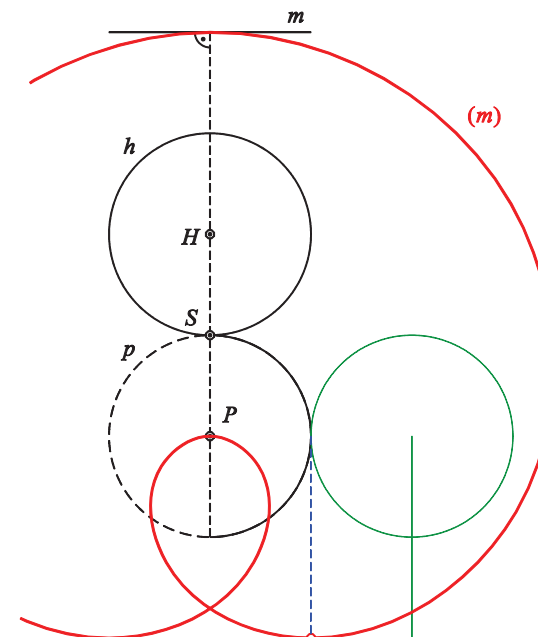
- **Exercise 3.18.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of circle  $k$ . Construct point of contact between the circle and its envelope ( $k$ ) at each position. Sketch the part of envelope ( $k$ ) determined by all positions of circle  $k$  and the corresponding points of contact. Consider the continuous part of fixed centrode  $p$  only.



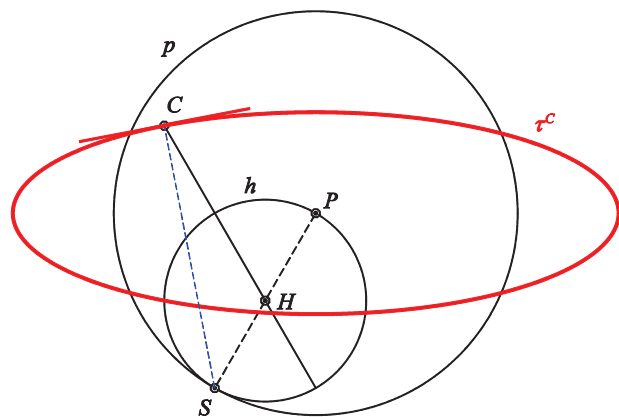
- **Exercise 3.19.** Motion is given by fixed centre  $p$  and moving centre  $h$ . Construct a sufficient number of new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of points  $C$  and the corresponding tangent lines.  
 Consider the continuous part of fixed centre  $p$  only.



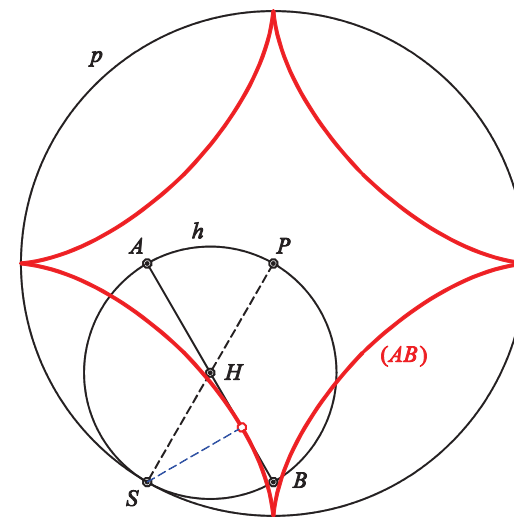
- **Exercise 3.20.** Motion is given by fixed centre  $p$  and moving centre  $h$ . Construct a sufficient number of new positions of straight line  $m$ . Construct point of contact between the line and its envelope  $(m)$  at each position. Sketch the part of envelope  $(m)$  determined by all positions of straight line  $m$  and the corresponding points of contact.  
 Consider the continuous part of fixed centre  $p$  only.



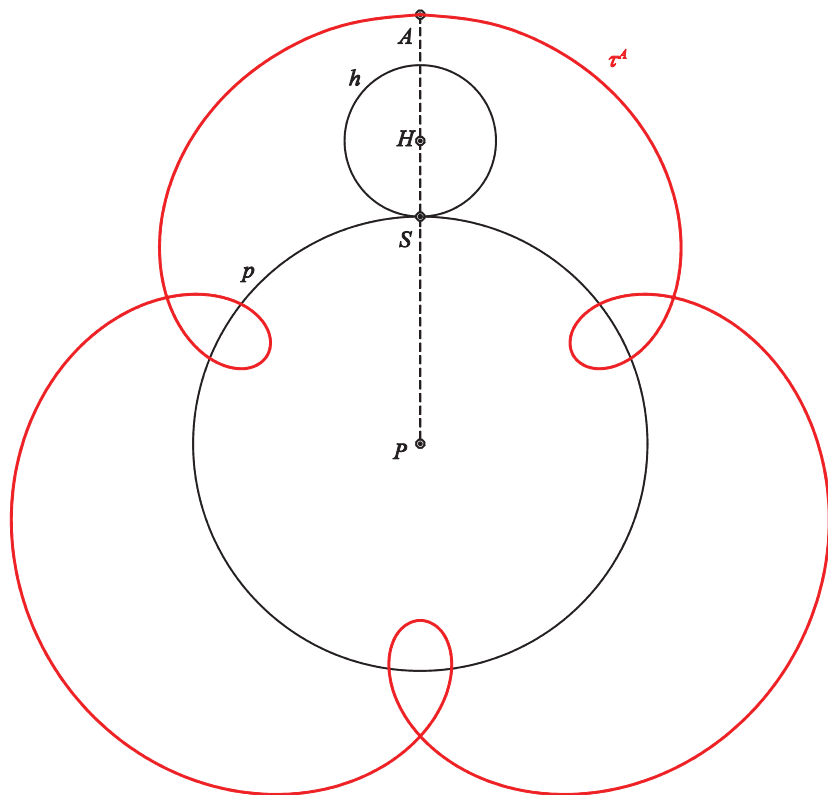
- **Exercise 3.21.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of points  $C$  and the corresponding tangent lines.



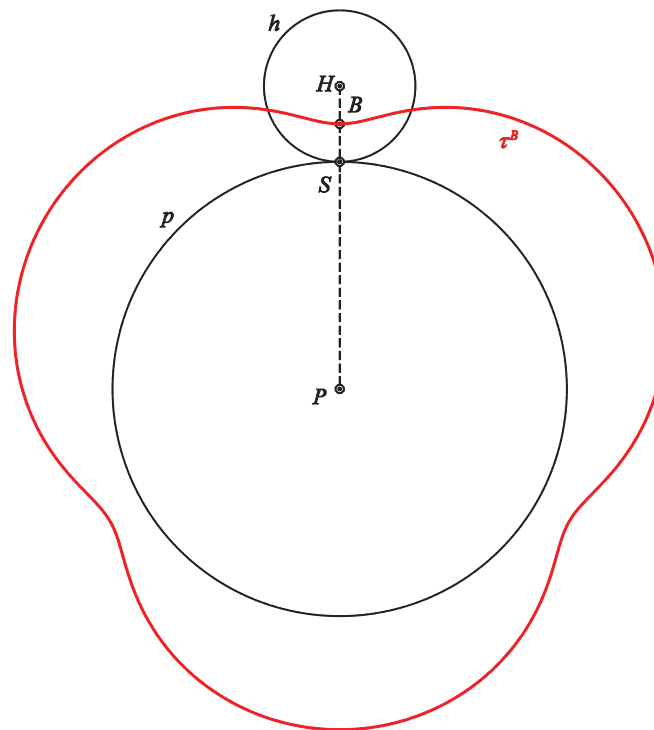
- **Exercise 3.22.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of straight line  $AB$ . Construct point of contact between the line and its envelope  $(AB)$  at each position. Sketch the part of envelope  $(AB)$  determined by all positions of straight line  $AB$  and the corresponding points of contact.



- **Exercise 3.23.** Motion is given by fixed centre  $p$  and moving centre  $h$ . Construct a sufficient number of new positions of point  $A$ . Construct tangent lines to the trajectory  $\tau^A$  at each position. Sketch the part of trajectory  $\tau^A$  determined by all positions of point  $A$  and the corresponding tangent lines. Consider the continuous part of fixed centre  $p$  only.

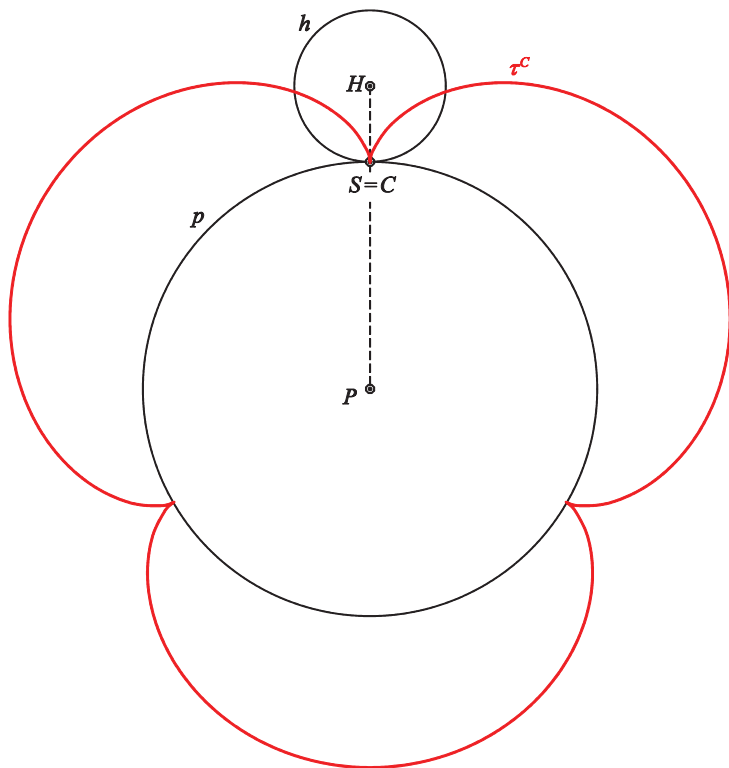


- **Exercise 3.24.** Motion is given by fixed centre  $p$  and moving centre  $h$ . Construct a sufficient number of new positions of point  $B$ . Construct tangent lines to the trajectory  $\tau^B$  at each position. Sketch the part of trajectory  $\tau^B$  determined by all positions of point  $B$  and the corresponding tangent lines.

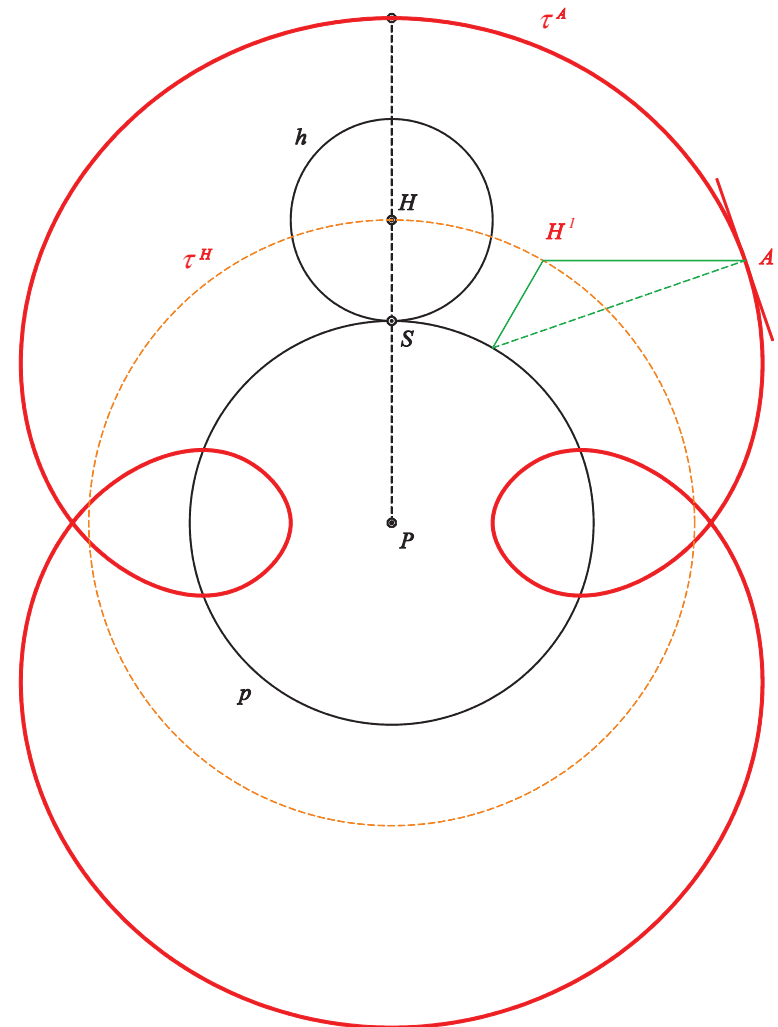




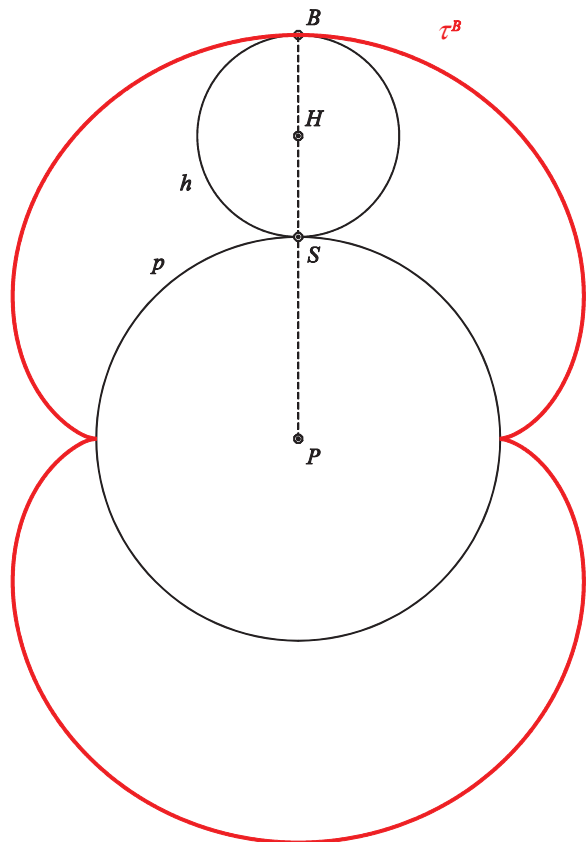
■ **Exercise 3.25.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines.



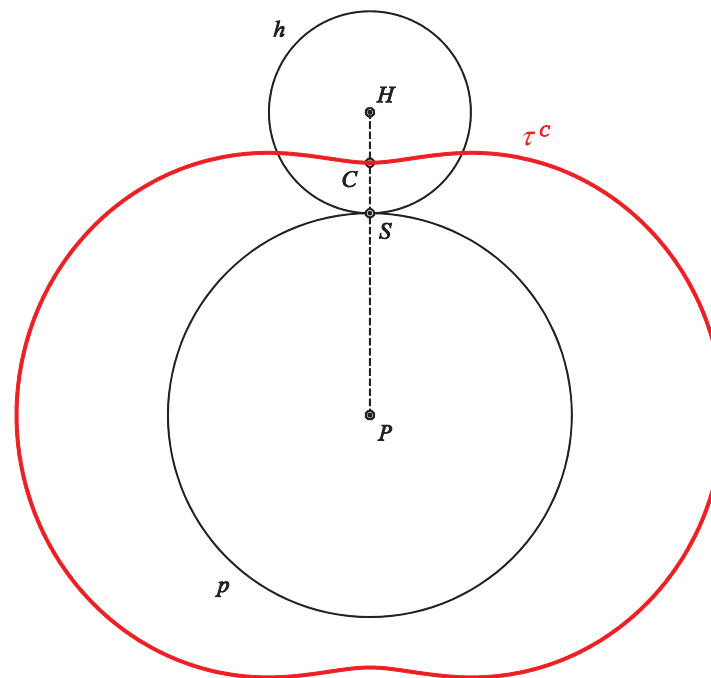
■ **Exercise 3.26.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $A$ . Construct tangent lines to the trajectory  $\tau^A$  at each position. Sketch the part of trajectory  $\tau^A$  determined by all positions of point  $A$  and the corresponding tangent lines.



- **Exercise 3.27.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $B$ . Construct tangent lines to the trajectory  $\tau^B$  at each position. Sketch the part of trajectory  $\tau^B$  determined by all positions of point  $B$  and the corresponding tangent lines.

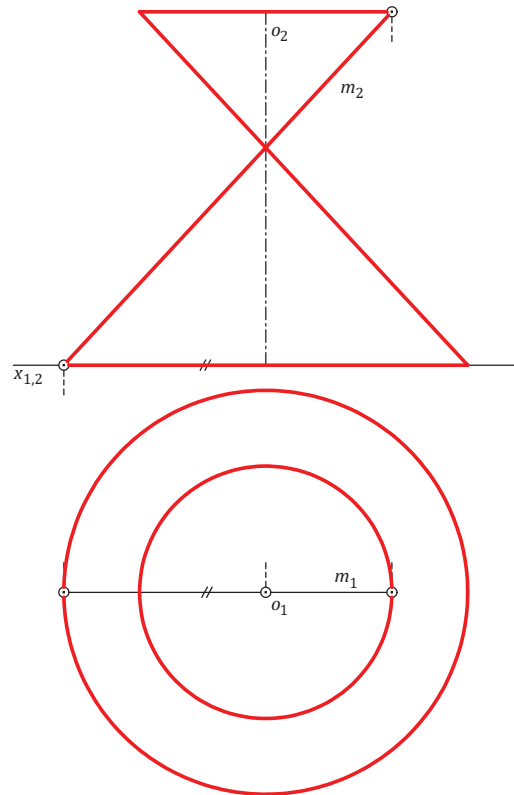


- **Exercise 3.28.** Motion is given by fixed centrode  $p$  and moving centrode  $h$ . Construct a sufficient number of new positions of point  $C$ . Construct tangent lines to the trajectory  $\tau^C$  at each position. Sketch the part of trajectory  $\tau^C$  determined by all positions of point  $C$  and the corresponding tangent lines.

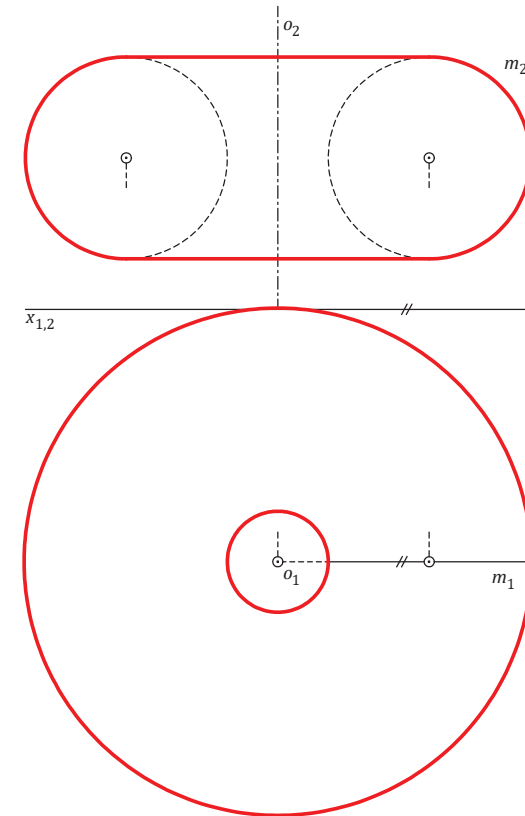


## 4 Surfaces of revolution

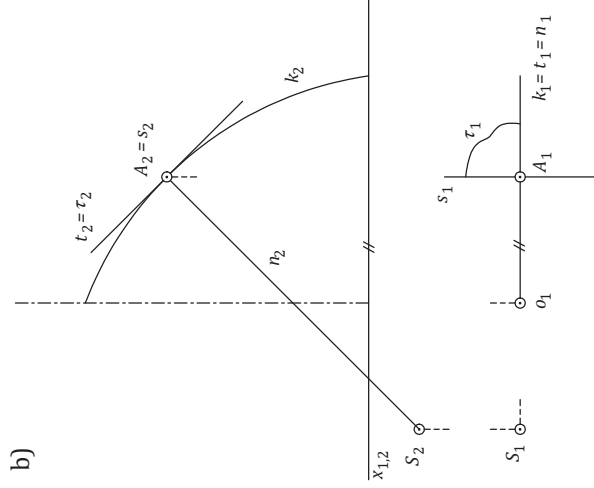
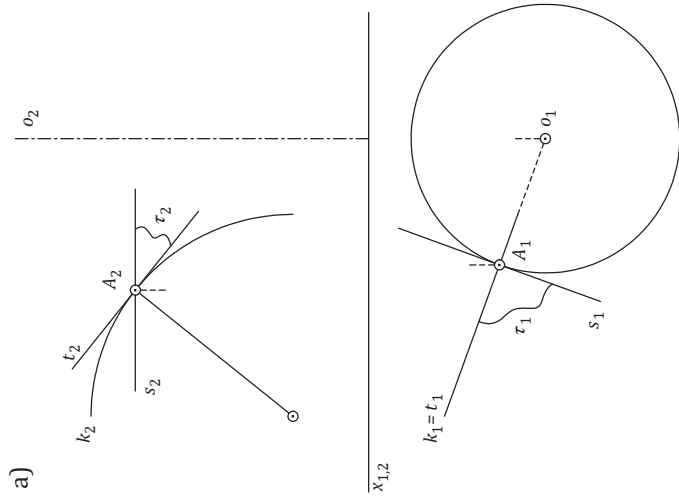
- **Exercise 4.1.** Surface of revolution (axis  $o$ , principal half-meridian  $m$ ) is given. Using Monge projection, construct the top view and the front view of the surface.



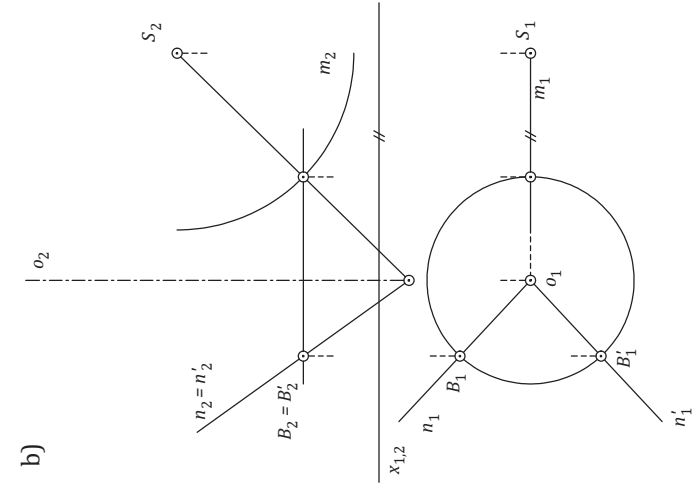
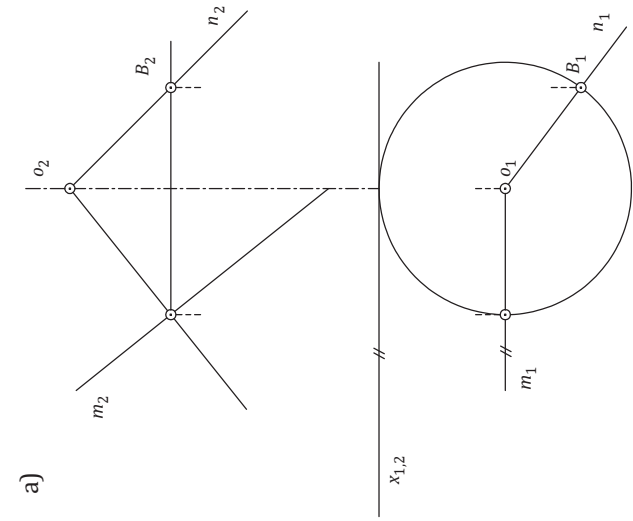
- **Exercise 4.2.** Surface of revolution (axis  $o$ , principal half-meridian  $m$ ) is given. Using Monge projection, construct the top view and the front view of the surface.



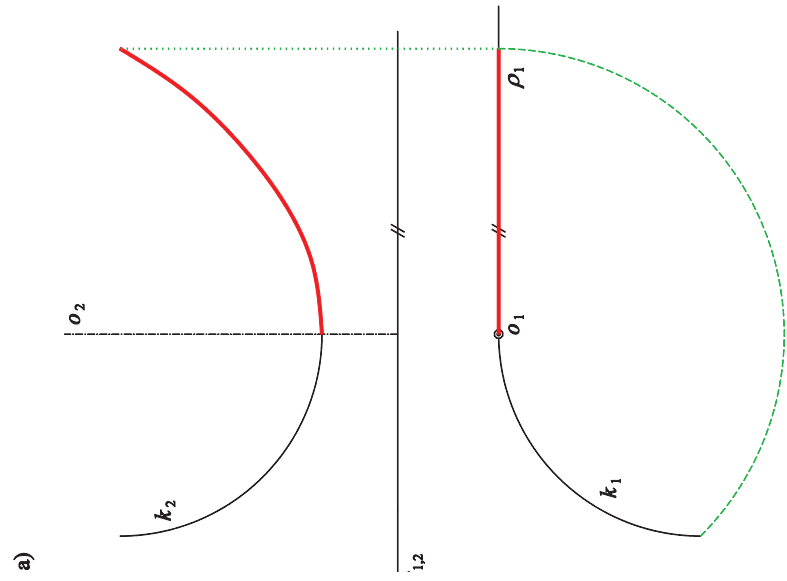
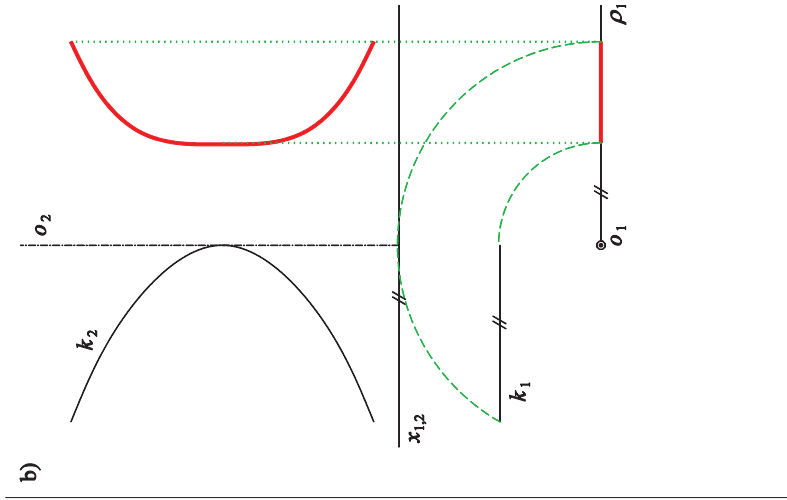
- **Exercise 4.3.** Surface of revolution (axis  $o$ , generating curve  $k$ ) is given. Using Monge projection,
- a) construct tangent plane  $\tau$  at point  $A \in k$ ,
  - b) construct tangent plane  $\tau$  and normal line  $n$  at point  $A \in k$ .



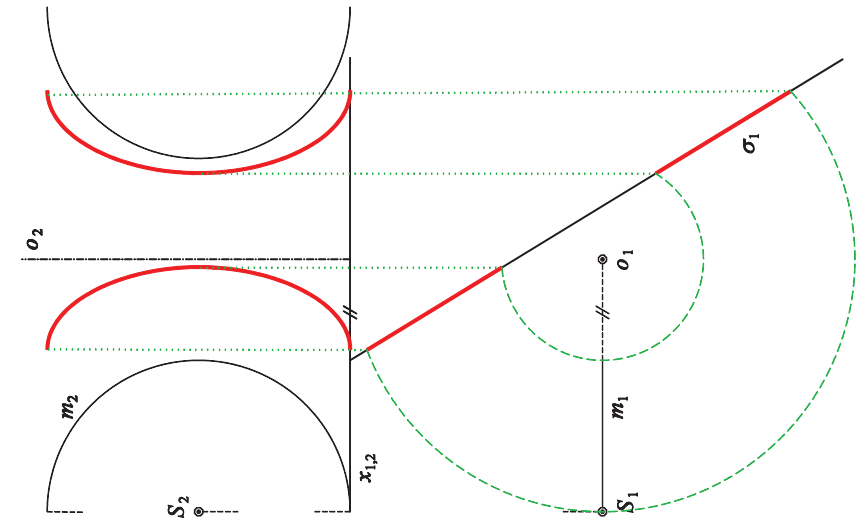
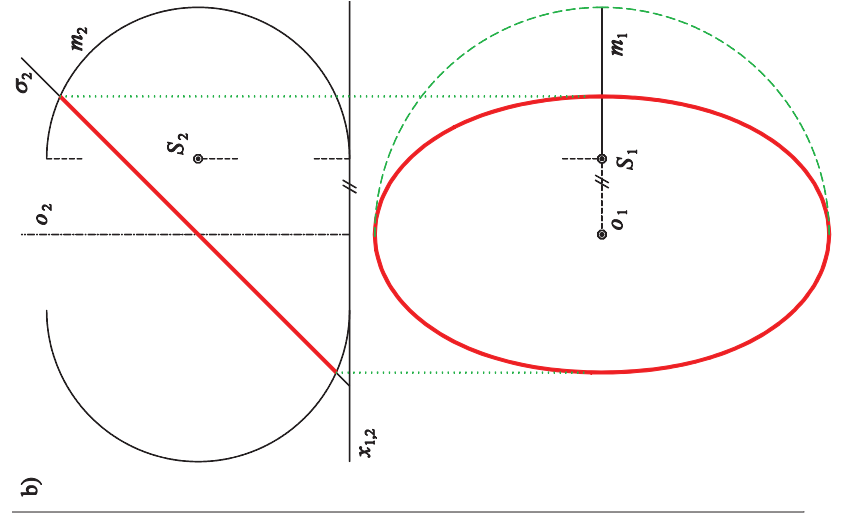
- **Exercise 4.4.** Surface of revolution (axis  $o$ , principal half-meridian  $m$ ) is given. Using Monge projection, construct a missing view of point  $B$ . Construct normal line  $n$  at point  $B$ .



■ **Exercise 4.5.** Surface of revolution (axis  $o$ , generating curve  $k$ ) is given. Using Monge projection, construct its principal half-meridian  $m$  in the given half-plane  $p$ .

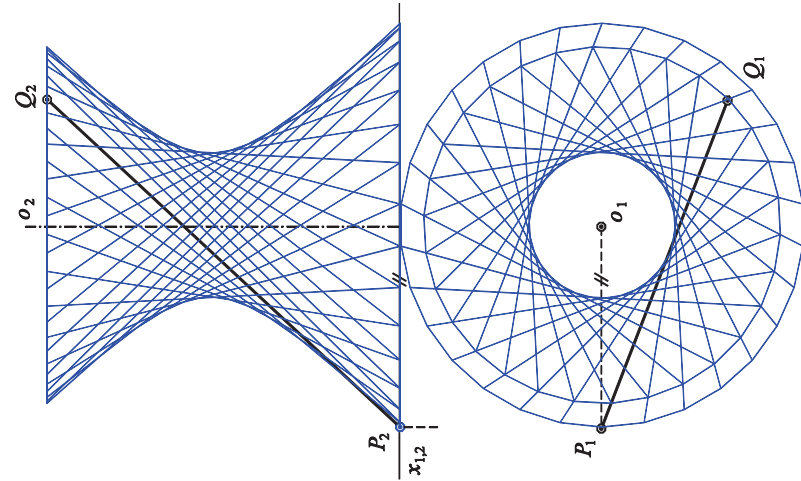


■ **Exercise 4.6.** Surface of revolution (axis  $o$ , principal half-meridian  $m$ ) is given. Using Monge projection, construct intersection curve  $p$  of the surface and the given plane  $\sigma$ . Construct normal line  $n$  at point  $M \in p$ ,  $z_M = 10$ .

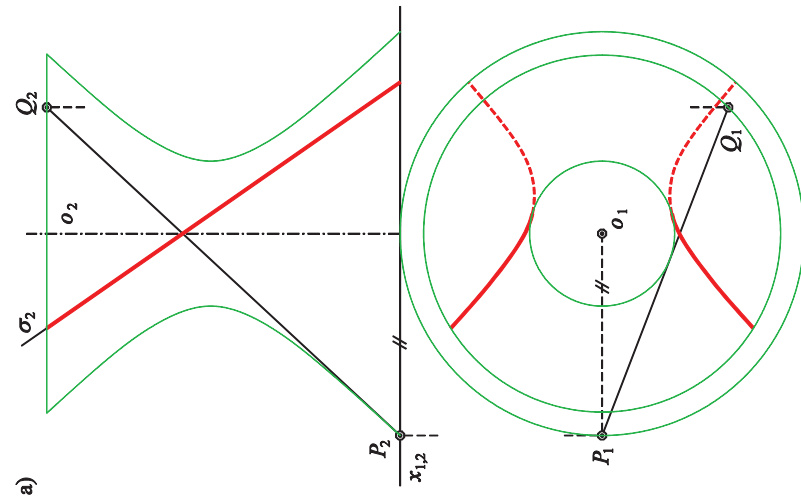


■ **Exercise 4.7.** Surface of revolution (axis  $o$ , generating straight line segment  $PQ$ ) is given. Using Monge projection

- a) construct intersection curve  $p$  of the surface of revolution and the given plane  $\sigma$ ,
- b) construct the top view and the front view of the surface, write the name of the surface and its equation.

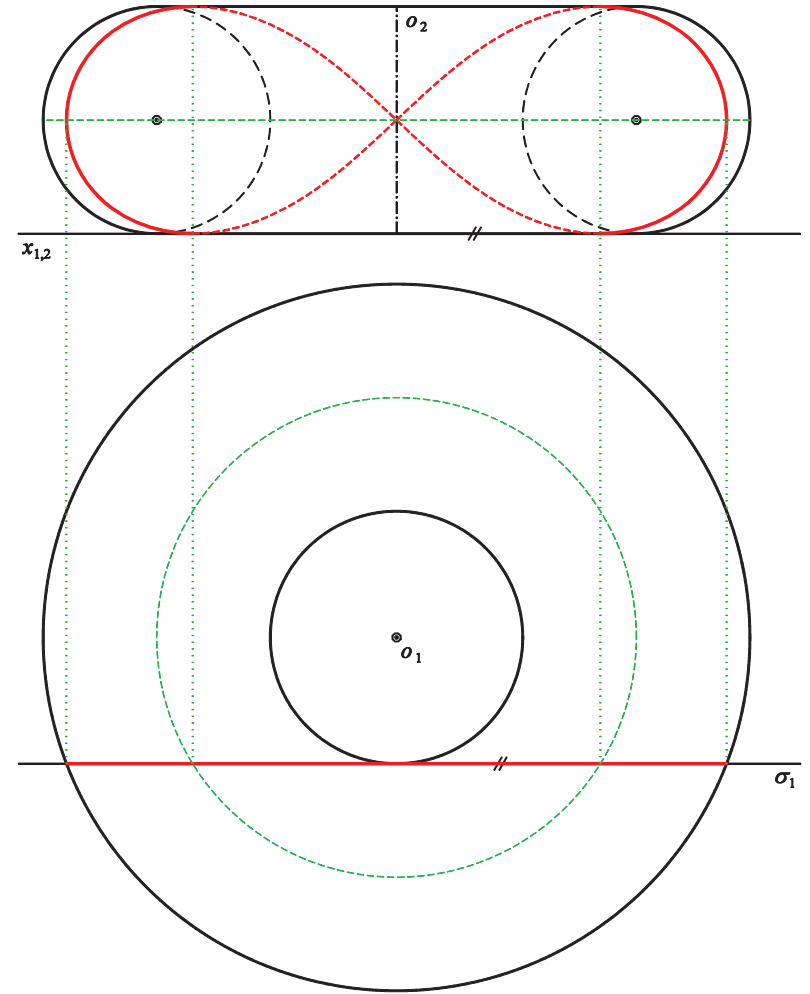


b)

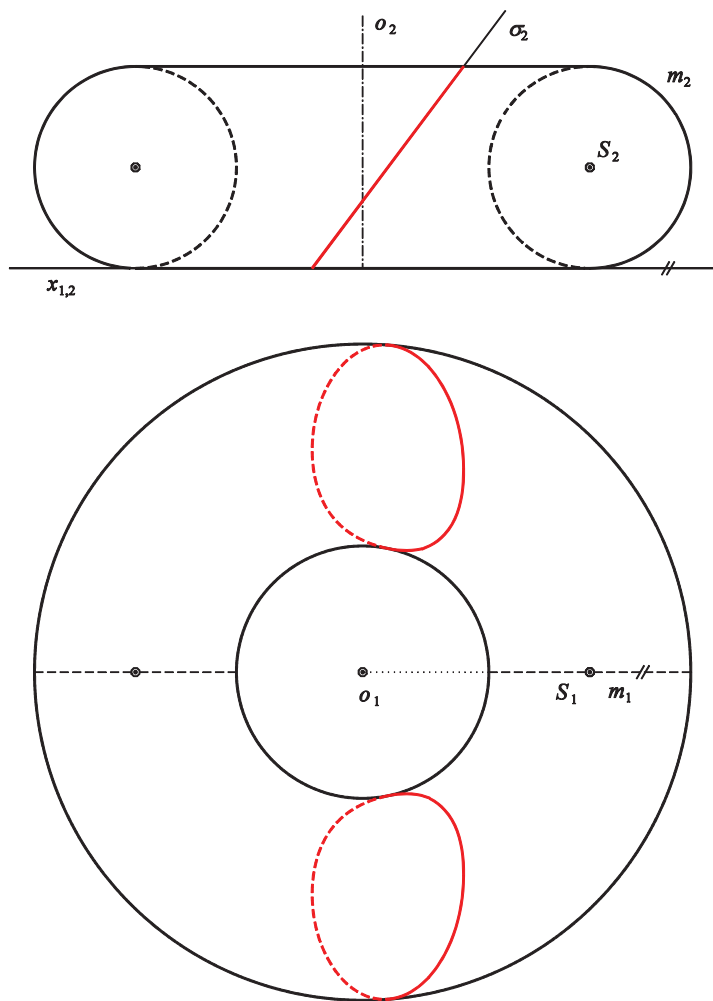


a)

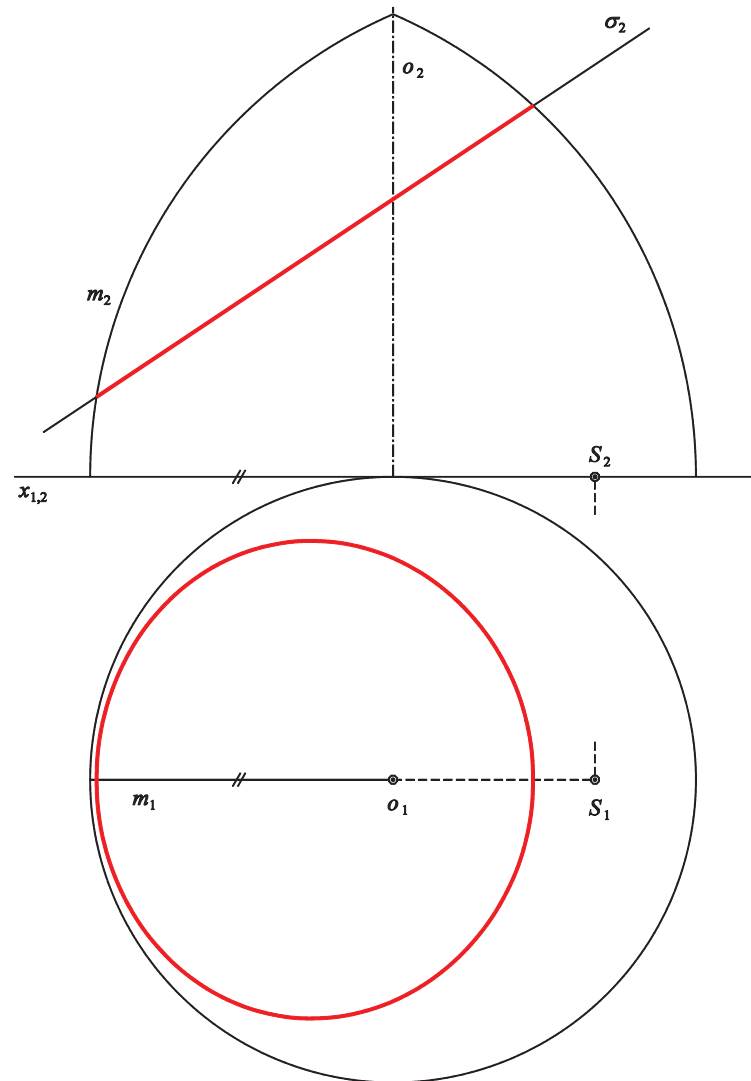
■ **Exercise 4.8.** Using Monge projection, construct intersection curve  $p$  of the torus and the given plane  $\sigma$ . Indicate the visibility. Construct normal line  $n$  at point of intersection  $M \in p, z_M = 35$ .



- **Exercise 4.9.** Using Monge projection, construct intersection curve  $p$  of the surface of revolution (axis  $o$ , principal half-meridian  $m$  - circle with centre at point  $S$ ) and the given plane  $\sigma$ . Indicate the visibility. Write the name of the surface of revolution.

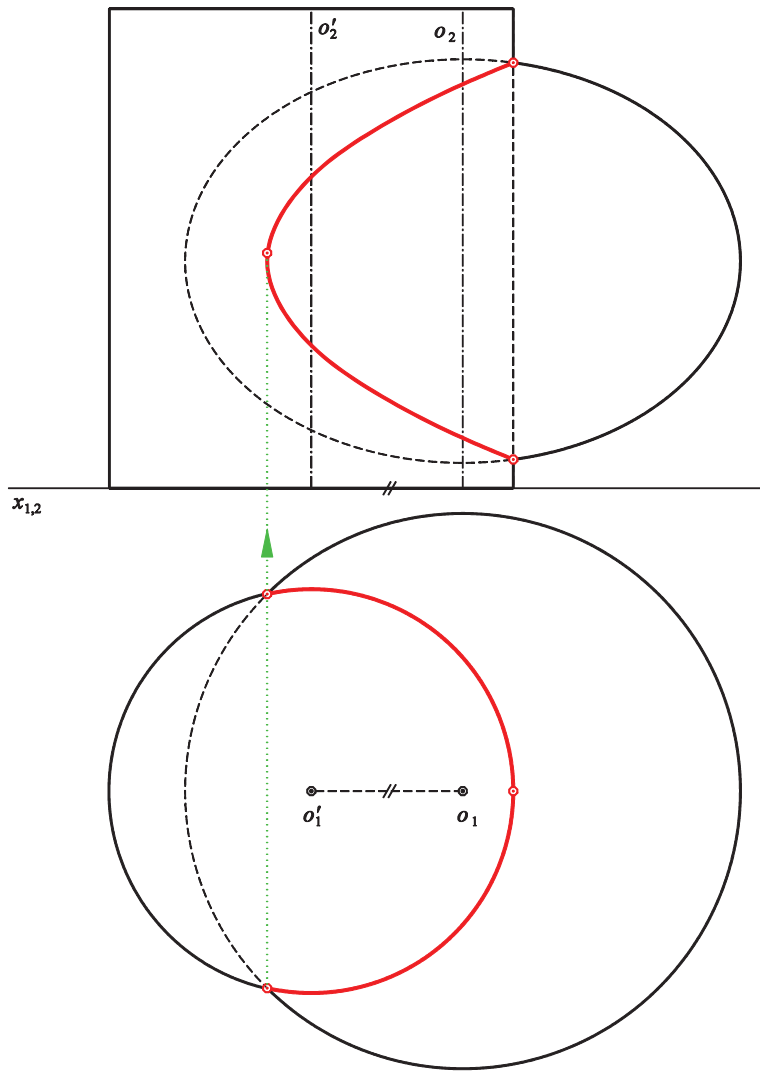


- **Exercise 4.10.** Surface of revolution (axis  $o$ , principal half-meridian  $m$  - circular arc with centre at point  $S$ ) is given. Using Monge projection, construct the top view and the front view of the surface. Construct intersection curve  $p$  of the surface of revolution and the given plane  $\sigma$ . Construct normal line  $n$  at point of intersection  $M \in p, z_M = 40$ .

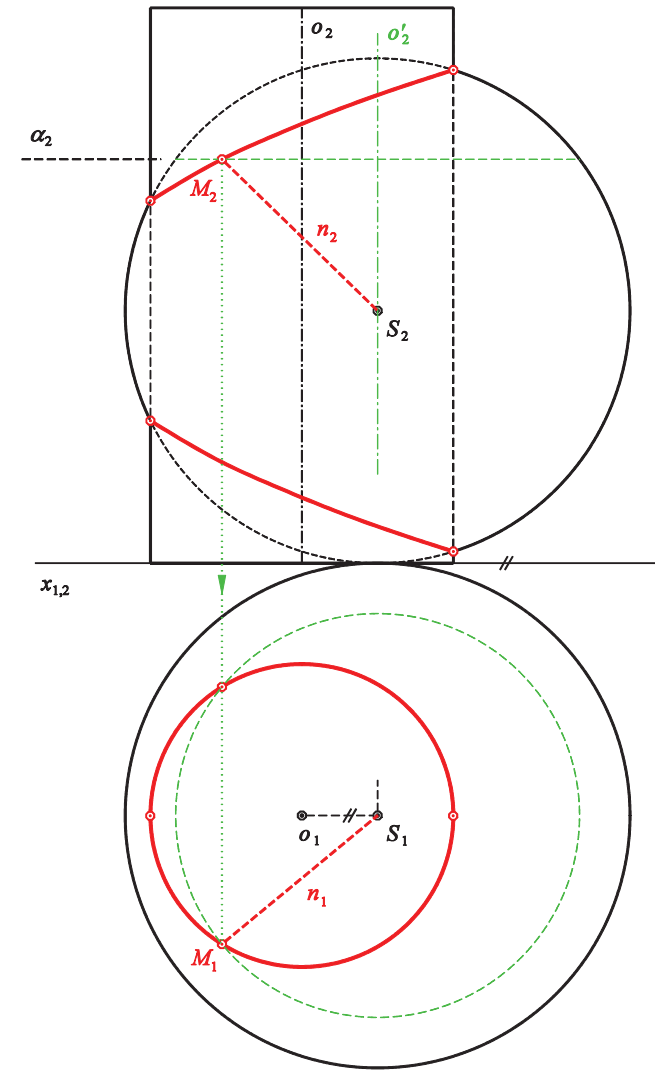


### 4.1 Intersection of surfaces of revolution

■ **Exercise 4.11.** Ellipsoid of revolution (axis  $o$ ) and cylinder of revolution (axis  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces.

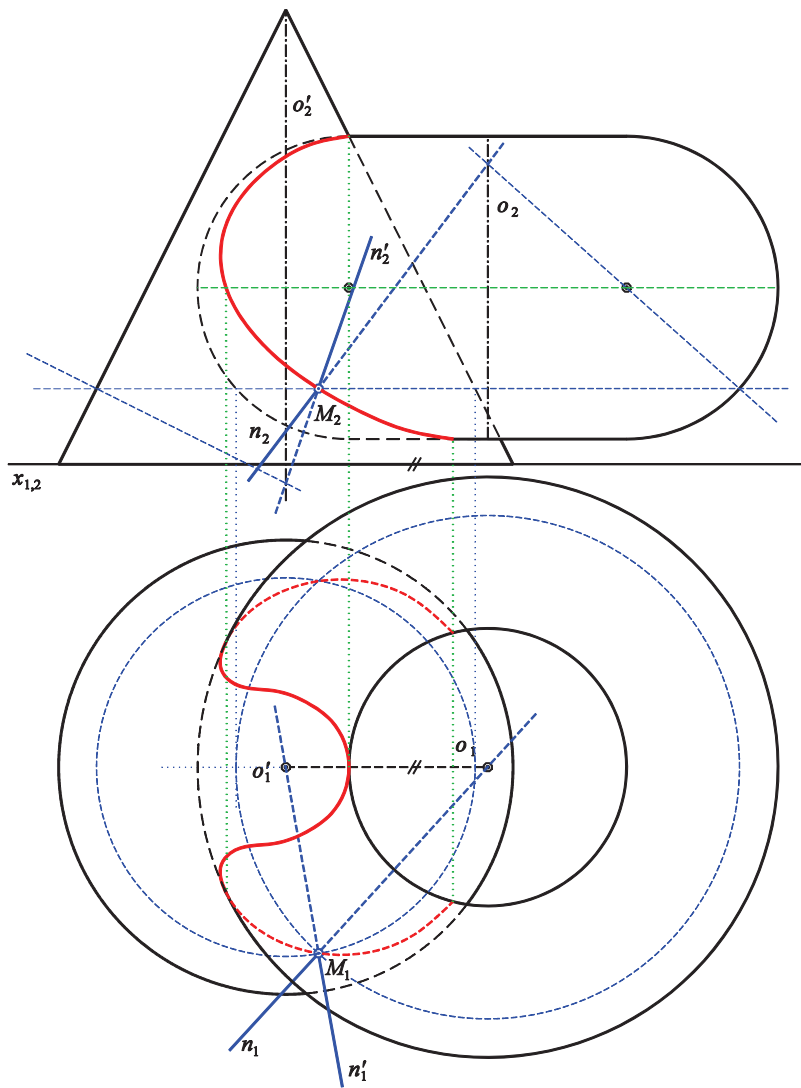


■ **Exercise 4.12.** Cylinder of revolution (axis  $o$ ) and sphere (centre  $S$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces. Construct normal lines of both surfaces at point  $M \in q$ ,  $M \in \alpha$ .

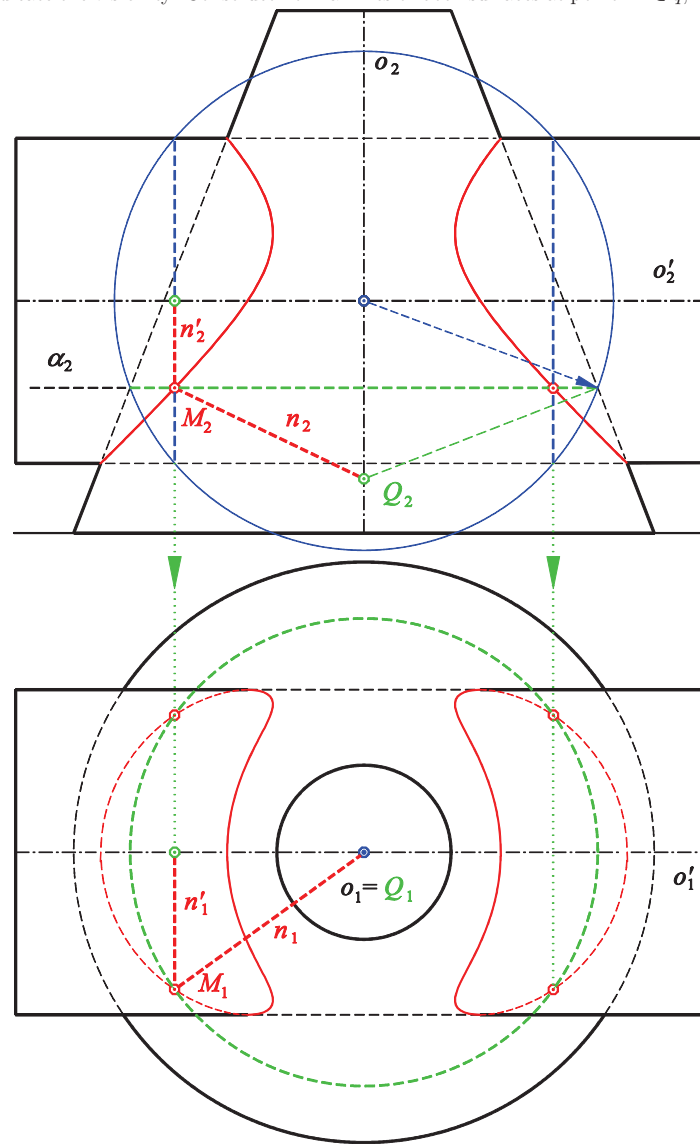




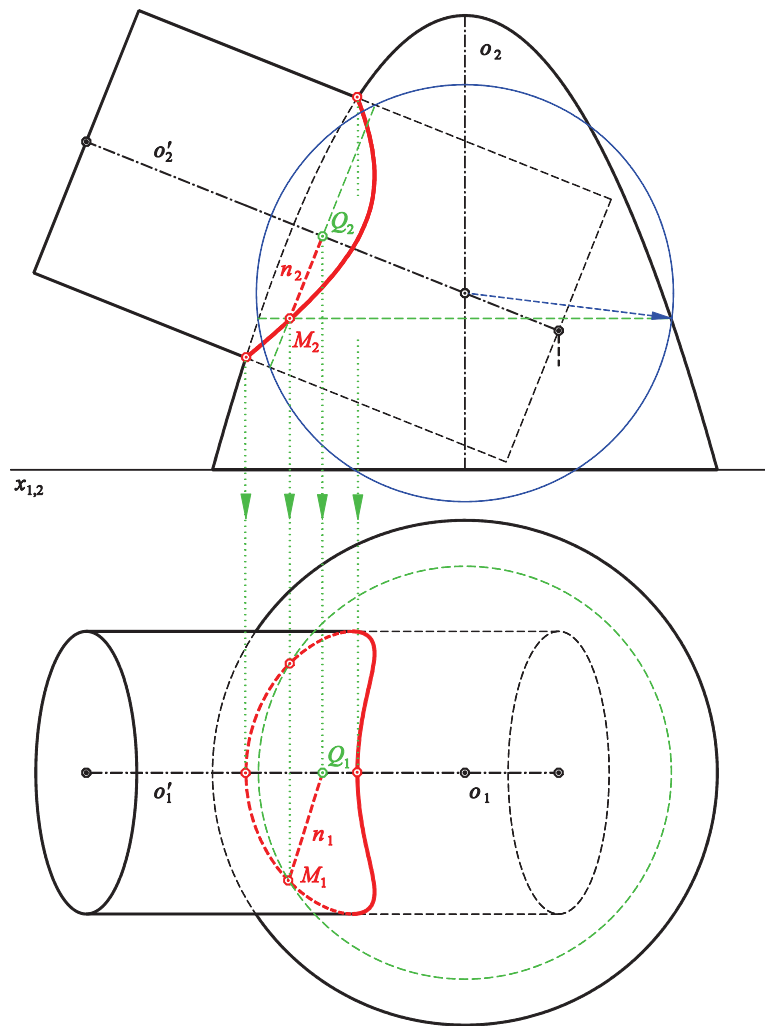
■ **Exercise 4.13.** Surface of revolution (axis  $o$ ) and cone of revolution (axis  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point  $M \in q, z_M = 15$ .



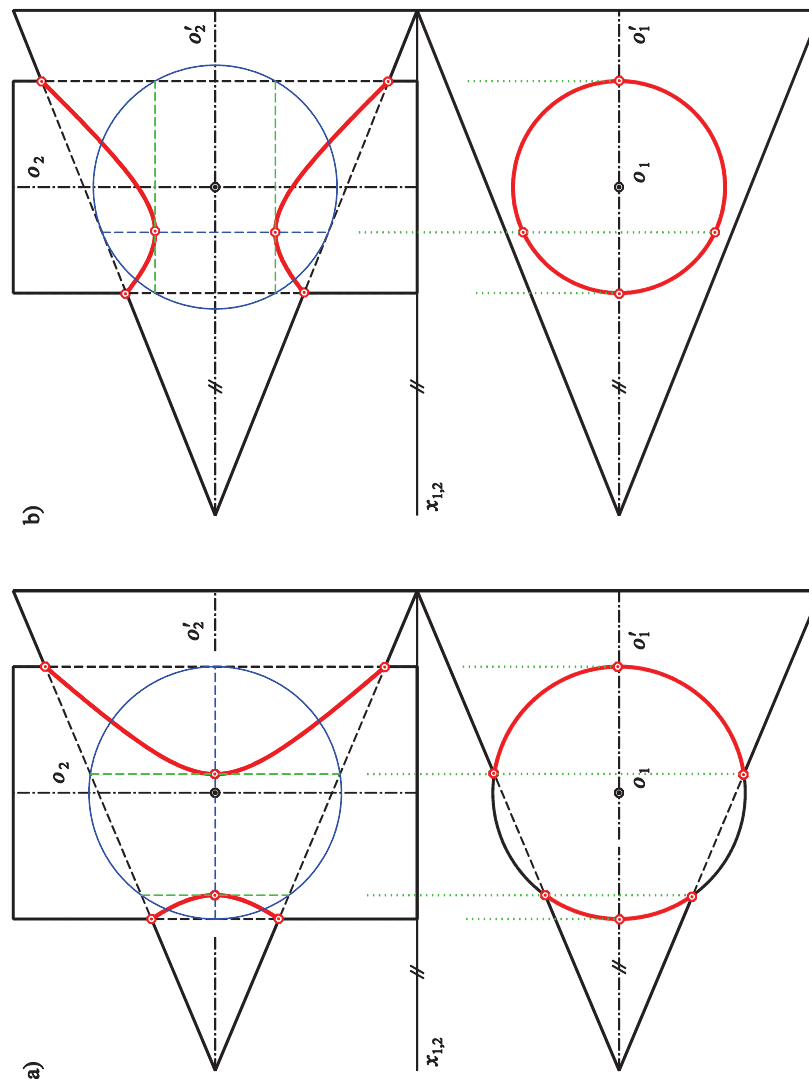
■ **Exercise 4.14.** Truncated cone of revolution (axis  $o$ ) and cylinder of revolution (axis  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point  $M \in q, M \in \alpha$ .



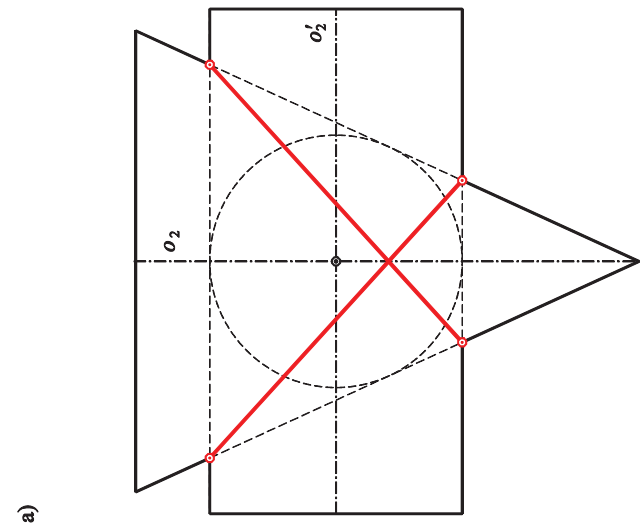
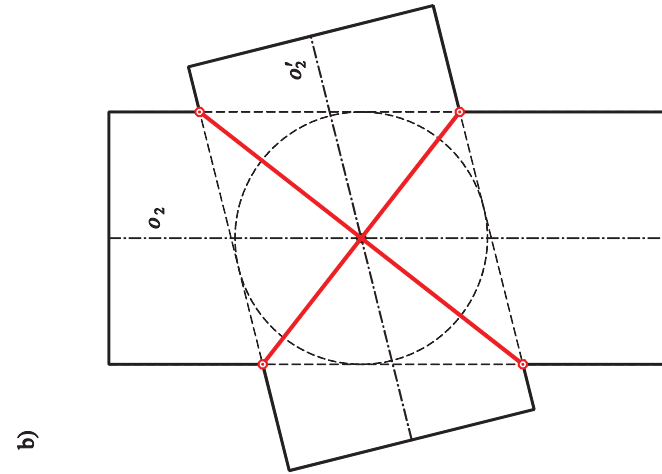
■ **Exercise 4.15.** Paraboloid of revolution (axis  $o$ ) and cylinder of revolution (axis  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point  $M \in q$ ,  $z_M = 30$  construct normal lines of both surfaces.



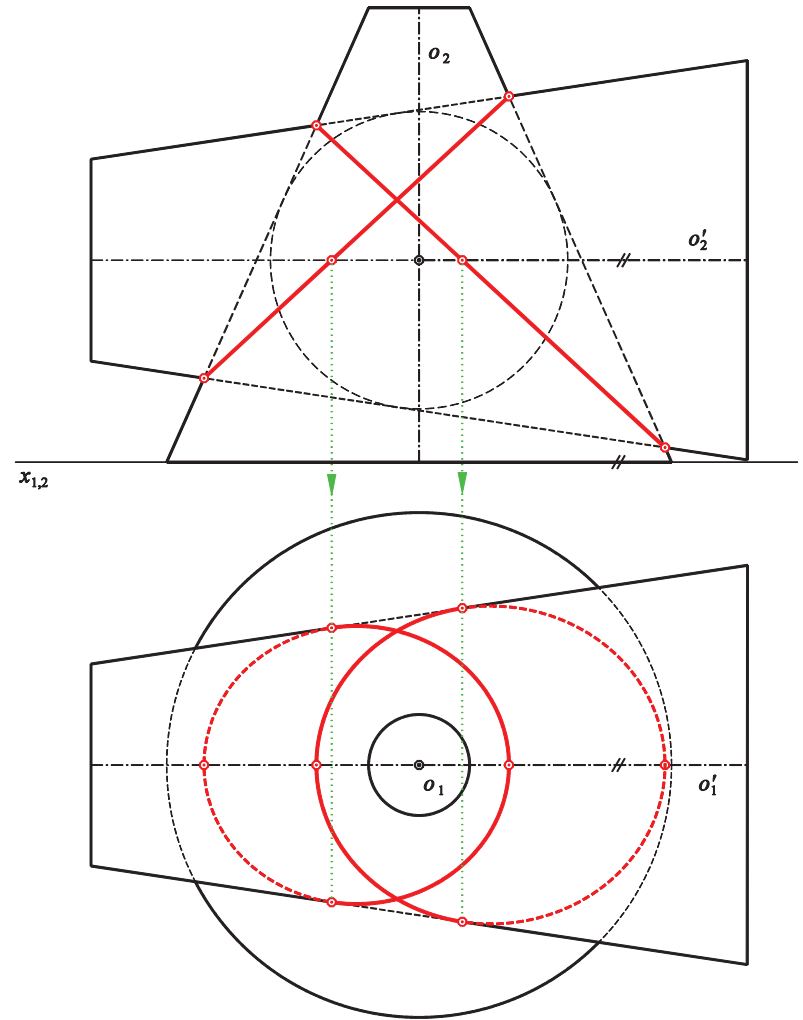
■ **Exercise 4.16.** Cylinder of revolution (axis  $o$ ) and cone of revolution (axis  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these two surfaces. Indicate the visibility.



- **Exercise 4.17.** Construct front view of intersection curve  $q$  of two surfaces of revolution.
  - a) Axial section of cone of revolution (axis  $o$ ) and axial section of cylinder of revolution (axis  $o'$ ) are given.
  - b) Axial sections of two cylinders of revolution (axes  $o$  and  $o'$ ) are given.



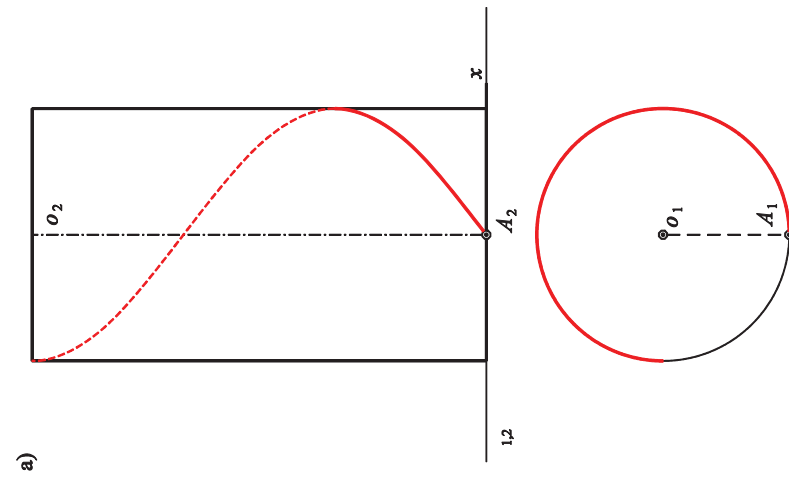
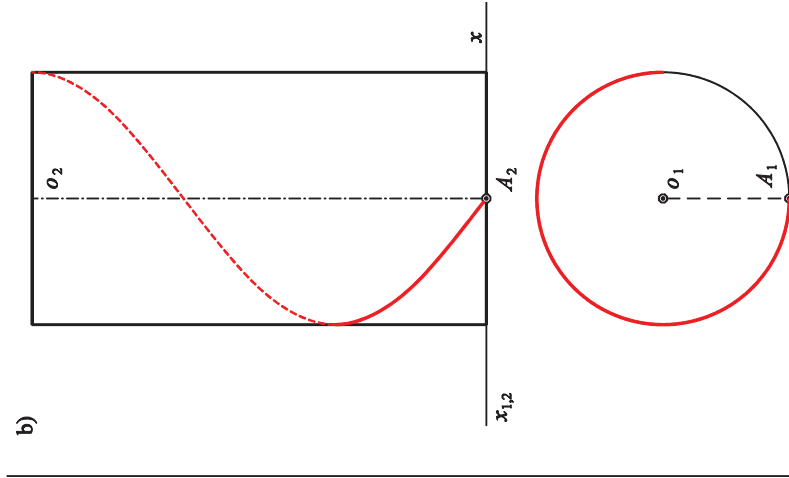
- **Exercise 4.18.** Two truncated cones of revolution (axes  $o$  and  $o'$ ) are given. Using Monge projection, construct intersection curve  $q$  of these surfaces. Indicate the visibility.



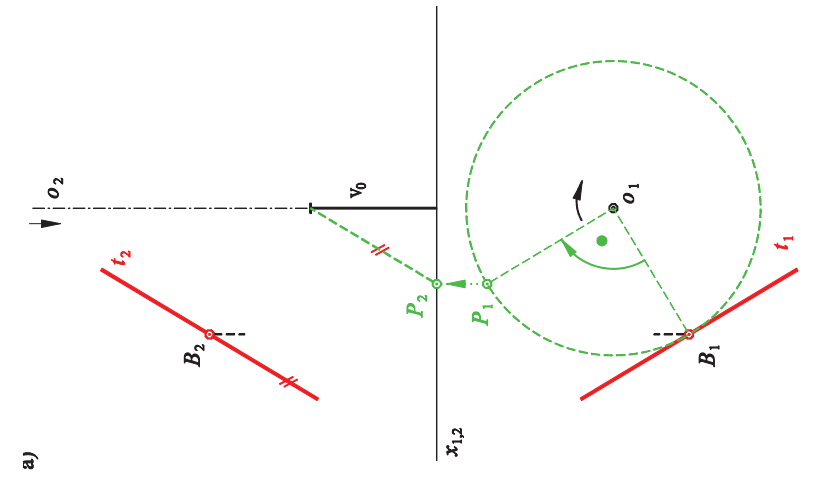
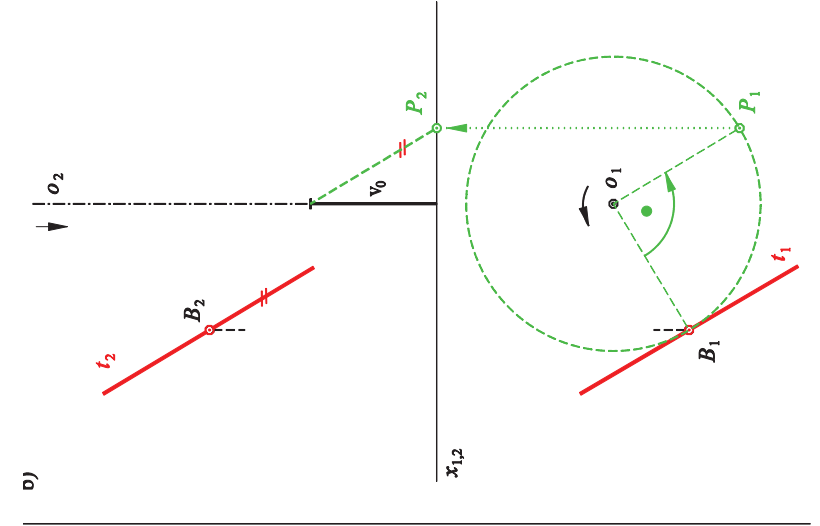
# 5 Helicoidal surfaces

## 5.1 Helix

■ **Exercise 5.1.** Considering the given cylinder of revolution (axis  $o$ ) draw a) right-handed, b) left-handed helix generated by screw motion of point  $A$  with lead  $v = 120$  mm. Use Monge projection.

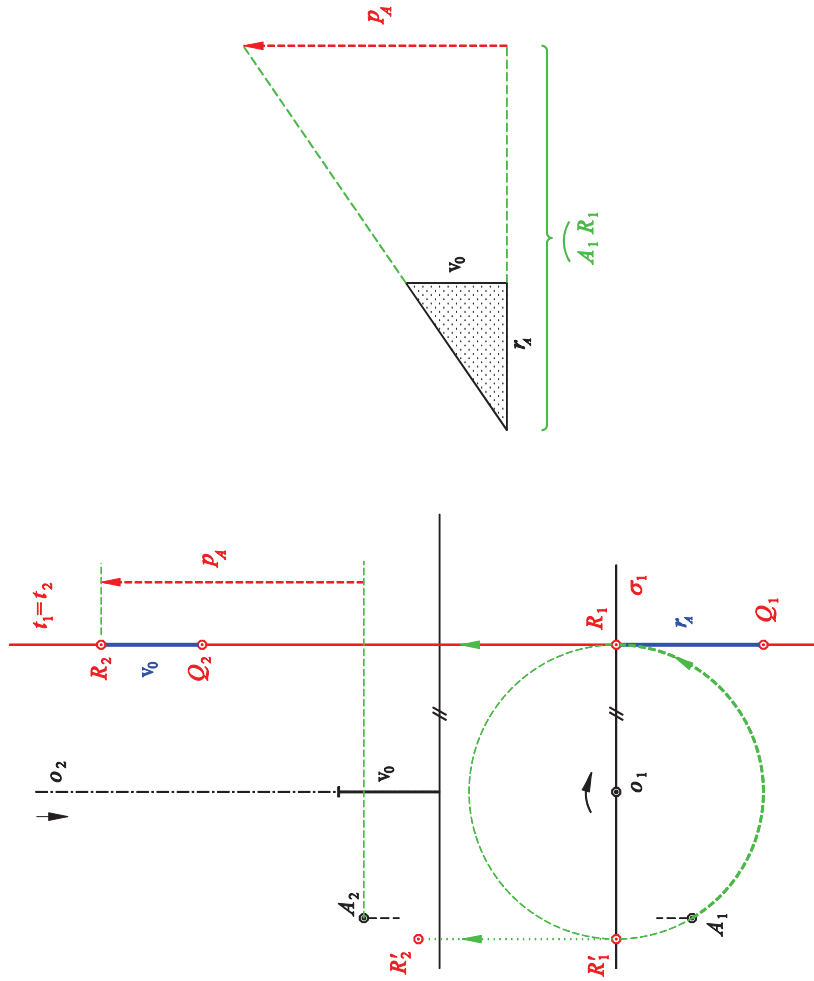


■ **Exercise 5.2.** Helix ( $B, o, v_0$ , a) right-handed, b) left-handed) is given. Using Monge projection, construct tangent line to the helix at its generating point  $B$ .

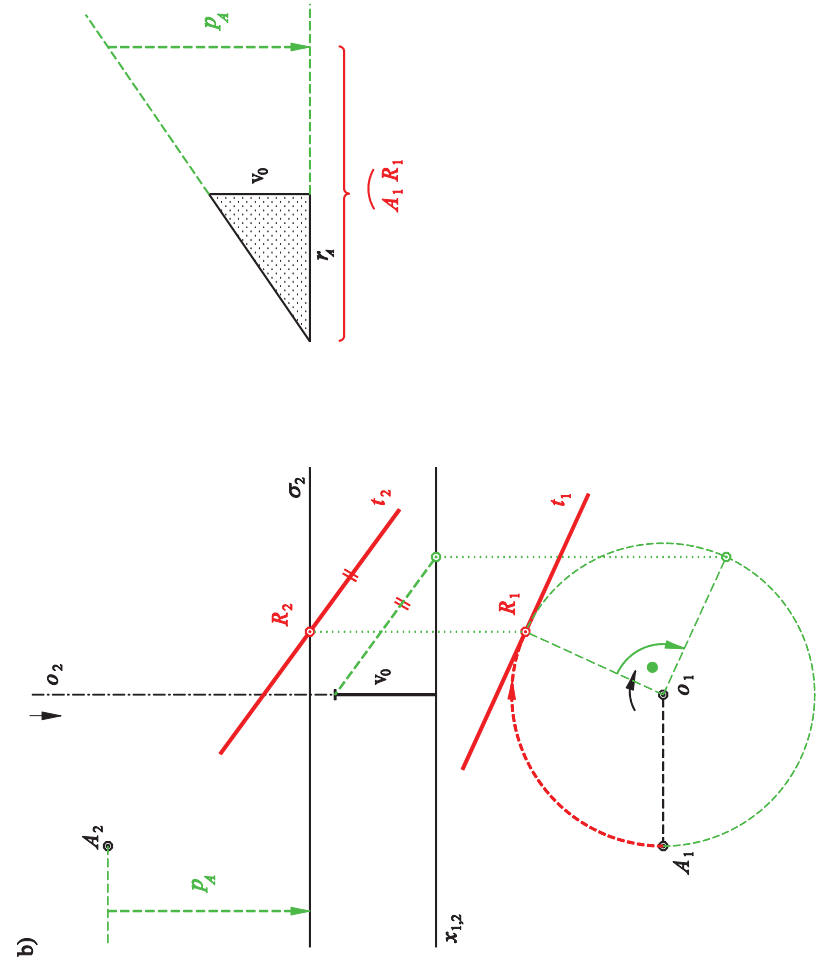


■ **Exercise 5.3.** Helix  $(A, o, v_0, \text{right-handed})$  is given. Using Monge projection, construct intersection  $R$  of the helix and the given plane  $\sigma$ . Construct tangent line to the helix at point  $R$ .

a)

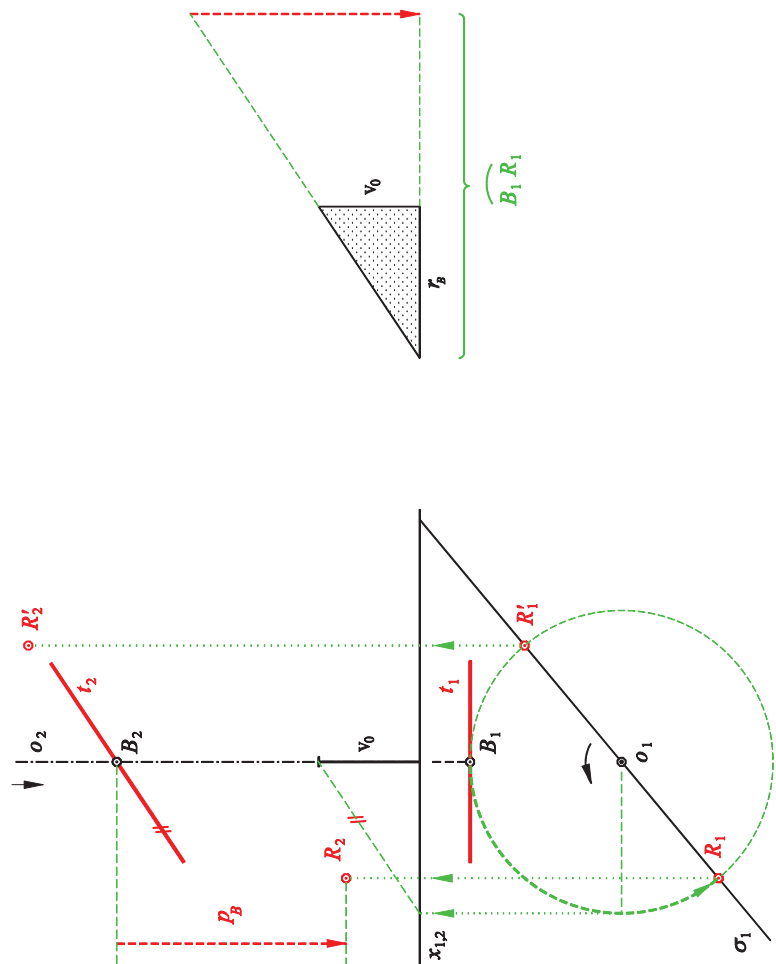


5.3 b)

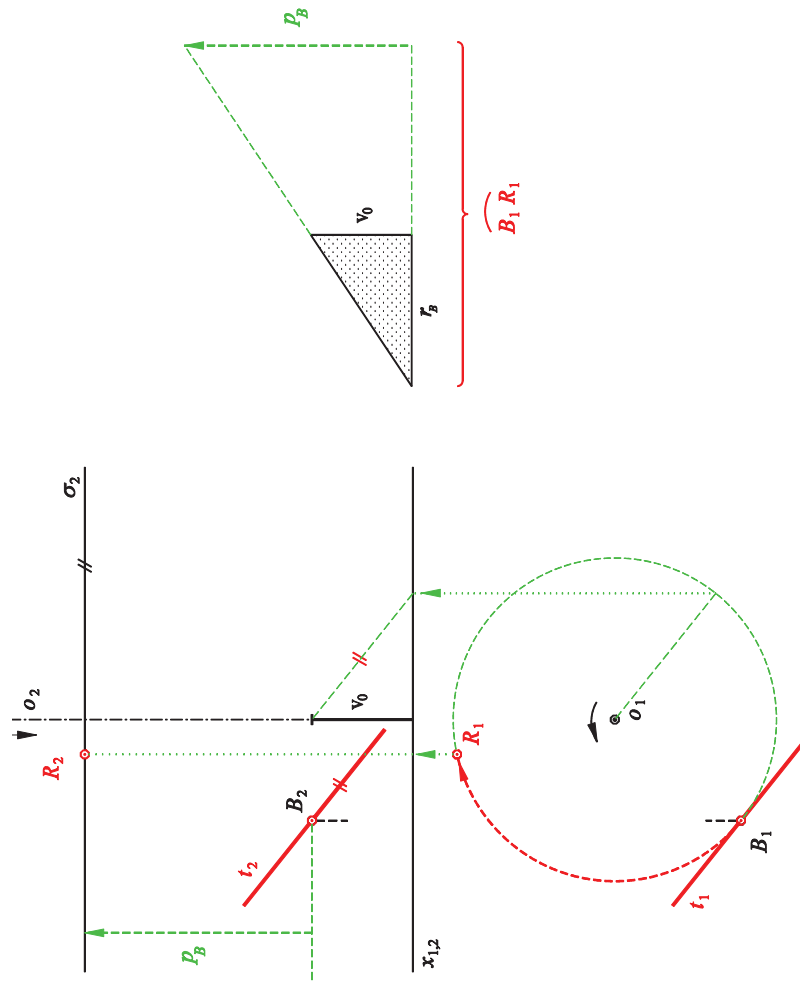


■ **Exercise 5.4.** Helix ( $B, o, v_0$ , left-handed) is given. Using Monge projection, construct intersection  $R$  of the helix and the given plane  $\sigma$ . Construct tangent line to the helix at point  $B$ .

a)

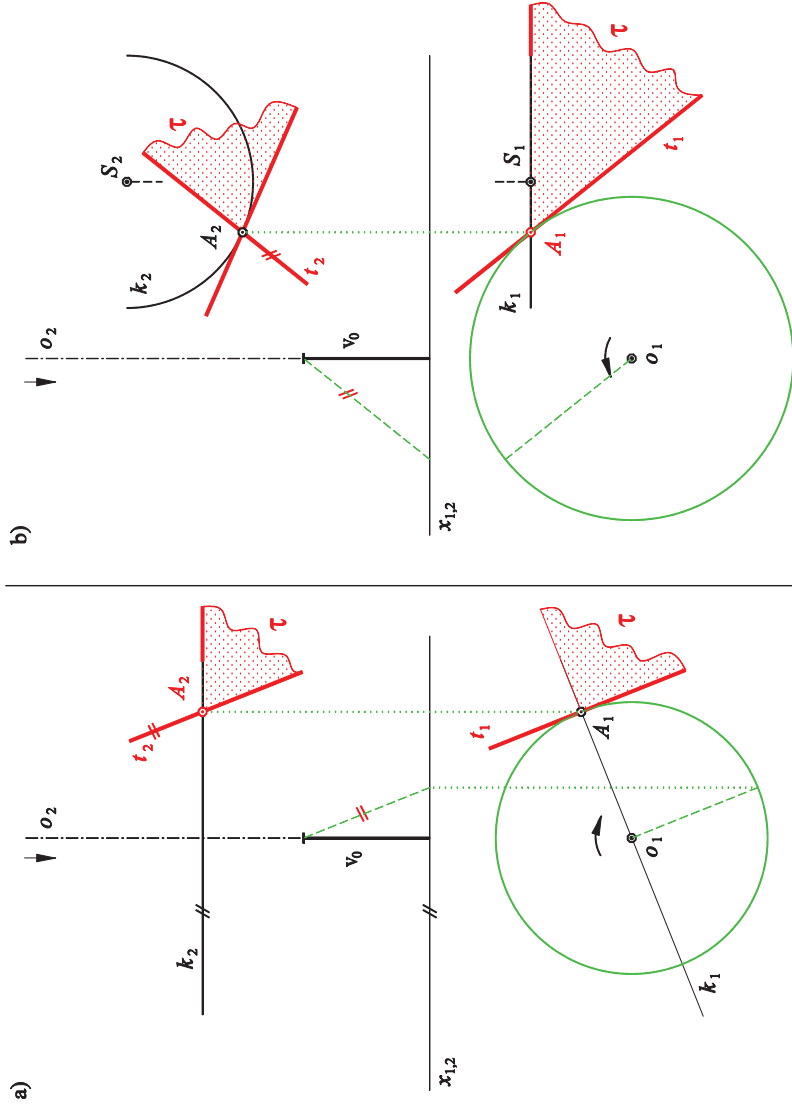


5.4 b)

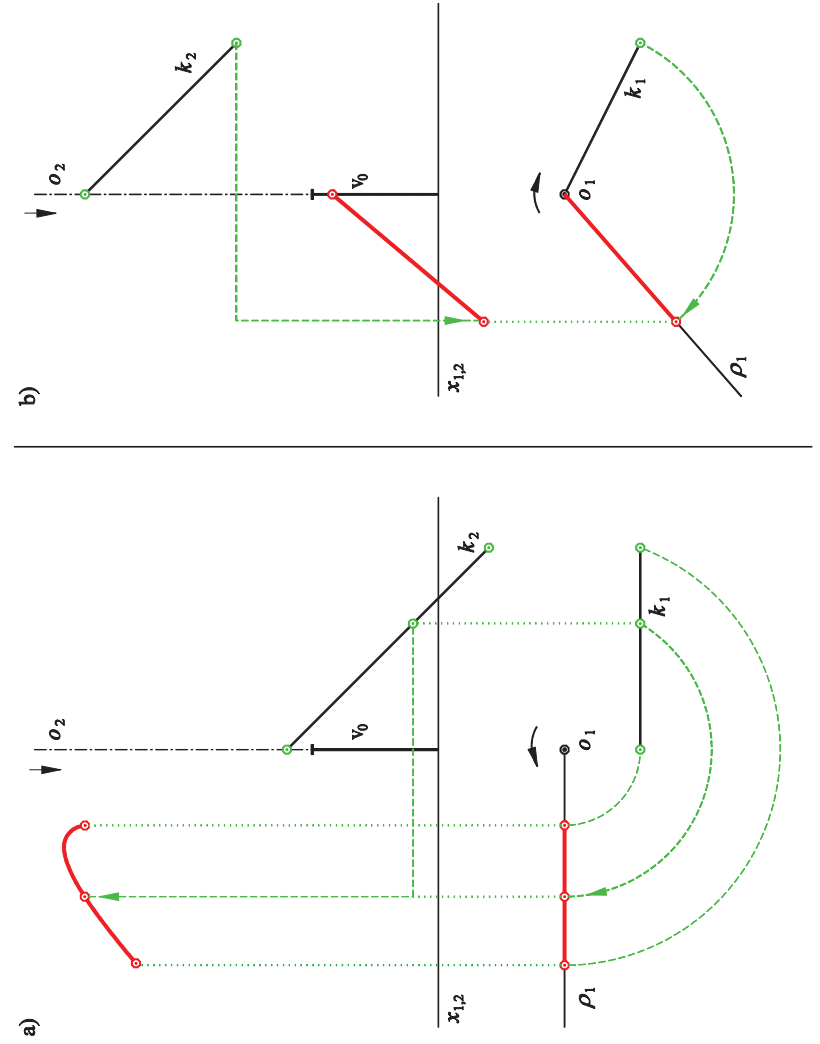


### 5.2 Helicoidal surfaces

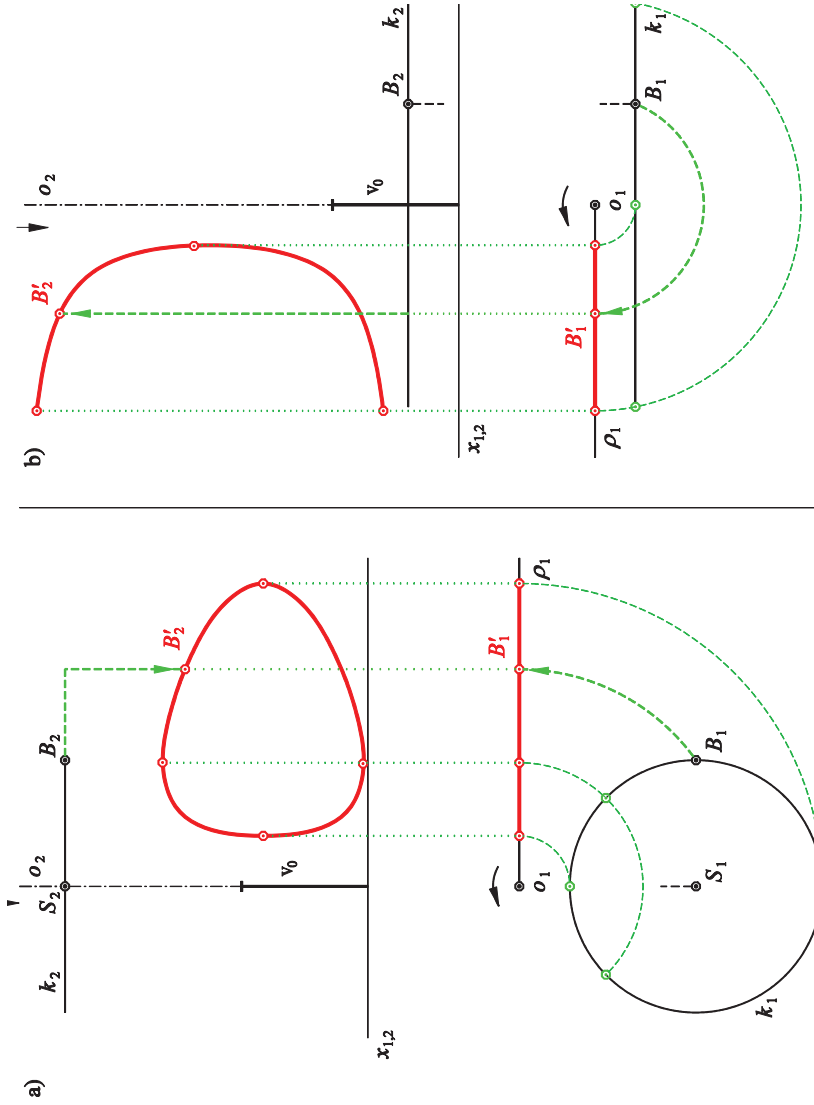
■ **Exercise 5.5.** Helicoidal surface ( $k$ ,  $o$ ,  $v_0$ , a) right-handed, b) left-handed) is given. Using Monge projection, construct tangent plane  $\tau$  at point  $A \in k$ .



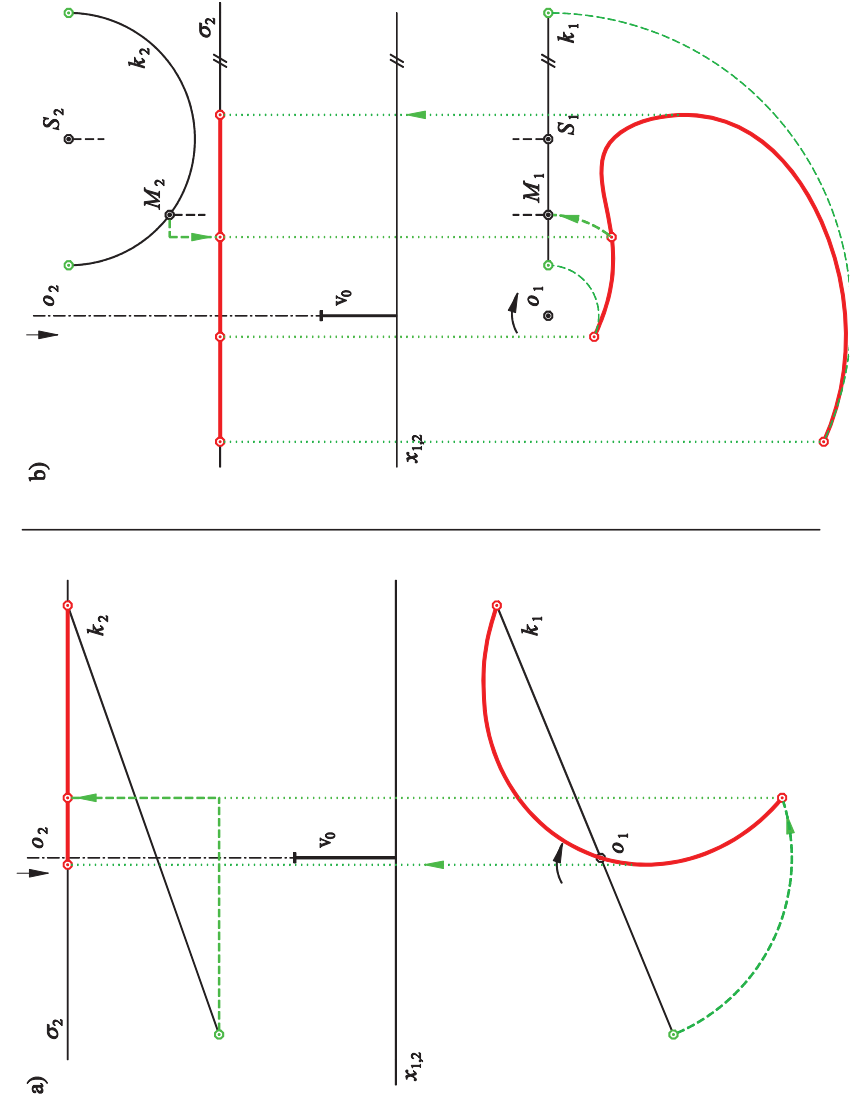
■ **Exercise 5.6.** Helicoidal surface ( $k$ ,  $o$ ,  $v_0$ , a) left-handed, b) right-handed) is given. Using Monge projection, construct the intersection of the surface and the given plane  $\rho$ .



■ **Exercise 5.7.** Helicoidal surface ( $k, o, v_0$ , left-handed) is given. Using Monge projection, construct tangent plane  $\tau$  of the surface at point  $B$ . Construct the curve of intersection  $m$  of the surface and the given plane  $\rho$  (principal half-meridian).

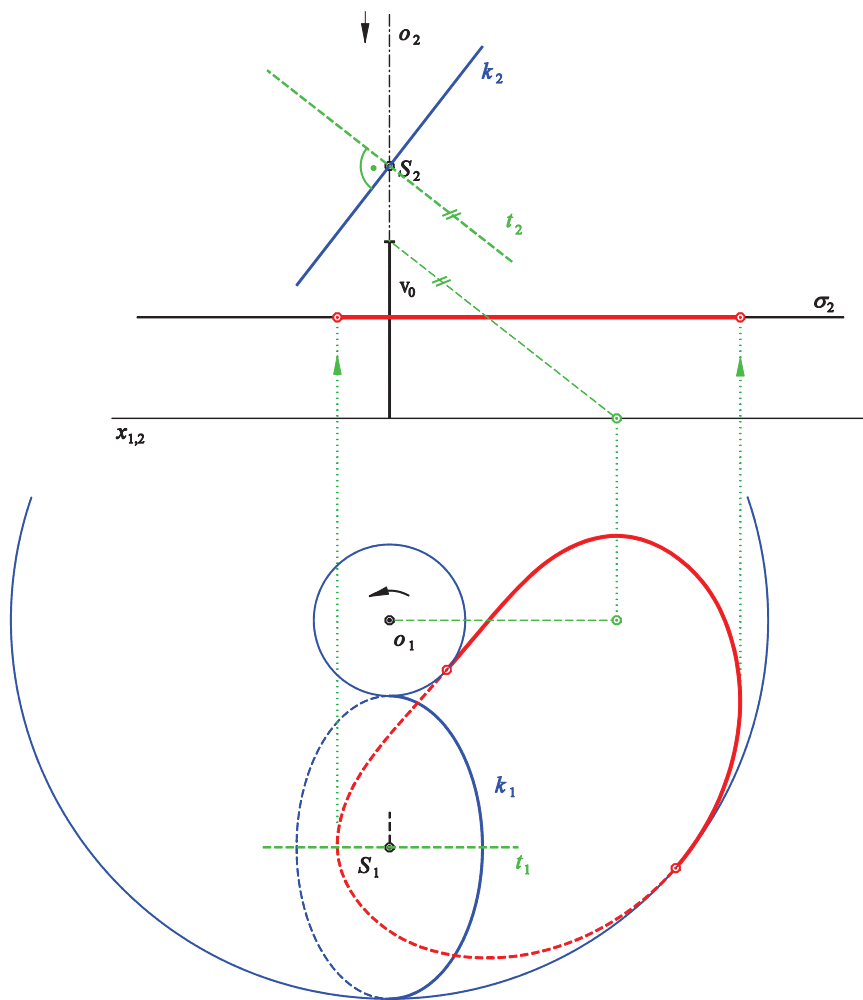


■ **Exercise 5.8.** Helicoidal surface ( $k, o, v_0$ , right-handed) is given. Using Monge projection, construct the curve of intersection  $n$  of the surface and the given plane  $\sigma$  (normal section).

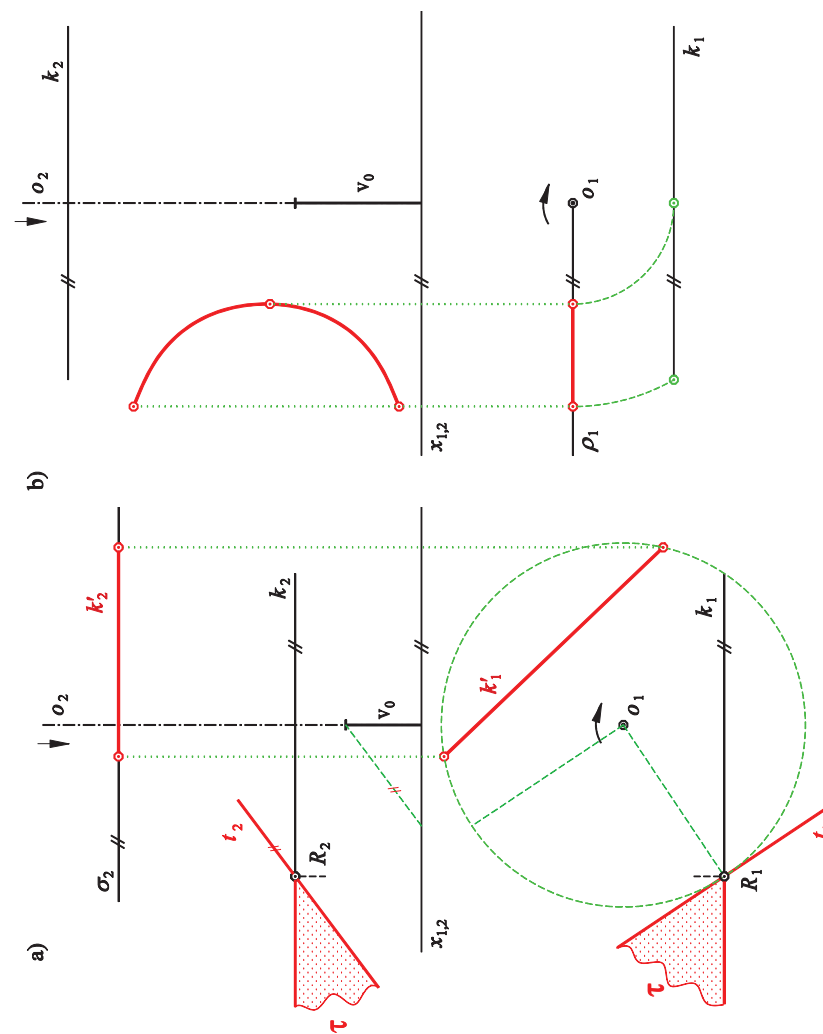




■ **Exercise 5.9.** Serpentine of Archimedes is given by centre  $S$ , radius  $r = 30$  mm and left-handed screw motion  $(o, v_0)$ . Using Monge projection, construct the generating circle. Construct the normal section  $n$  of the helicoidal surface by the given plane  $\sigma$ .

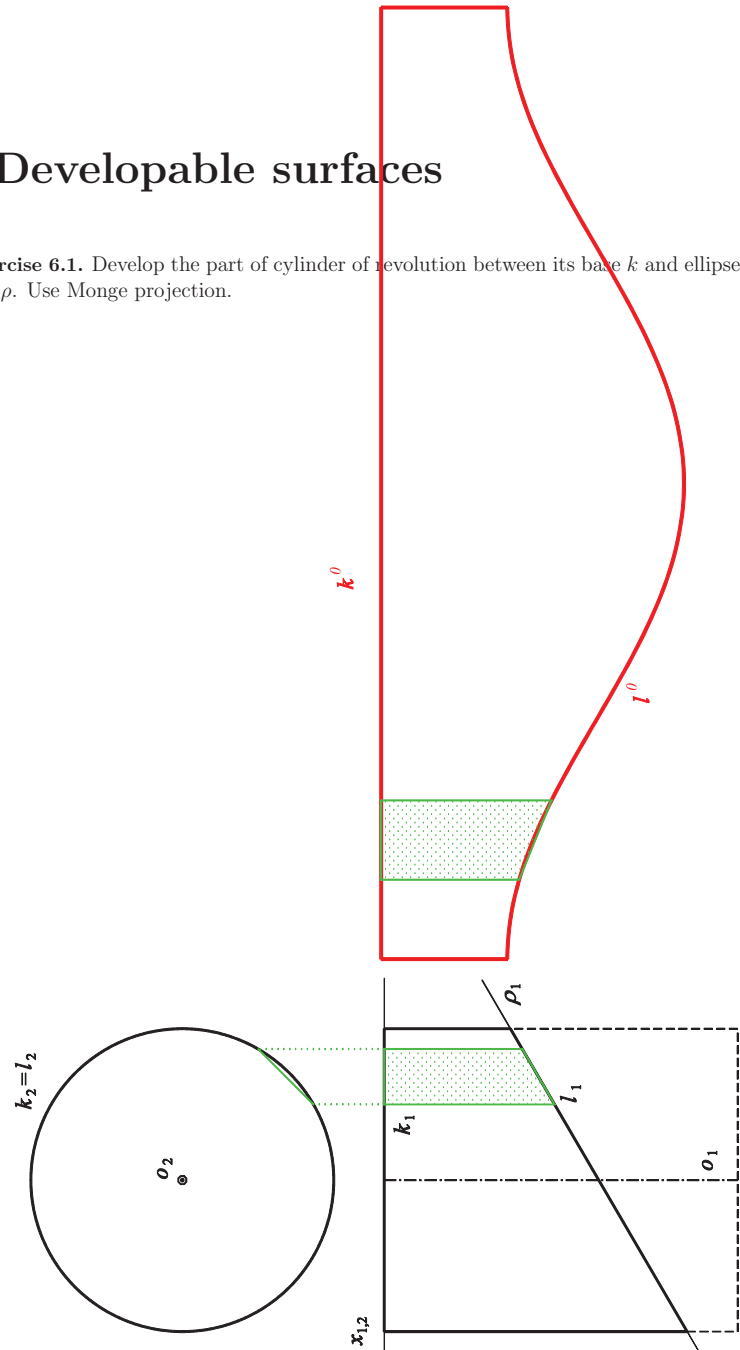


■ **Exercise 5.10.** Helicoidal surface  $(k, o, v_0, \text{right-handed})$  is given.  
 a) Using Monge projection, construct tangent plane  $\tau$  of the surface at point  $B$ . Construct the normal section  $c$  of the surface and the given plane  $\sigma$ .  
 b) Using Monge projection, construct principal half-meridian of the surface in the given plane  $\rho$ .

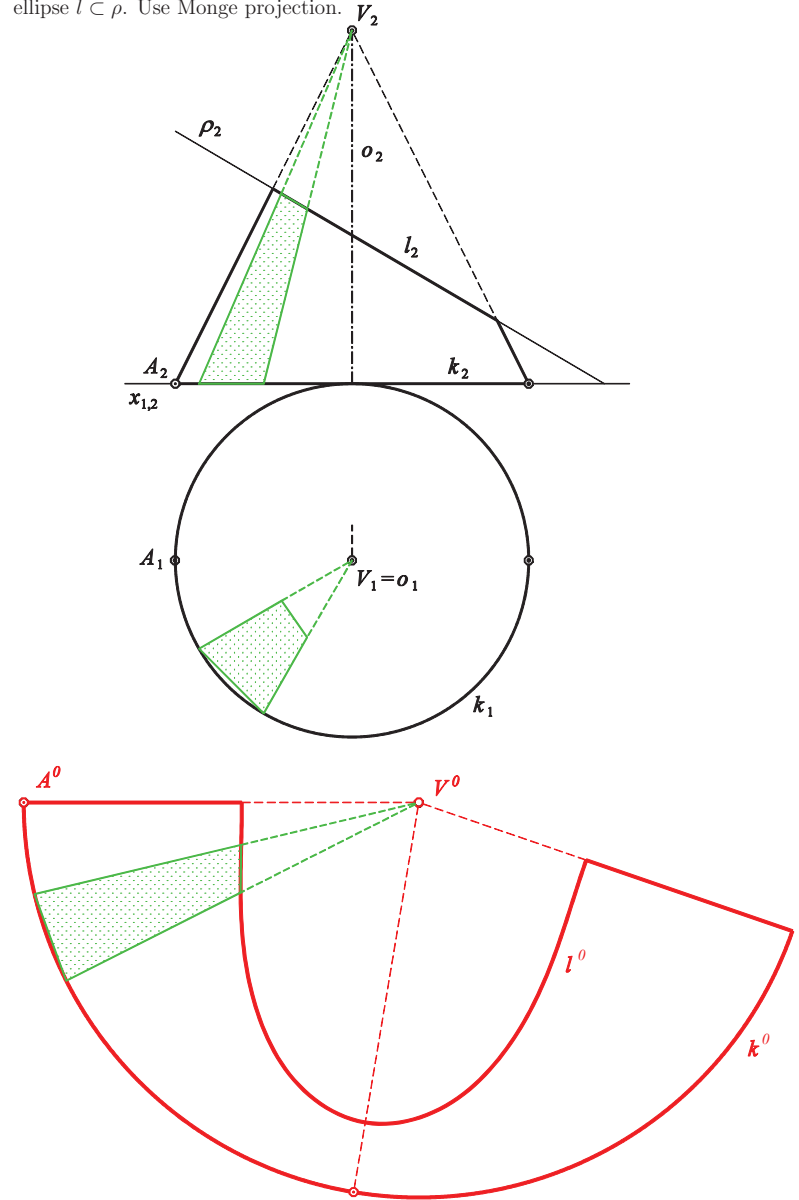


## 6 Developable surfaces

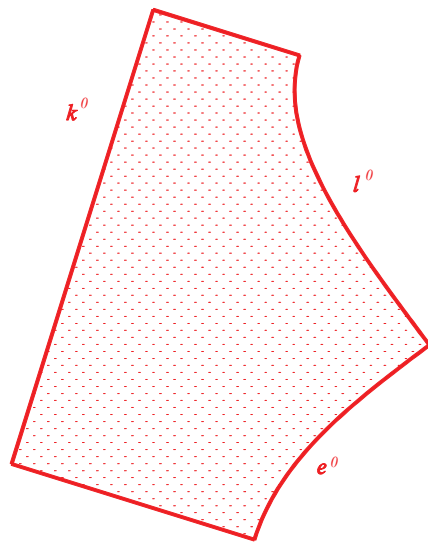
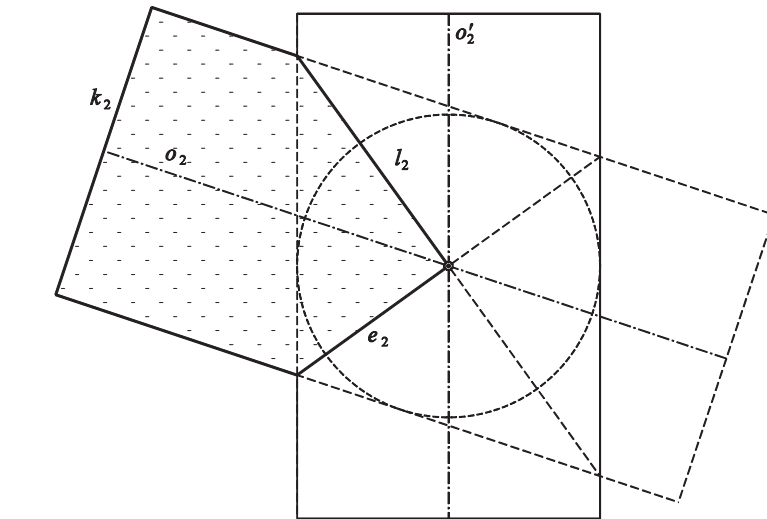
- **Exercise 6.1.** Develop the part of cylinder of revolution between its base  $k$  and ellipse  $e \subset \rho$ . Use Monge projection.



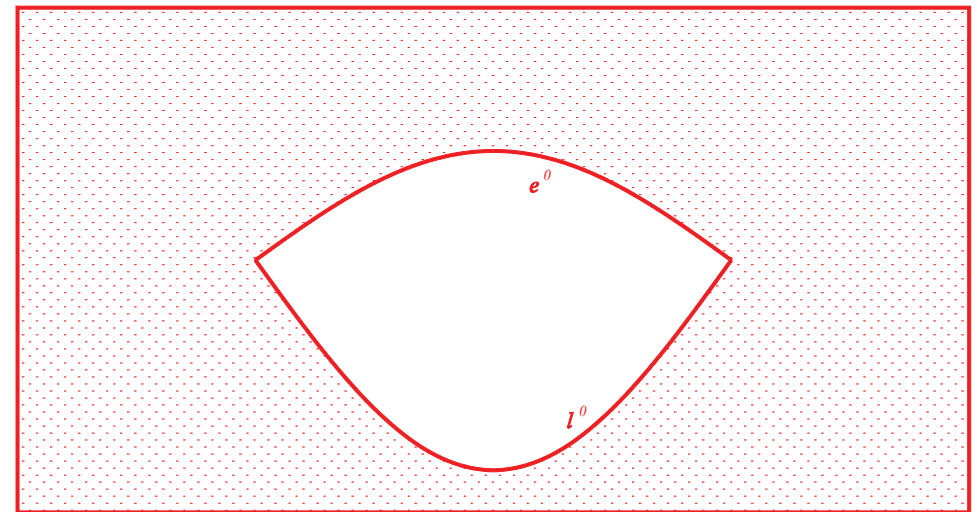
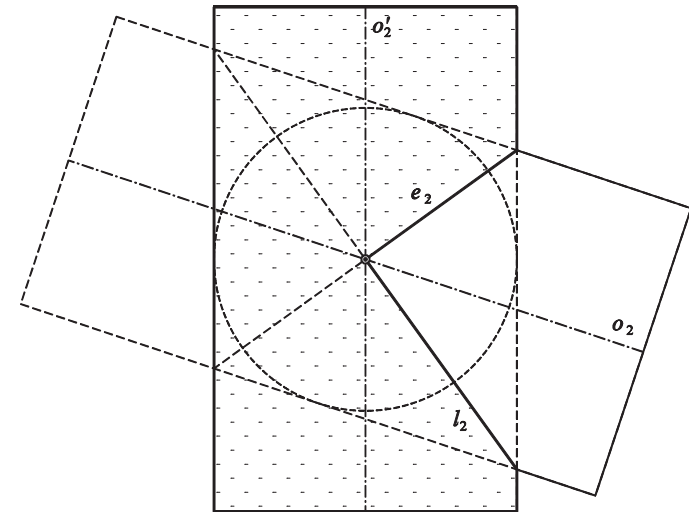
- **Exercise 6.2.** Develop the part of cone of revolution  $(V, k)$  between its base  $k$  and ellipse  $l \subset \rho$ . Use Monge projection.



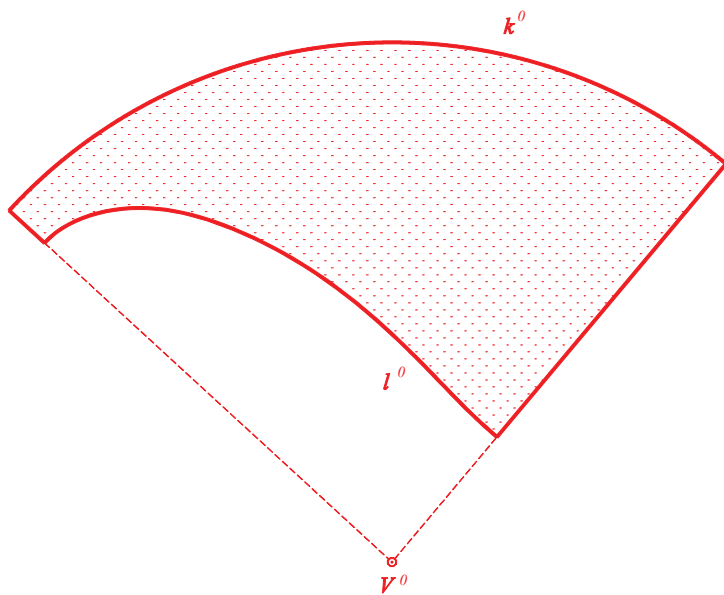
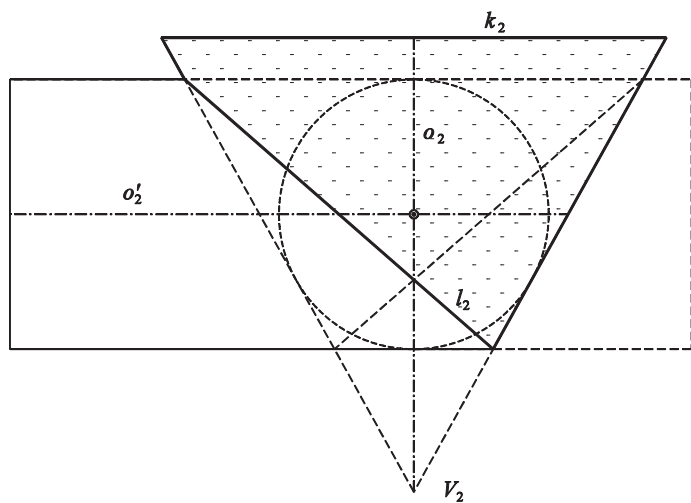
■ **Exercise 6.3.** Degenerated intersection of two cylinders of revolution is given. Develop the dotted part of the cylinder of revolution.



■ **Exercise 6.4.** Degenerated intersection of two cylinders of revolution is given. Develop the dotted part of the cylinder of revolution.



■ **Exercise 6.5.** Degenerated intersection of cylinder of revolution and cone of revolution is given. Develop the dotted part of the cone of revolution.



■ **Exercise 6.6.** Degenerated intersection of cylinder of revolution and cone of revolution is given. Develop the dotted part of the cone of revolution.  $k_2$

