Czech Technical University in Prague
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CONSTRUCTIVE GEOMETRY - EXERCISES

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Contents

Introduction

This textbook contains a collection of examples in Constructive Geometry course (first semester, Faculty of Mechanical Engineering, Czech Technical University in Prague). The examples given here are solved on tutorials of the course or can be used for self-study training.

The textbook is organised as follows. Chapter 1 Analytic geometry brings problems of vector operations and analytic representations of straight line, conic sections, plane and quadratic surfaces. A few examples for training two dimensional representation of three dimensional objects is given in 2 Technical isometry. Chapter 3 Planar kinematic geometry is focused on investigation and constructive solution of trajectories generated by planar motion of points and envelopes generated by planar motion of curves. Chapters 4 to 6 are devoted to geometrical properties of surfaces in engineering practice. In particular, 4 Surfaces of revolution is focused on surfaces generated by revolution of a curve about axis and on intersection of surfaces of revolution. Chapter 5 Helicoidal surfaces brings basic problems on helix and surfaces generated by screw motion of a curve. In chapter 6 Developable surfaces, problems for the graphical solution of construction of planar figure into which is possible to unfold or unroll special types of ruled surfaces are given.

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1 Analytic geometry

1.1 Planar analytic geometry

■ **Exercise 1.1.** Points $\mathbf{A} = (2, 5)$, $\mathbf{B} = (2, 1)$ and $\mathbf{C} = (4, 7)$ are given. Line p passes through point $\mathbf{A}$ and is parallel to line $\mathbf{BC}$, line q passes through point $\mathbf{C}$ and is perpendicular to line $\mathbf{AB}$. Determine parametric equations of straight line p and q, general equation of both straight lines, parametric equations of straight line segment $\mathbf{AB}$ and parametric equations of ray $\mathbf{AB}$ with initial point $\mathbf{A}$.

■ **Exercise 1.2.** Points $\mathbf{A} = (-5, 18)$ and $\mathbf{B} = (-14, 6)$ are given. Determine parametric, general, slope and intercept equations of straight line $\mathbf{AB}$.

■ **Exercise 1.3.** Determine equation of line p passing through point $\mathbf{A} = (-1, -2.5)$ with slope $90°$.

■ **Exercise 1.4.** Straight line p is given by $y = -1$. Determine its parametric equations.

■ **Exercise 1.5.** Determine slope equation of straight line p that passes through point $\mathbf{A} = (2, -2\sqrt{2})$ and is perpendicular to straight line $q : y = x\sqrt{2} - 3$.

■ **Exercise 1.6.** Determine the distance between point $\mathbf{A} = (3, -2)$ and straight line $p : 2x - y + 3 = 0$.

■ **Exercise 1.7.** Determine the mutual position of straight lines

$$\begin{aligned} p : x(t) &= 5 - 7t, \\ y(t) &= 4 - 14t, t \in \mathbb{R} \end{aligned}$$

and

$$\begin{aligned} q : x(s) &= 18 - 3s, \\ y(s) &= 17 - s, s \in \mathbb{R} \end{aligned}$$

In case of intersecting lines calculate the angle between them and coordinates of their intersection.

■ **Exercise 1.8.** Determine equation of line p passing through point $\mathbf{B} = (2, 3)$ and forming angle $45°$ with line $q : 2x + 5y - 5 = 0$.

■ **Exercise 1.9.** The following conic sections are given. Determine the type of conic section and its basic characteristics (coordinates of centre/vertex, major and minor vertices, length of radius/semiaxes, equation of axis/axes). Calculate coordinates of intersection of the conic section with coordinate axes and sketch the conic section.

1. $x^2 - 6x - 2y^2 - 16y - 27 = 0$
2. $6x^2 + 2y^2 + 4y = 6$
3. $3xy + 12 = 0$

4. $2x + 4y^2 - 2y + 6 = 0$

5. $x^2 + 4x + y + 1 = 0$

6. $3x + 3xy + 6y + 10 = 0$

7. $-6x^2 - 7x + 6xy + 15y + 20 = 0$

8. $y^2 + 4y - 12 = 0$

9. $x^2 + 6x + y^2 - 2y + 1 = 0$

10. $4x^2 + 8x + 9y^2 - 54y + 49 = 0$

11. $-x^2 + 6x + 2y^2 - 8y - 5 = 0$

■ **Exercise 1.10.** Determine the type, basic characteristics (coordinates of centre/vertex, major and minor vertices, length of radius/semiaxes, equation of axis/axes) and intersections with coordinate axes of conic section $x^2 - 2x + y^2 - 4y + 3 = 0$ and sketch the conic section. Determine intersections of conic section and straight line passing through points $\mathbf{A} = (3, 0)$ and $\mathbf{B} = (2, 4)$.

■ **Exercise 1.11.** Determine the type, characteristics (coordinates of centre/vertex, major and minor vertices, length of radius/semiaxes, equation of axis/axes) and intersections with coordinate axes of conic section $x^2 + 4x + 2y^2 = 0$ and sketch the conic section. Determine intersections of conic section and straight line passing through points $\mathbf{A} = (-3, 2)$ and $\mathbf{B} = (0, -4)$.

■ **Exercise 1.12.** Planar region D is given by boundaries c_1 and c_2. Calculate the intersections of the boundaries and the intersections of the boundaries and coordinate axes. Sketch the boundaries, designate all intersections and indicate the region.

1. $c_1 : 4x^2 - 16x + y^2 + 6y + 9 = 0,$
   $c_2 : 2xy - y^2 - 6y + 6x - 9 = 0$

2. $c_1 : y^2 - 2y - 4x^2 - 16x = 19 = 0,$
   $c_2 : y^2 - 4y - 32 = 0$

3. $c_1 : 4x^2 - 32x + y + 60,$
   $c_2 : 4y^2 - 4x^2 + 24x = 36$

4. $c_1 : x^2 - 2x + 5y^2 - 8 = 0,$
   $c_2 : x^2 - 10x + y^2 - 4y + 24 = 0$

5. $c_1 : x^2 - 2x + 5y^2 - 8 = 0,$
   $c_2 : x^2 - 10x + y^2 - 4y + 24 = 0$

■ **Exercise 1.13.** Planar region D is given by the following sets. Determine the individual boundaries of the region D, calculate the intersections of the boundaries and the intersections of the boundaries and coordinate axes. Sketch the boundaries, designate all intersections and indicate region D.

1. $D : x^2 + 6x + 4y^2 - 8y - 32 < 0,\ 0 \leq \frac{1}{2}y^2 - 2y - x$
2. $D : -x^2 + 2x + 2y^2 - 3 < 0,\ x^2 - 2x + 2y^2 - 8y < 9$
3. $D : x^2 + 10x + y^2 - 2y - 3 \leq 0,\ 2x^2 + 8x - y^2 + 2y + 3 \leq= 0$
4. $D : 4x^2 - 24x + y^2 + 6y + 41 \leq 0,\ 4x^2 - 24x - y^2 - 2y + 35 \leq 0$
5. $D : x^2 + 6x + 9y^2 - 1 < 0,\ x + y^2 + 2 < 0$
6. $D : 9x^2 - 36x + 4y^2 - 40y + 100 > 0,\ x^2 - 8x + 4y^2 - 16y - 4 \leq 0$
7. $D : 4x^2 + 36x + 4y^2 - 16y - 88 < 0,\ 2x^2 - 12x - y + 16 \leq 0$
8. $D : -x + xy + 2y - 4 \geq 0,\ 5y + x^2 - 2x - 10 \leq 0$
9. $D : 3x + 3y - xy - 12 < 0,\ 8x^2 - 32x + 4y^2 - 24y + 32 \leq 0$
10. $D : y^2 - x^2 - 8x - 4y - 16 \geq 0,\ x^2 + 14x - 2y^2 + 12y + 29 \geq 0$
11. $D : 2x^2 + 8x - y + 5 \leq 0,\ 16x^2 - 24x + 16y^2 + 72y - 160 \leq 0$
12. $D : x - 4y^2 + 16y - 19 \geq 0,\ x^2 - 8x + 4y^2 - 16y - 4 \leq 0$

1.2 Spatial analytic geometry

■ **Exercise 1.14.** Prove that triangle with vertices $\mathbf{A} = (1, 1, 2)$, $\mathbf{B} = (1, 4, 2)$ and $\mathbf{C} = (3, 1, 5)$ is a right angle triangle.

■ **Exercise 1.15.** Calculate the area of the triangle **ABC** given by vertices $\mathbf{A} = (0, 0, 2)$, $\mathbf{B} = (1, 3, 2)$ and $\mathbf{C} = (3, 1, 4)$.

■ **Exercise 1.16.** Determine whether the following points are collinear.

1. $\mathbf{A} = (1, 2, 3)$, $\mathbf{B} = (0, 4, 4)$, $\mathbf{C} = (2, 0, 2)$, $\mathbf{D} = (-1, 6, 5)$
2. $\mathbf{A} = (-1, 2, 2)$, $\mathbf{B} = (1, 5, 3)$, $\mathbf{C} = (-3, -1, 1)$, $\mathbf{D} = (3, 6, 4)$

■ **Exercise 1.17.** Determine whether the following points are coplanar.

1. $\mathbf{A} = (2, 0, 0)$, $\mathbf{B} = (0, -2, -2)$, $\mathbf{C} = (3, 5, 1)$, $\mathbf{D} = (5, 2, 4)$
2. $\mathbf{A} = (0, 1, 0)$, $\mathbf{B} = (1, -2, 0)$, $\mathbf{C} = (-1, 1, 2)$, $\mathbf{D} = (2, 1, 4)$

■ **Exercise 1.18.** Calculate the volume of the pyramid **ABCD** given by vertices $\mathbf{A} = (0, -2, 0)$, $\mathbf{B} = (2, 1, 0)$, $\mathbf{C} = (-1, 2, 0)$ and $\mathbf{D} = (0, 0, 4)$.

■ **Exercise 1.19.** Determine the mutual position of planes α and β. If the planes are intersecting, determine the equation of intersection and angle formed by the given planes. Draw the planes in technical isometry.

1. $\alpha : 2x + 2z = 8,\ \beta : 2x - 2y - 3z + 9 = 0$
2. $\alpha : x - 2y = 3,\ \beta : z = 2$

■ **Exercise 1.20.** Determine the mutual position of planes α, β and γ. If the planes are intersecting, determine the coordinates of intersection. Draw the planes in technical isometry.

1. $\alpha : x + 4z = 2$, $\beta : 2x + 3y + z + 2 = 0$, $\gamma : x = -2$
2. $\alpha : -3x + 4y - z - 2 = 0$, $\beta : x - 2y + 3z - 6 = 0$, $\gamma : 2x + 2y + z - 9 = 0$

■ **Exercise 1.21.** Determine the general equation of plane α passing through points **A**, **B** and **C**. Determine whether point **D** lies in the plane α and if not calculate the distance $d(\mathbf{D}, \alpha)$. Draw all figures in technical isometry.

1. $\mathbf{A} = (1, 1, 0)$, $\mathbf{B} = (1, 3, 1)$, $\mathbf{C} = (4, 4, -1)$, $\mathbf{D} = (2, 3, 1)$
2. $\mathbf{A} = (0, 1, 2)$, $\mathbf{B} = (3, -2, 4)$, $\mathbf{C} = (\frac{1}{2}, \frac{1}{6}, 3)$, $\mathbf{D} = (7, 7, 1)$

■ **Exercise 1.22.** Calculate the intersection of plane α and straight line p. Straight line p passes through point **M** and is perpendicular to plane β.

1. $\alpha : x + 2y + 3z = 0$, $\mathbf{M} = (2, 3, 0)$, $\beta : y + z = 8$
2. $\alpha : x + y - z = 2$, $\mathbf{M} = (2, 1, 1)$, $\beta : 2x - 2y = 3$

■ **Exercise 1.23.** Quadratic surface σ is given. Determine the type and basic characteristics (coordinates of centre/vertex, length of radius/semiaxes/altitudes, equation of axis/axes) of surface σ. Determine the type and equation of the intersection of surface σ with coordinate planes.

1. $\sigma : 4x^2 + 4y^2 - 8y + 4 - z^2 = 0$
2. $\sigma : y^2 - 4y + z^2 + 6z + 5 = 0$
3. $\sigma : 4x^2 - 12x - z^2 - 4z + 5 = 0$
4. $\sigma : x^2 - 4x + y^2 - 10y + 9z^2 - 72z + 164 = 0$
5. $\sigma : 4x^2 + y^2 - 2y + 4z^2 - 16z + 1 = 0$
6. $\sigma : \sqrt{z^2 + 4z + 4y^2 + 4} - x = 0$
7. $\sigma : x^2 - 2x - 4y^2 + 8y - 7 = 0$
8. $\sigma : 2y^2 + 2z^2 - 4z - x + 4 = 0$
9. $\sigma : 2x^2 - 4x + 4y^2 + 8y + z + 1 = 0$
10. $\sigma : -4x^2 + 16x + y^2 + z^2 - 4z - 16 = 0$
11. $\sigma : 2x^2 + 4x + y^2 - 6y - 4z^2 + 16z - 9 = 0$
12. $\sigma : 4x^2 + 8x - y^2 - 6y - z^2 - 9 = 0$
13. $\sigma : -8x^2 - y^2 - 2y + 2z^2 + 12z + 9 = 0$

■ **Exercise 1.24.** Determine the type and basic characteristics (coordinates of centre/vertex, length of radius/semiaxes/altitudes, equation of axis/axes) of quadratic surface

$\sigma : 4x^2 + 4x - y^2 - 2y - 2z + 2 = 0.$

Determine type and equation of the intersections of quadratic surface σ with planes $x = 0$, $y = -2$, $z = -3$ and $z = 1$. Draw surface σ.

■ **Exercise 1.25.** Draw spatial region D bounded by surfaces $x^2 + 6x + 2z^2 + 5 = 0$ and $2x^2 - y^2 - xy = 0$. Write the name of the boundaries and determine their intersection.

■ **Exercise 1.26.** Draw spatial region D bounded by surfaces $z = \sqrt{x^2 + y^2}$ and $z = 2 - x^2 - y^2$. Write the name of the boundaries and determine the intersections of region D and planes $z = 0$ and $z = 2$.

■ **Exercise 1.27.** Draw spatial region D bounded by surfaces $3z = x^2+y^2$, $x^2+-y^2 = z^2$, $z = 0$ and $x = 0$. Write the name of all boundaries.

■ **Exercise 1.28.** Draw spatial region D bounded by surfaces $x = 0$, $y = 0$, $z = 0$ and $2x + 3y + 6z = 6$. Write the name of all boundaries and determine the intersection of the region D and planes $z = 0$ and $x = 0$.

■ **Exercise 1.29.** Draw spatial region D bounded by surfaces $x = 0$, $y = 0$, $z = 0$, $2x + 3z = 6$ and $y = 4$. Write the name of all boundaries. Determine the intersection of the region D and planes $z = 0$ and $x = 0$.

■ **Exercise 1.30.** Spatial region D is bounded by surfaces σ_1, σ_2 and σ_3. Write the name of all boundaries. Determine the intersections of the boundaries. Draw region D.

1. $\sigma_1 : x^2 - 2y^2 - 4 = 0,$

   $\sigma_2 : z^2 - 4z - 12 = 0,$

   $\sigma_3 : y^2 + 2y - 8 = 0$

2. $\sigma_1 : x^2 - 8x + y^2 - 6y + 10z^2 - 20z + 25 = 0,\ z \in (0, 1),$

   $\sigma_2 : x^2 - 8x + y^2 - 6y - z^2 + 8z + 8 = 0,\ z \in (1, 5),$

   $\sigma_3 : x^2 - 8x + y^2 - 6y + z^2 - 10z + 48 = 0,\ z \in (5, 10)$

3. $\sigma_1 : z = x^2 + y^2,$

   $\sigma_2 : x^2 + y^2 = 2,$

   $\sigma_3 : z - 6 = -\sqrt{2x^2 + 2y^2}$

4. $\sigma_1 : 9x^2 - 36x - 4y^2 + 24y - 36 = 0,$

   $\sigma_2 : z^2 - z - 6 = 0,$

   $\sigma_3 : 10x + 5xy + 6y^2 - 18y - 60 = 0$

5. $\sigma_1 : x - \sqrt{-y^2 - z^2 - 6z - 5} = 0,$

   $\sigma_2 : x^2 + y^2 - 4 = 0,$

   $\sigma_3 : z + \sqrt{x^2 + y^2} - 4 = 0$

6. $\sigma_1 : z - \sqrt{2x^2 + 4x + 2y^2 + 4y + 4} + 5 = 0,$
$\sigma_2 : x^2 + 2x + y^2 - 2y + 4z^2 - 8z + 2 = 0,$
$\sigma_3 : x^2 + 2x + y^2 - 2y = 0$

■ **Exercise 1.31.** Spatial region D is given by the following sets. Write the name of all boundaries and draw region D.

1. $D : x^2 + 4x + y^2 - 2y + z^2 + 2z - 3 \leq 0,\ x^2 + 2x + y^2 - 3 \geq 0$
2. $D : y^2 - 4y + z^2 - 8z + 12 \leq 0,\ 4x^2 - 32x + 9z^2 - 72z + 172 \leq 0$
3. $D :: x^2 - 2x + y^2 + 2y - 2 \leq 0,\ -3 \leq z \leq x^2 - 2x + y^2 + 2y + 4$
4. $D : 4x^2 - 16x - y^2 + 2y - z^2 + 8z - 5 \leq 0,\ y^2 - 2y + z^2 - 8z + 8 \leq 0$
5. $D : x^2 + 4x + z^2 - 8z + 16 \geq 0,\ 2x^2 + 8x - y^2 - 2y + 2z^2 - 16z + 27 \leq 0,\ -5 \leq y \leq 2$
6. $D : x^2 + y^2 + 4y + 2 \leq 0,\ 9x^2 + 9y^2 - 36y + 4z^2 - 8z + 4 \geq 0,\ -3 \leq z \leq 5$
7. $D : 2y^2 - 4y + z^2 - 2z + 2 \leq 0,\ x^2 - 10x - 2y^2 + 4y - z^2 + 2z + 22 \geq 0,\ 1 \leq x \leq 5$
8. $D : x^2 + y^2 + 4y + 2 \leq 0,\ 9x^2 + 9y^2 - 36y + 4z^2 - 8z + 4 \geq 0,\ -3 \leq z \leq 5$
9. $D : x^2 - 6x + y^2 + 2z^2 + 8z + 13 \leq 0,\ x^2 - 6x + y^2 - 3z^2 + 12z - 3 \geq 0$
10. $D : 9x^2 + 18x + y^2 - 4y + z^2 - 4z + 13 \leq 0,\ -x - 2y^2 + 8y - 2z^2 + 8z - 12 \geq 0$
11. $D : x^2 + 2x + y^2 - 2y + 4z^2 + 8z + 2 \leq 0,\ x^2 + 2x + y^2 - 2y - z^2 - 2z \geq 0$
12. $D : x^2 + 2y^2 - 12y + z^2 - 4z + 31 \leq 0,\ -x^2 + y^2 - 4y - z^2 + 4z - 1 \leq 0$
13. $D : 9x^2 + y^2 - 6y + z^2 - 4z + 4 \leq 0,\ x^2 + 4y^2 - 16y + 4z^2 - 16z + 16 \geq 0$
14. $D : 4x - 2y^2 - 4y - z^2 + 6z - 3 \geq 0,\ x^2 - 12x - 2y^2 - 4y - z^2 + 6z + 25 \geq 0,\ x \leq 6$
15. $D : x^2 - 4x + y^2 - 6y - z + 10 \geq 0,\ x^2 - 4x + y^2 - 6y + 2z + 1 \leq 0$
16. $D : x^2 - 6x + y^2 - 2y - z^2 + 4z + 5 \leq 0,\ x^2 - 6x + y^2 - 2y - z + 9 \leq 0,\ z \leq 6$
17. $D : x^2 + y^2 - 4y + z - 1 \leq 0,\ -x^2 - y^2 + 4y + z^2 - 4z - 1 \leq 0$
18. $D : x^2 + 6x + y^2 - 4y - 4z + 13 \leq 0,\ x^2 + 6x + y^2 - 4y + z^2 - 12z + 29 \geq 0$
19. $D : 2x^2 + 12x - y^2 + 4y - z^2 + 6z + 5 \leq 0,\ x^2 + 6xy^2 - 4y + z^2 - 6z + 13 \leq 0$

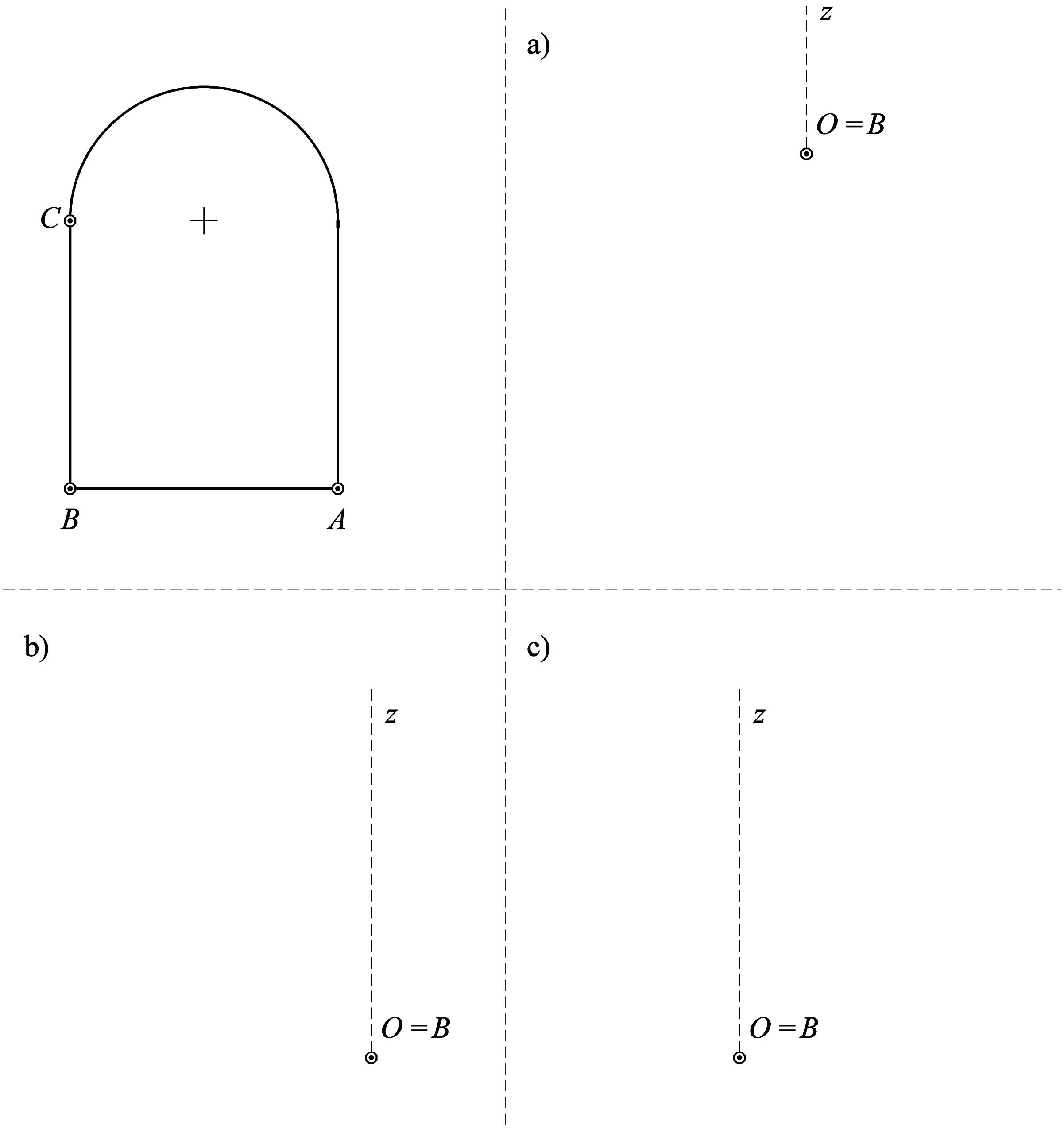
2 Technical isometry

■ **Exercise 2.1.** In technical isometry, construct the planar figure given by its orthogonal projection. Point B lies at origin O. The figure lies

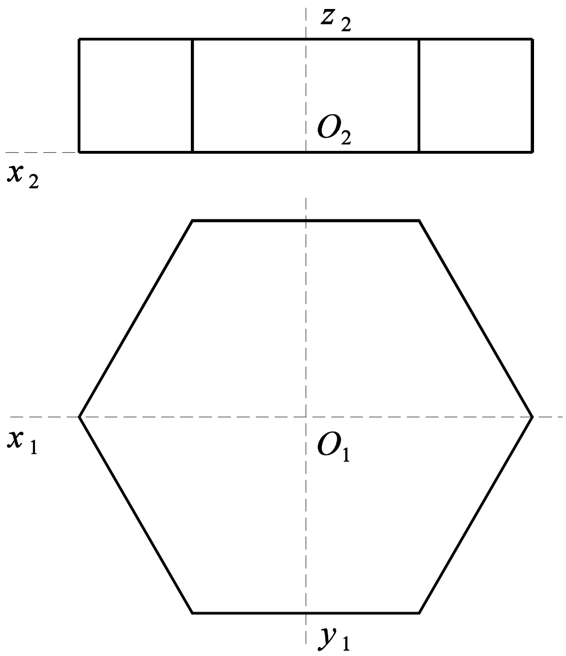
a) in the plane (x, y), $A \in x$, choose solution for $x_A > 0$ and $y_C > 0$,

b) in the plane (x, z), $A \in z$, choose solution for $z_A > 0$ and $x_C > 0$,

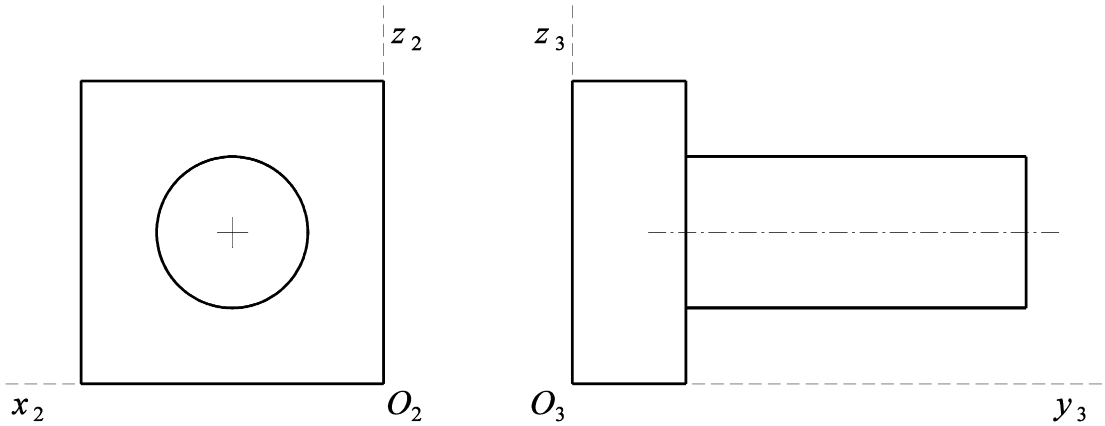
c) in the plane (y, z), $A \in y$, choose solution for $y_A > 0$ and $z_C > 0$.



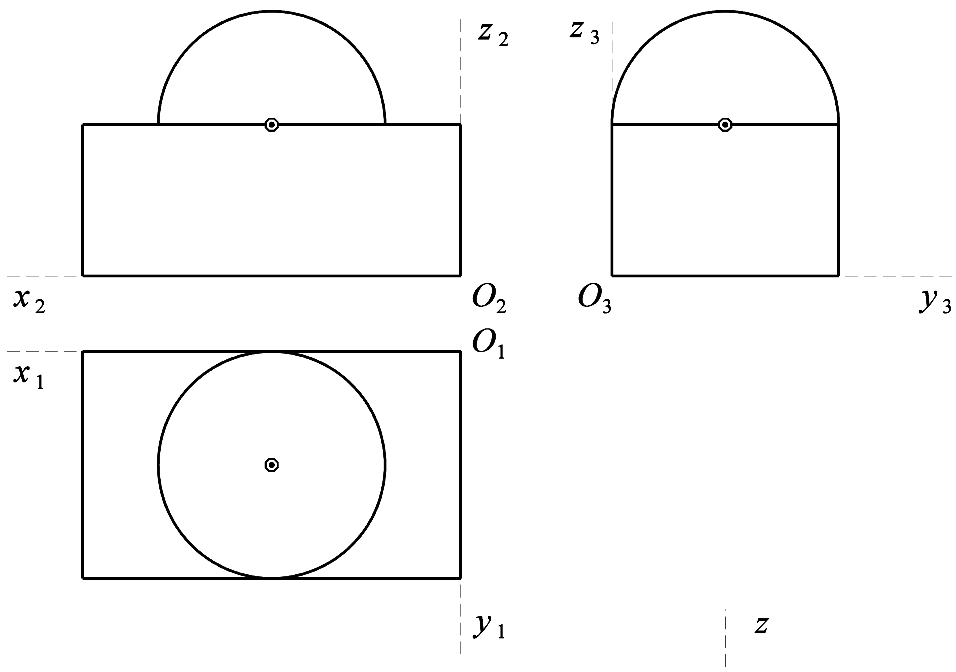
■ **Exercise 2.2.** In technical isometry, construct the regular hexagonal prism given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.

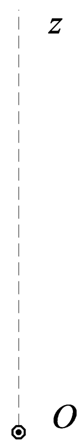


■ **Exercise 2.3.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.

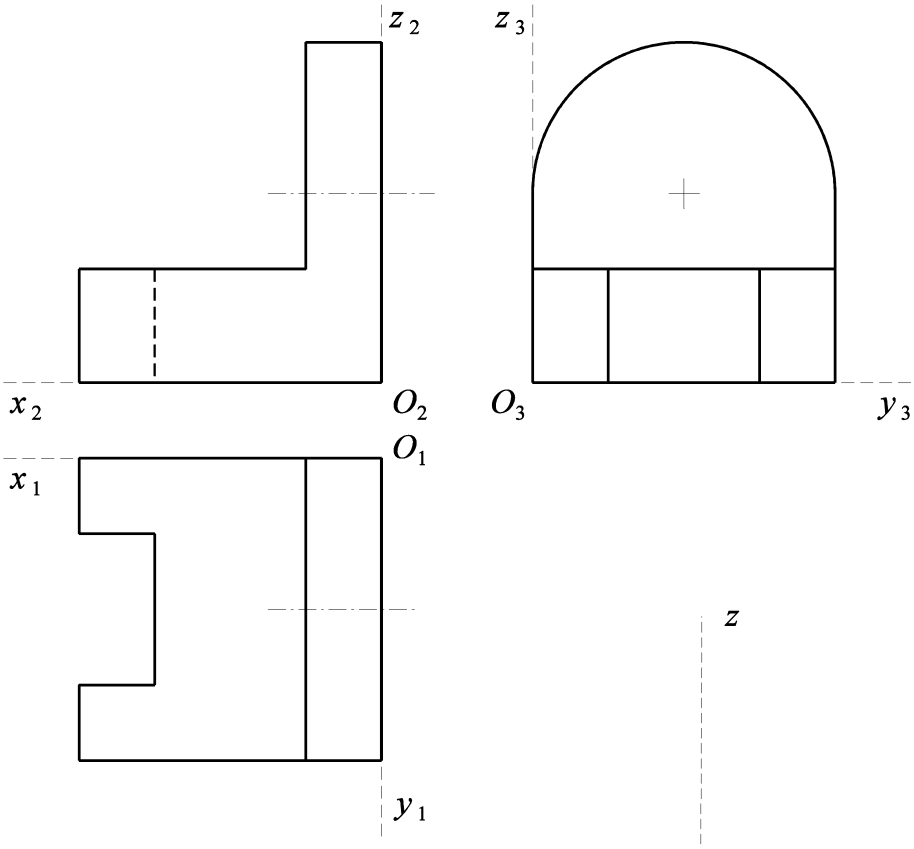


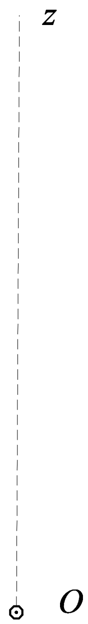
■ **Exercise 2.4.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.



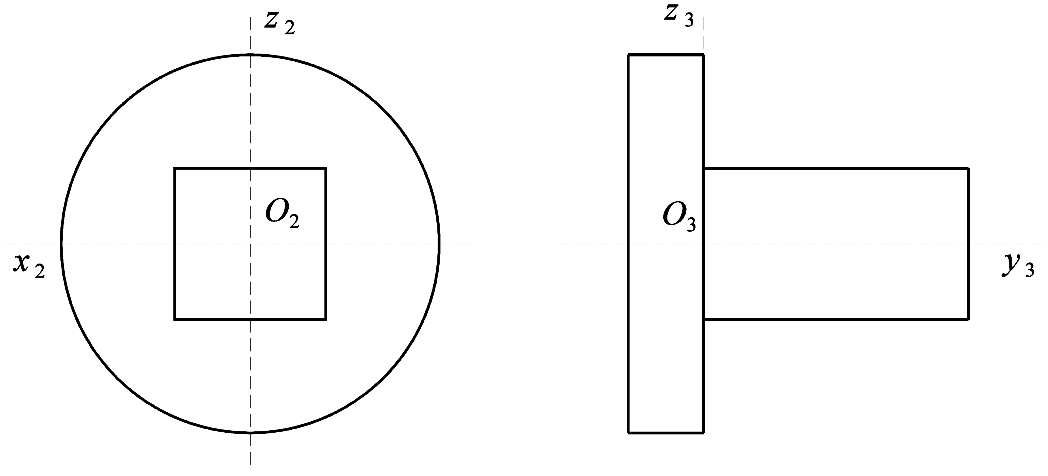


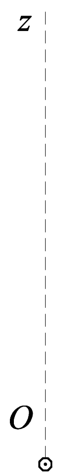
■ **Exercise 2.5.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.





■ **Exercise 2.6.** In technical isometry, construct the object given by its orthogonal projections in scale 1:2. Measure all dimensions you need. Indicate the visibility.



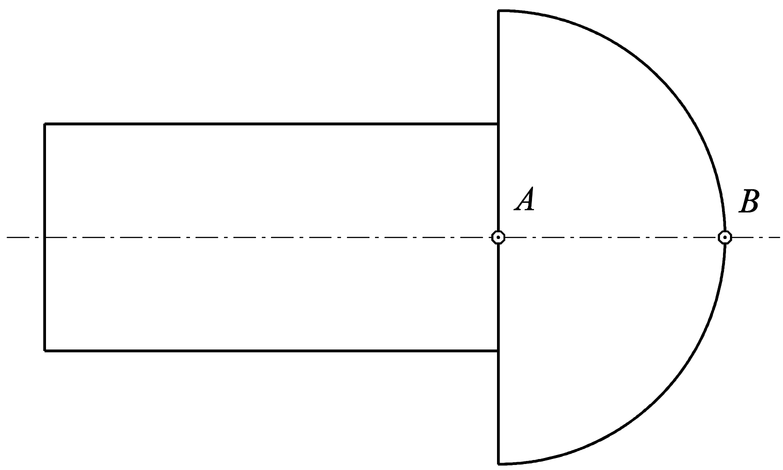


■ **Exercise 2.7.** In technical isometry, construct the rotary object (hemisphere and cylinder of revolution) given by its orthogonal projection in scale 1:1. Measure all dimensions you need. Indicate the visibility. Centre A of the hemisphere lies at origin O, the axis of revolution is identical with

a) x-axix, choose solution for $x_B < 0$,

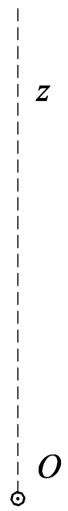
b) z-axis, choose solution for $z_B < 0$,

c) y-axis, choose solution for $y_B > 0$.

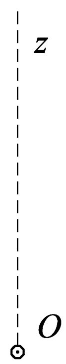


a)

b)



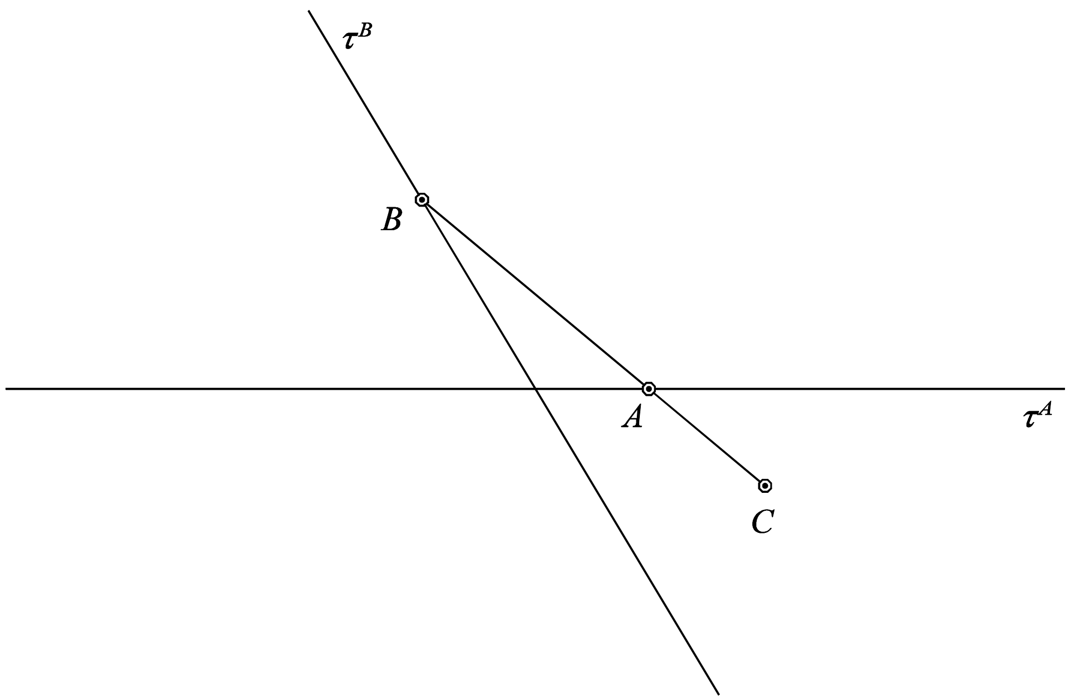
c)



3 Planar kinematic geometry

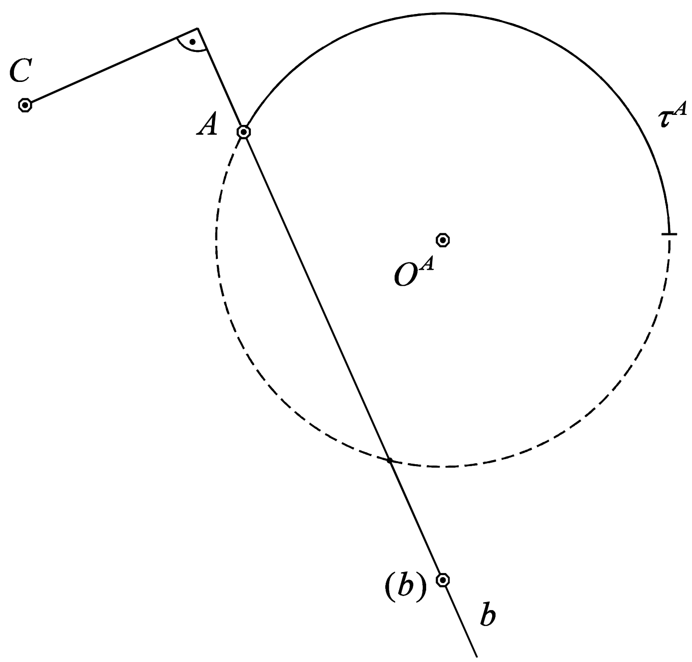
3.1 Motion given by trajectories and envelopes

■ **Exercise 3.1.** Motion is given by trajectory τ^A of point A and trajectory τ^B of point B. Construct at least three new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.



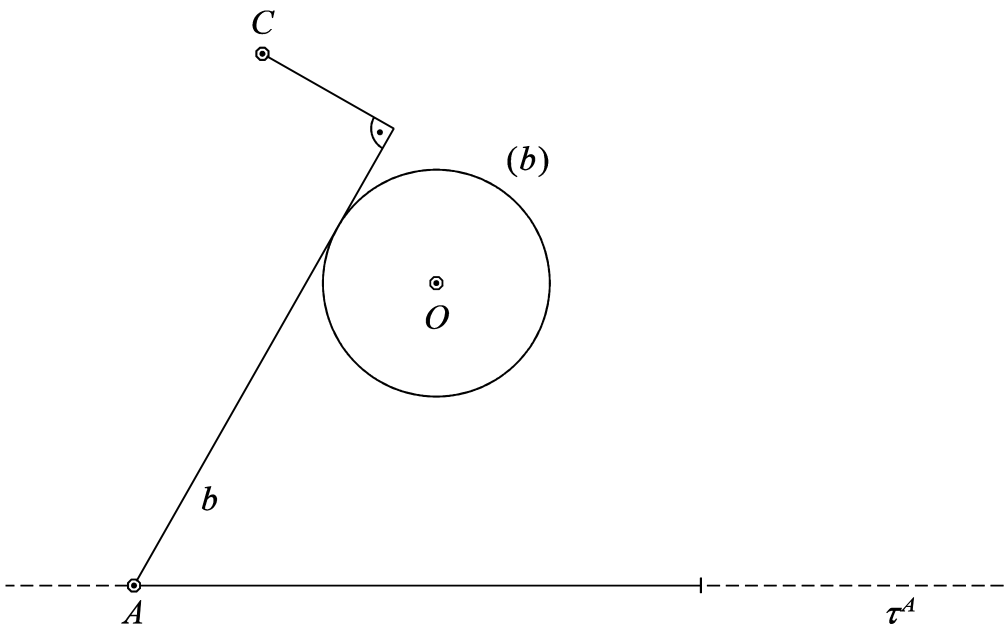
■ **Exercise 3.2.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Construct at least three new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.

Consider the continuous part of trajectory τ^A only.



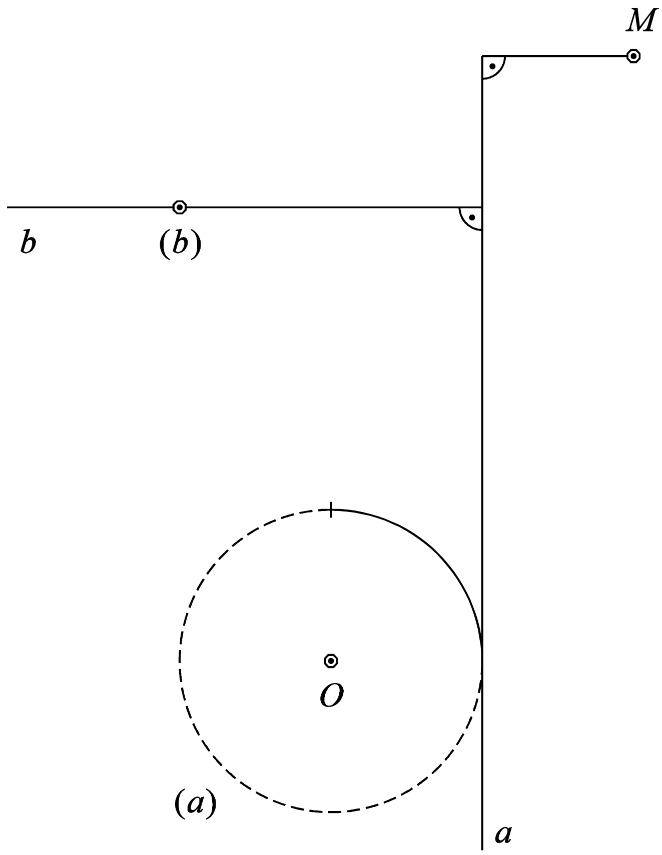
■ **Exercise 3.3.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Construct at least three new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.

Consider the continuous part of trajectory τ^A only.

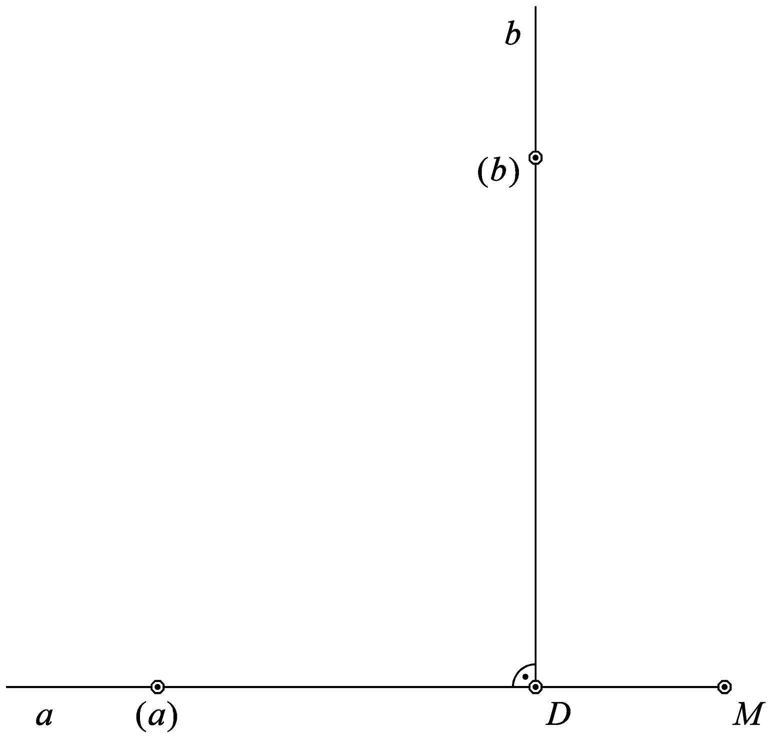


■ **Exercise 3.4.** Motion is given by envelope (a) of straight line a and envelope (b) of straight line b. Construct at least three new positions of point M. Construct tangent lines to the trajectory τ^M at each position. Sketch the part of trajectory τ^M determined by all positions of point M and the corresponding tangent lines.

Consider the continuous part of envelope (a) only.

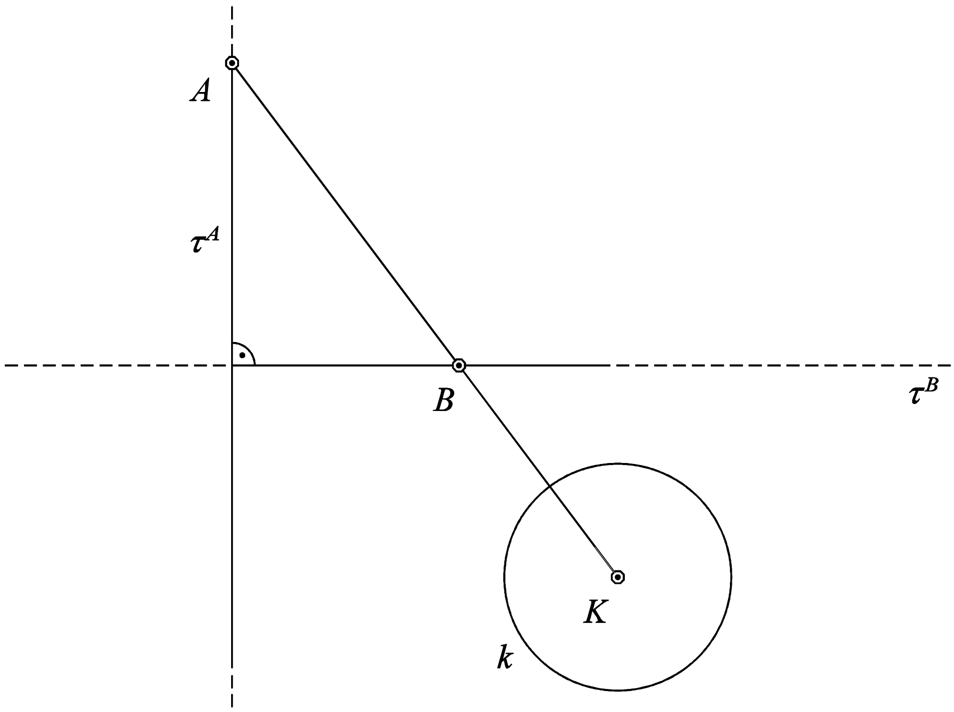


■ **Exercise 3.5.** Motion is given by envelope (a) of straight line a and envelope (b) of straight line b. Construct at least three new positions of point M. Construct tangent lines to the trajectory τ^M at each position. Sketch the part of trajectory τ^M determined by all positions of point M and the corresponding tangent lines.



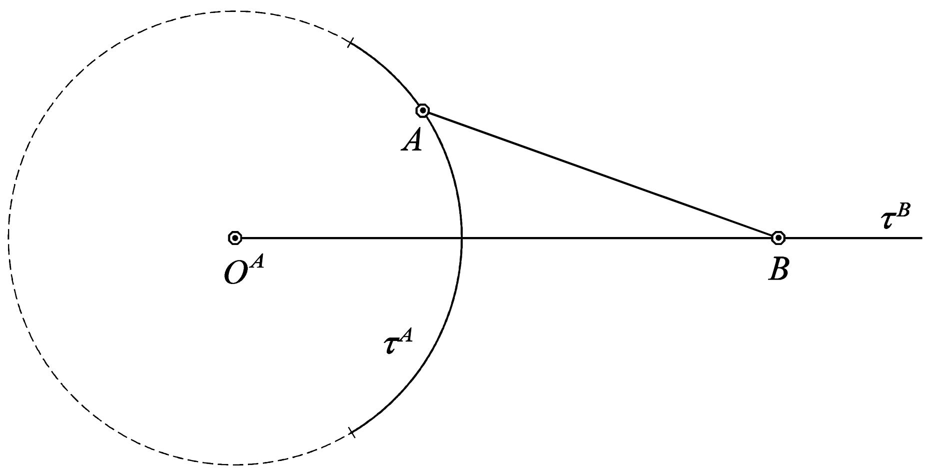
■ **Exercise 3.6.** Motion is given by trajectory τ^A of point A and trajectory τ^B of point B. Construct at least three new positions of circle k. Construct point of contact between the circle and its envelope (k) at each position. Sketch the part of envelope (k) determined by all positions of circle k and the corresponding points of contact.

Consider the continuous parts of both trajectories only.

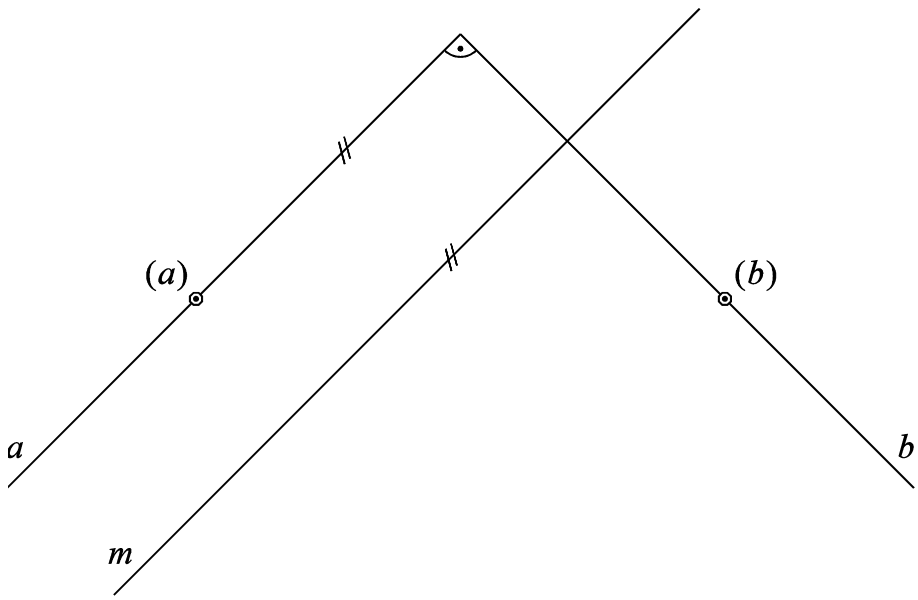


■ **Exercise 3.7.** Motion is given by trajectory τ^A of point A and trajectory τ^B of point B. Construct at least three new positions of straight line AB. Construct point of contact between straight line AB and its envelope (AB) at each position. Sketch the part of envelope (AB) determined by all positions of straight line AB and the corresponding points of contact.

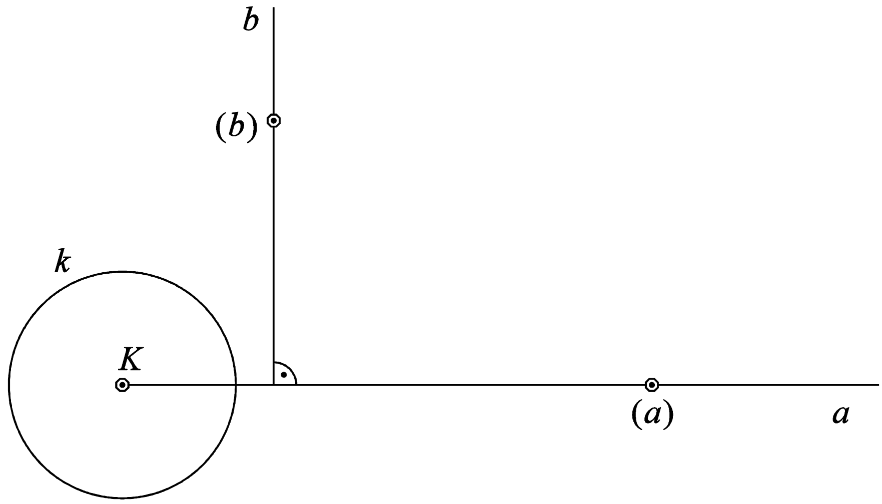
Consider the continuous parts of both trajectories only.



■ **Exercise 3.8.** Motion is given by envelope (a) of straight line a and envelope (b) of straight line b. Construct at least three new positions of straight line m. Construct point of contact between straight line m and its envelope (m) at each position. Sketch the part of envelope (m) determined by all positions of straight line m and the corresponding points of contact.

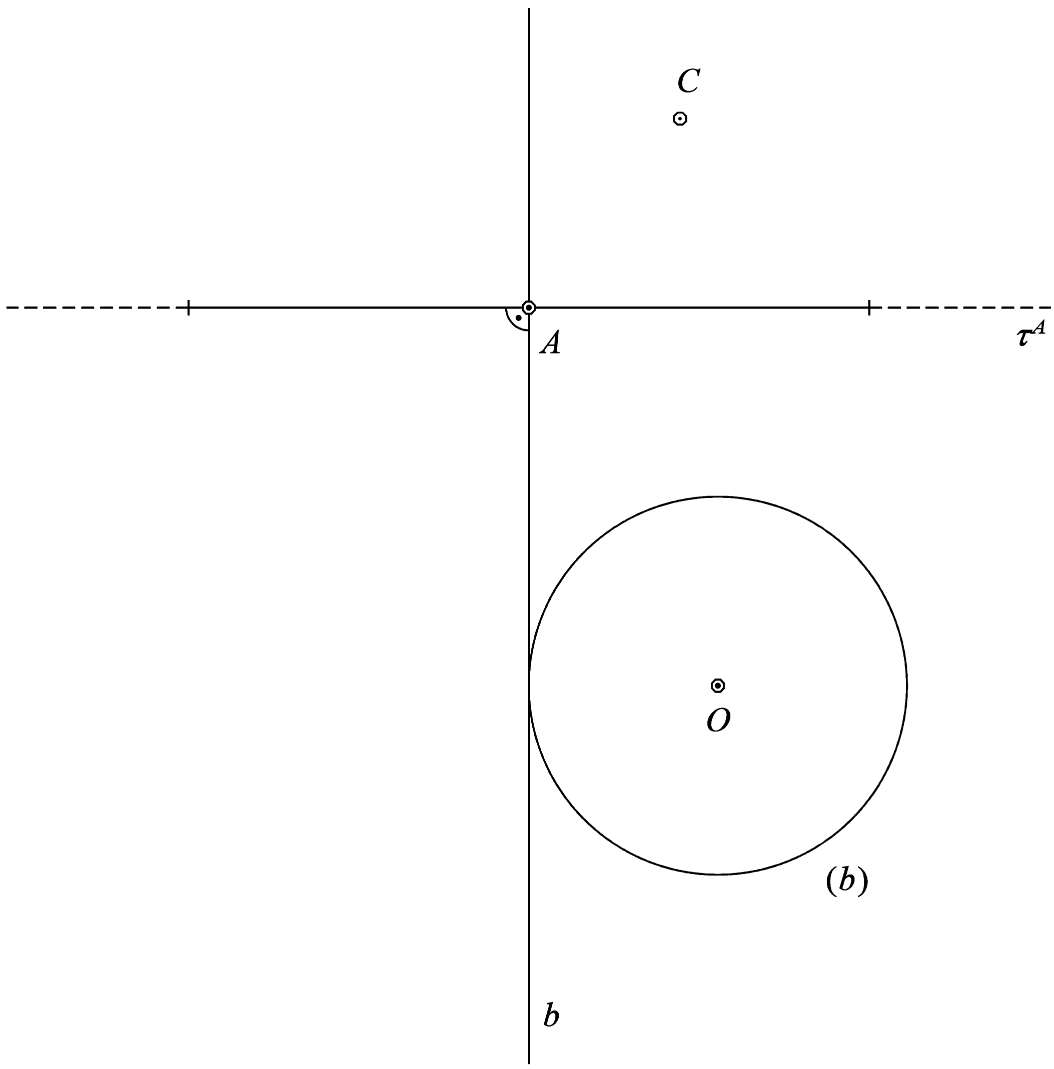


■ **Exercise 3.9.** Motion is given by envelope (a) of straight line a and envelope (b) of straight line b. Construct at least three new positions of circle k. Construct point of contact between circle k and its envelope (k) at each position. Sketch the part of envelope (k) determined by all positions of circle k and the corresponding points of contact.



■ **Exercise 3.10.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Construct at least three new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.

Consider the continuous parts of trajectory τ^A only.

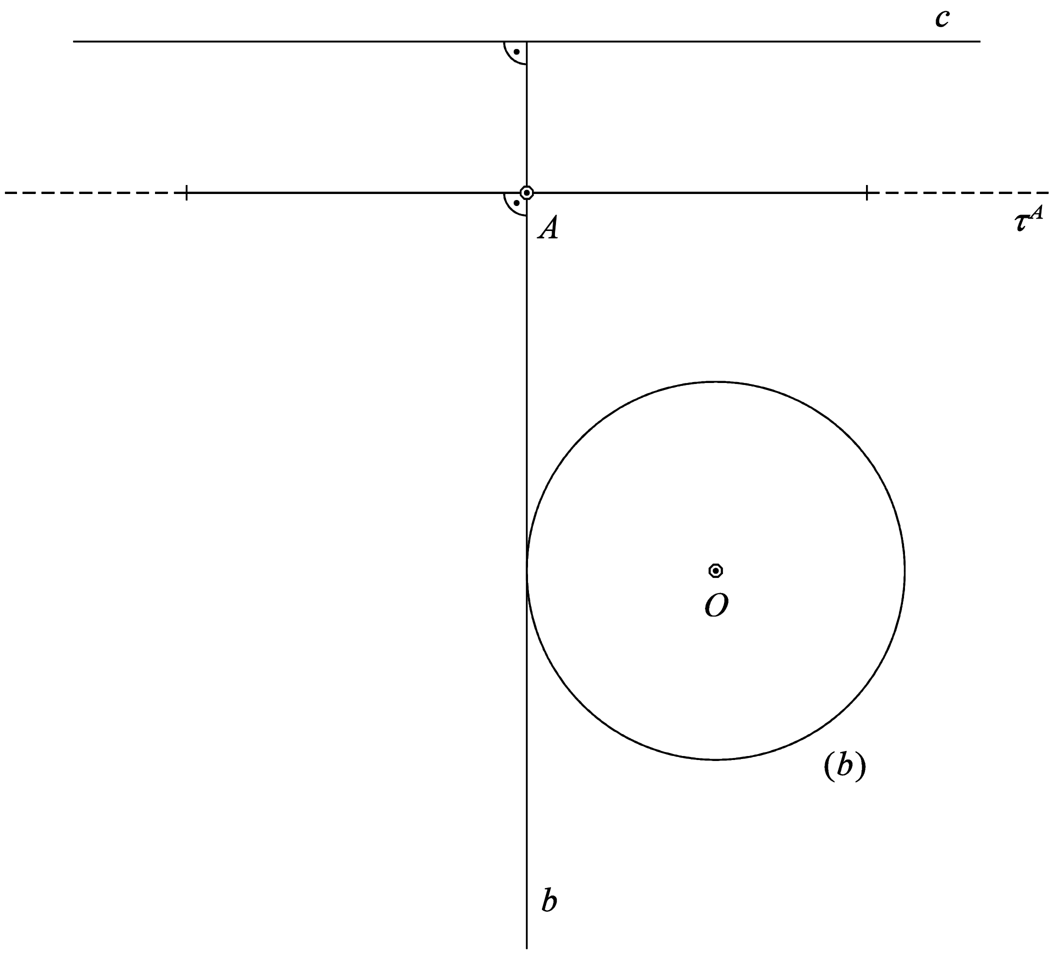


■ **Exercise 3.11.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Construct at least three new positions of straight line c. Construct point of contact between the line and its envelope (c) at each position. Sketch the part of envelope (c) determined by all positions of straight line c and the corresponding points of contact.

Construct the corresponding part of fixed centrode p.

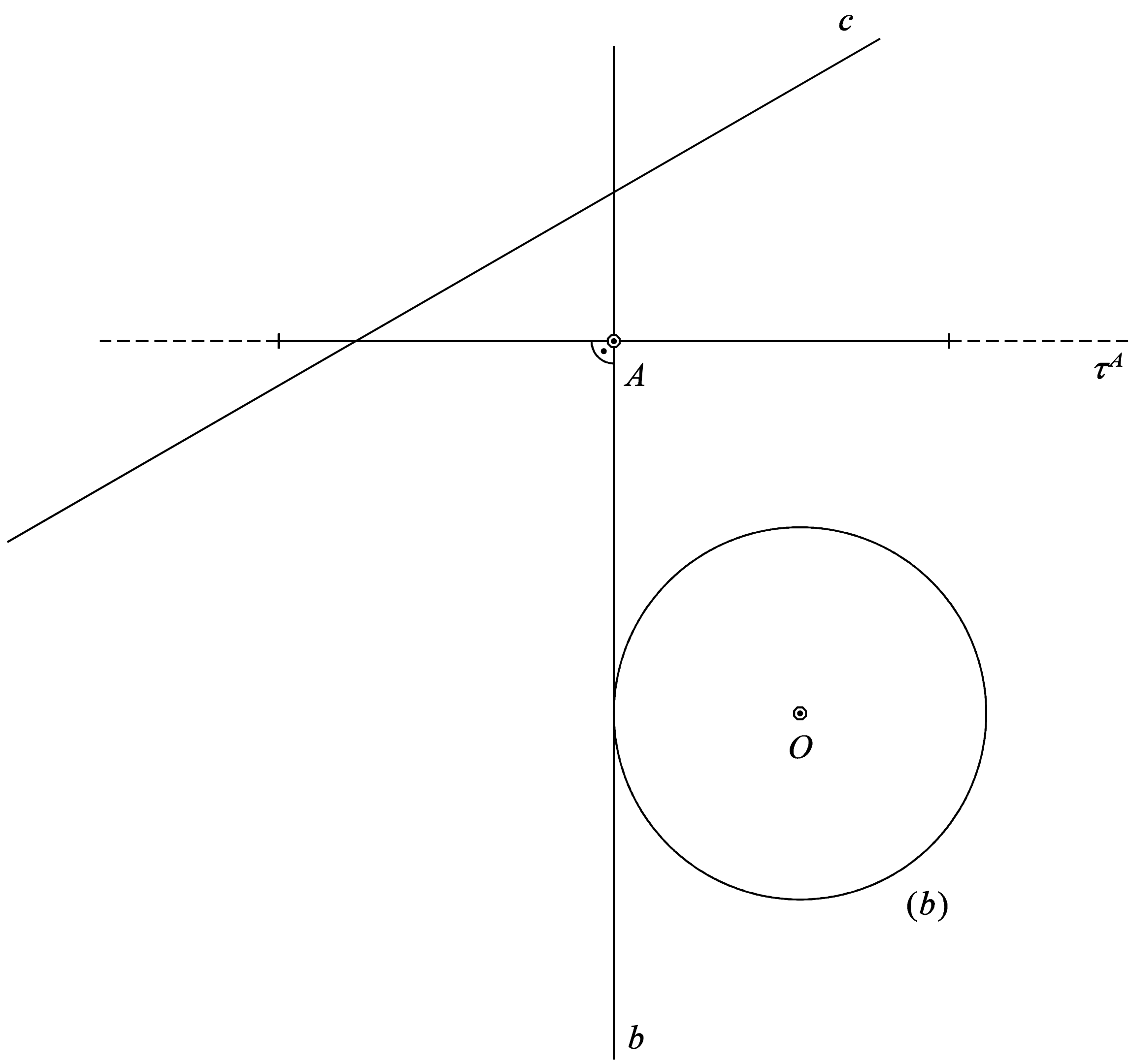
Construct the corresponding part of moving centrode h^0 at the given instant.

Consider the continuous parts of trajectory τ^A only.



■ **Exercise 3.12.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Construct at least three new positions of straight line c. Construct point of contact between the line and its envelope (c) at each position. Sketch the part of envelope (c) determined by all positions of straight line c and the corresponding points of contact.

Consider the continuous parts of trajectory τ^A only.

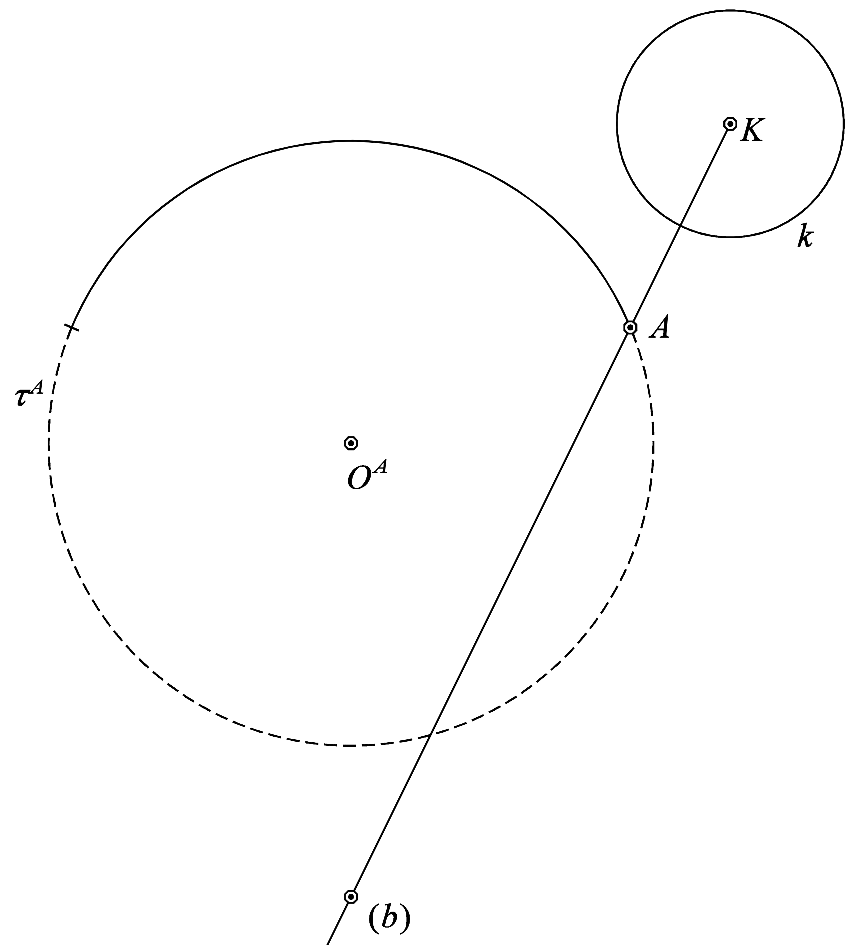


■ **Exercise 3.13.** Motion is given by trajectory τ^A of point A and envelope (b) of straight line b. Consturct at least three new positions of circle k. Construct point of contact between circle k and its envelope (k) at each position. Sketch the part of envelope (k) determined by all positions of circle k and the corresponding points of contact.

Construct the corresponding part of fixed centrode p

Construct the corresponding part of moving centrode h^0 at the given instant.

Consider the continuous part of trajectory τ^A only.

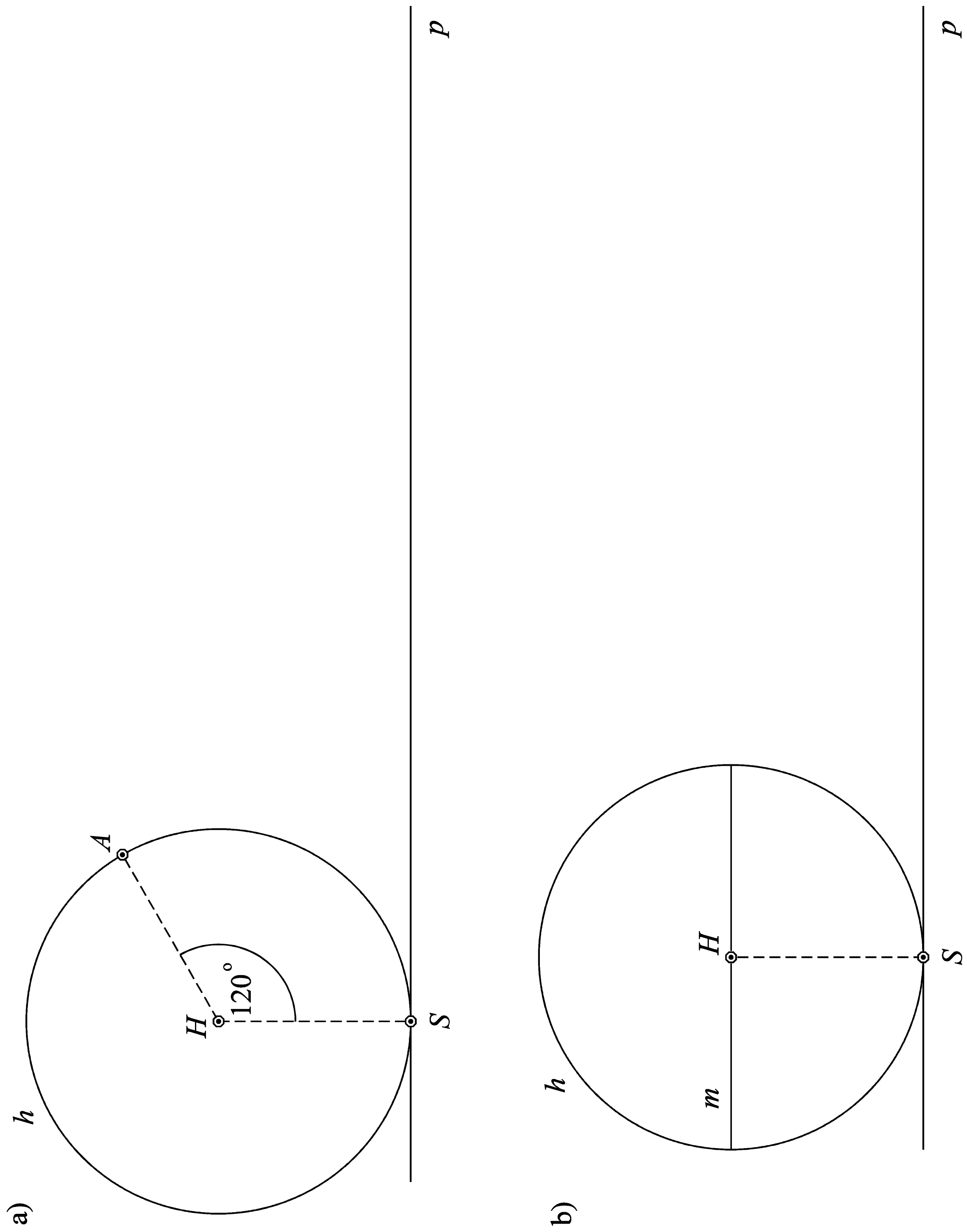


3.2 Cyclic motion

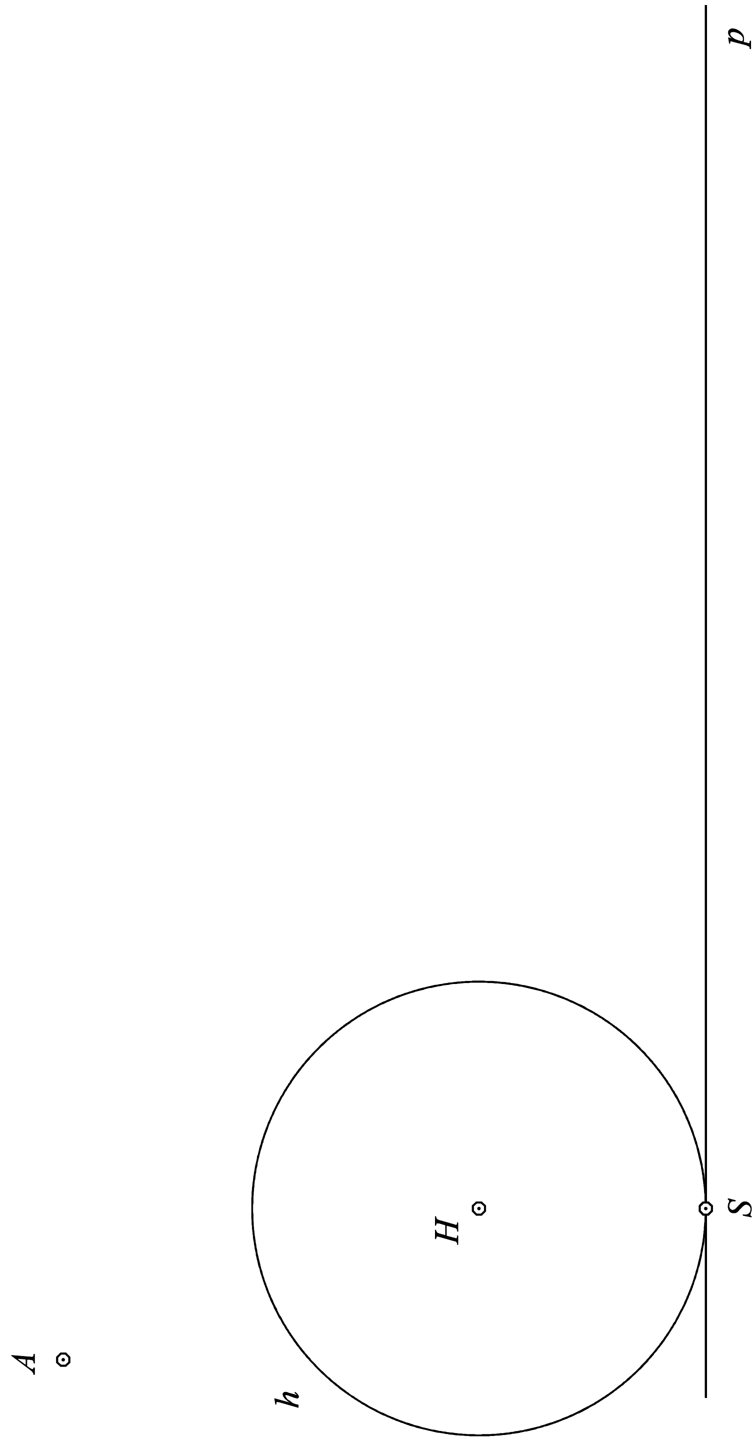
■ **Exercise 3.14.** Motion is given by fixed centrode p and moving centrode h.

a) Construct a sufficient number of new positions of point A. Construct tangent lines to the trajectory τ^A at each position. Sketch the part of trajectory τ^A determined by all positions of point A and the corresponding tangent lines.

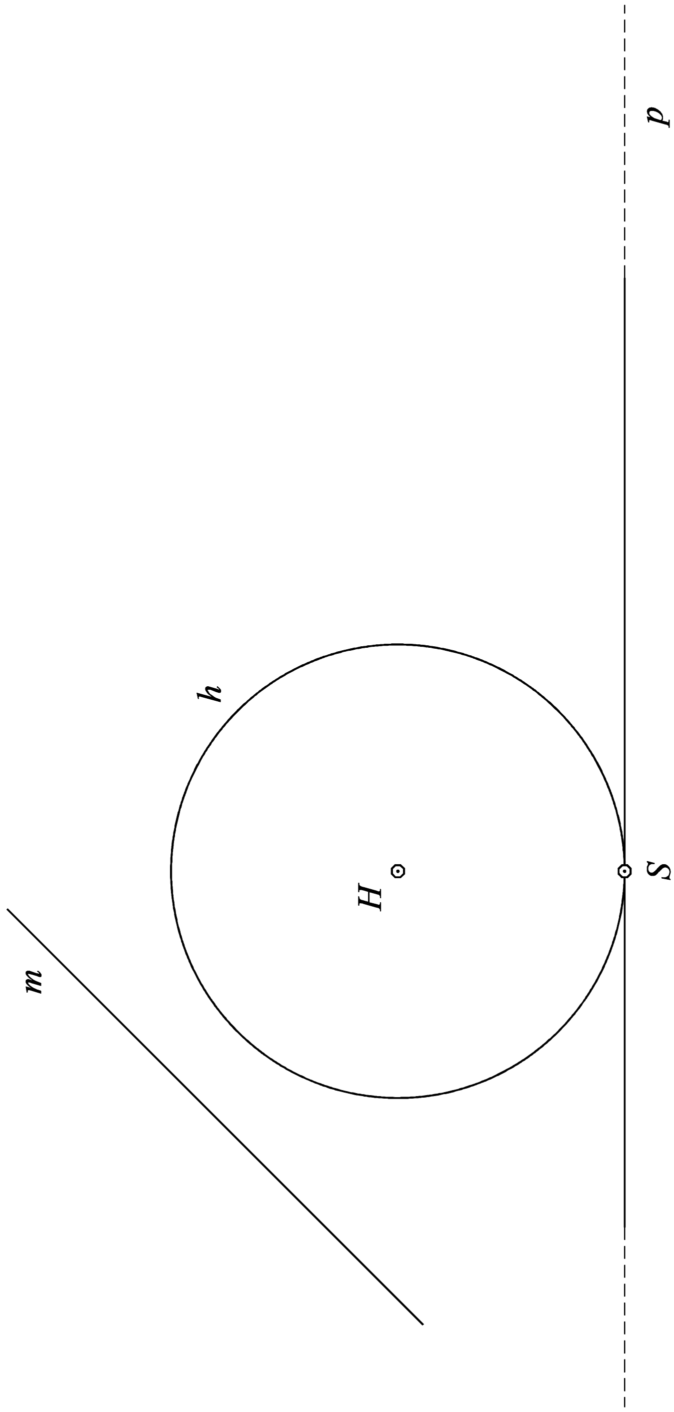
b) Construct point of contact between line m and its envelope (m) at each position. Sketch the part of envelope (m) determined by all positions of straight line m and the corresponding points of contact.



■ **Exercise 3.15.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point A. Construct tangent lines to the trajectory τ^A at each position. Sketch the part of trajectory τ^A determined by all positions of point A and the corresponding tangent lines.

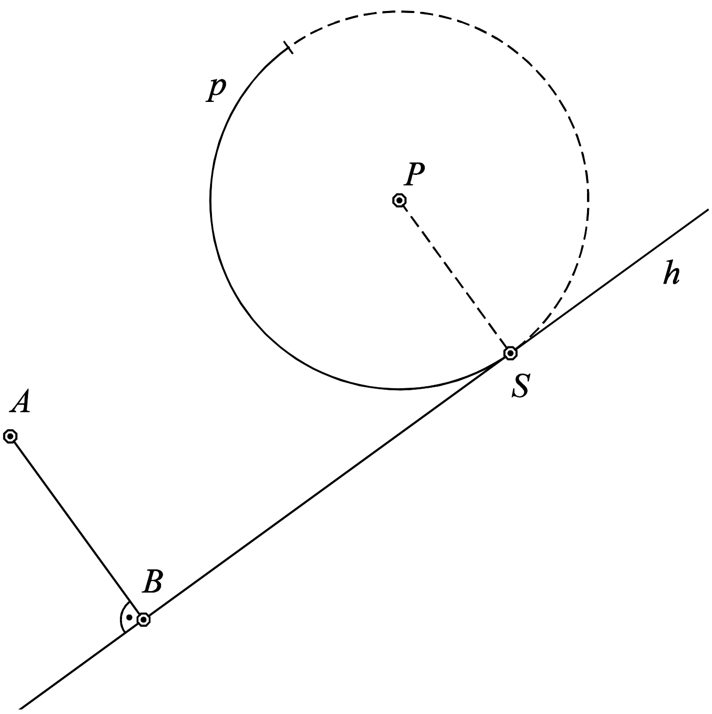


■ **Exercise 3.16.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of straight line m. Construct point of contact between line m and its envelope (m) at each position. Sketch the part of envelope (m) determined by all positions of straight line m and the corresponding points of contact. Consider the continuous part of fixed centrode p only.



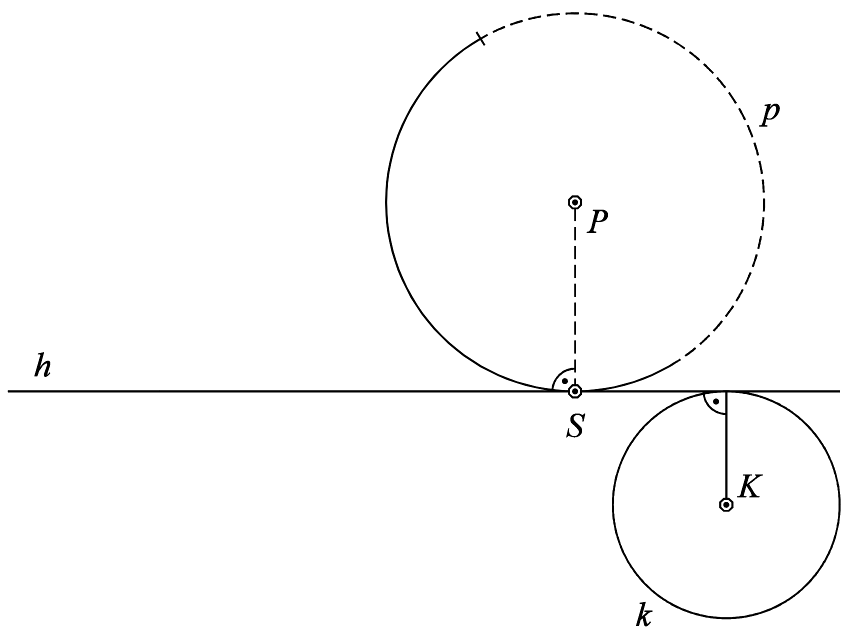
■ **Exercise 3.17.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of points A and B. Construct tangent lines to the trajectories τ^A and τ^B at each position. Sketch the part of trajectories τ^A and τ^B determined by all positions of points A and B and the corresponding tangent lines.

Consider the continuous part of fixed centrode p only.



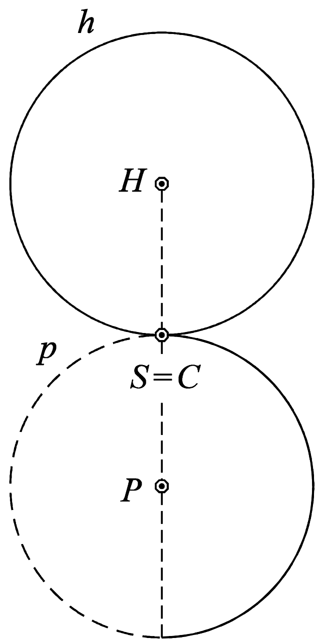
■ **Exercise 3.18.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of circle k. Construct point of contact between the circle and its envelope (k) at each position. Sketch the part of envelope (k) determined by all positions of circle k and the corresponding points of contact.

Consider the continuous part of fixed centrode p only.



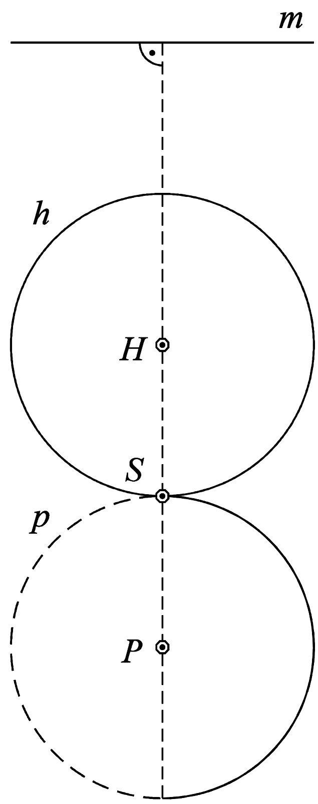
■ **Exercise 3.19.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of points C and the corresponding tangent lines.

Consider the continuous part of fixed centrode p only.

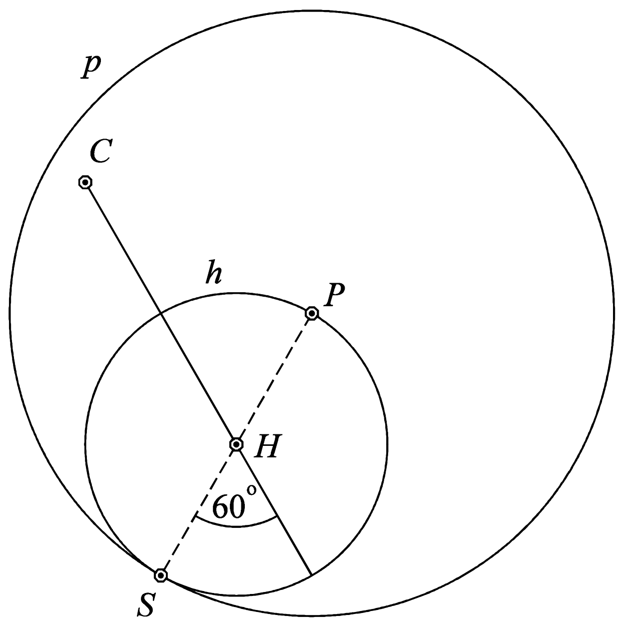


■ **Exercise 3.20.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of straight line m. Construct point of contact between the line and its envelope (m) at each position. Sketch the part of envelope (m) determined by all positions of straight line m and the corresponding points of contact.

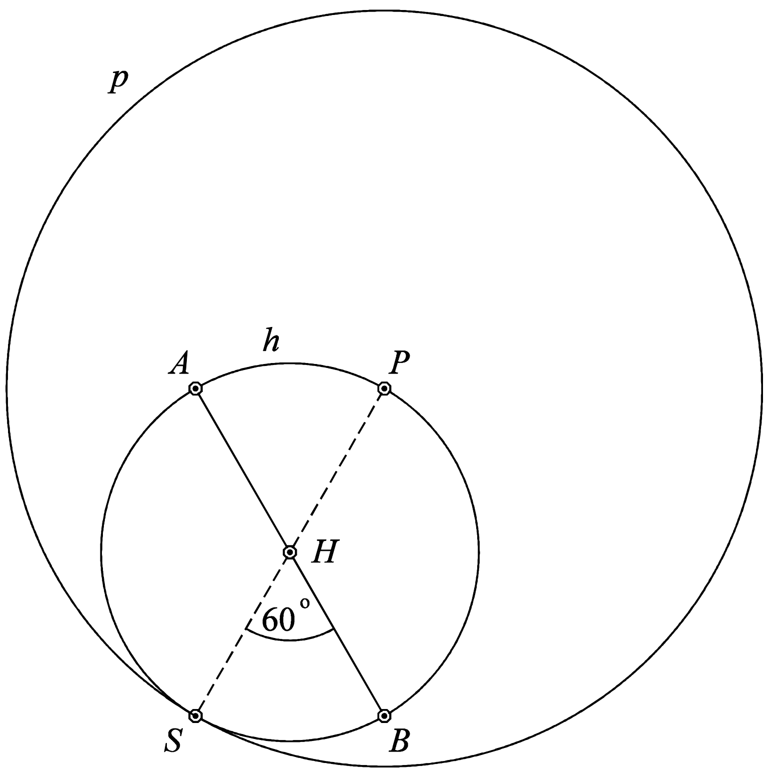
Consider the continuous part of fixed centrode p only.



■ **Exercise 3.21.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of points C and the corresponding tangent lines.

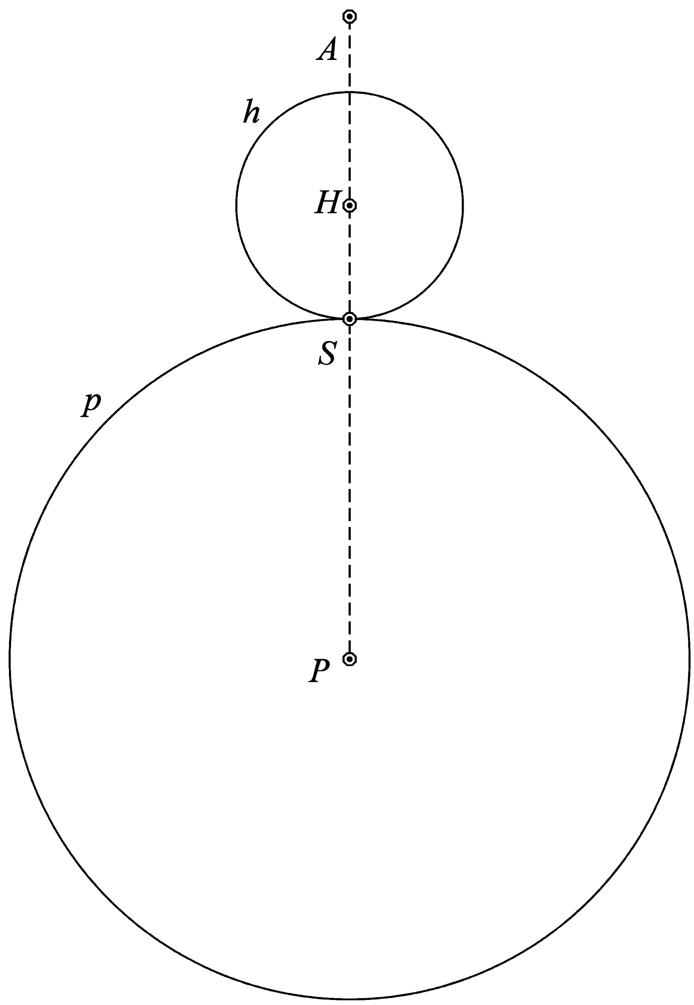


■ **Exercise 3.22.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of straight line AB. Construct point of contact between the line and its envelope (AB) at each position. Sketch the part of envelope (AB) determined by all positions of straight line AB and the corresponding points of contact.

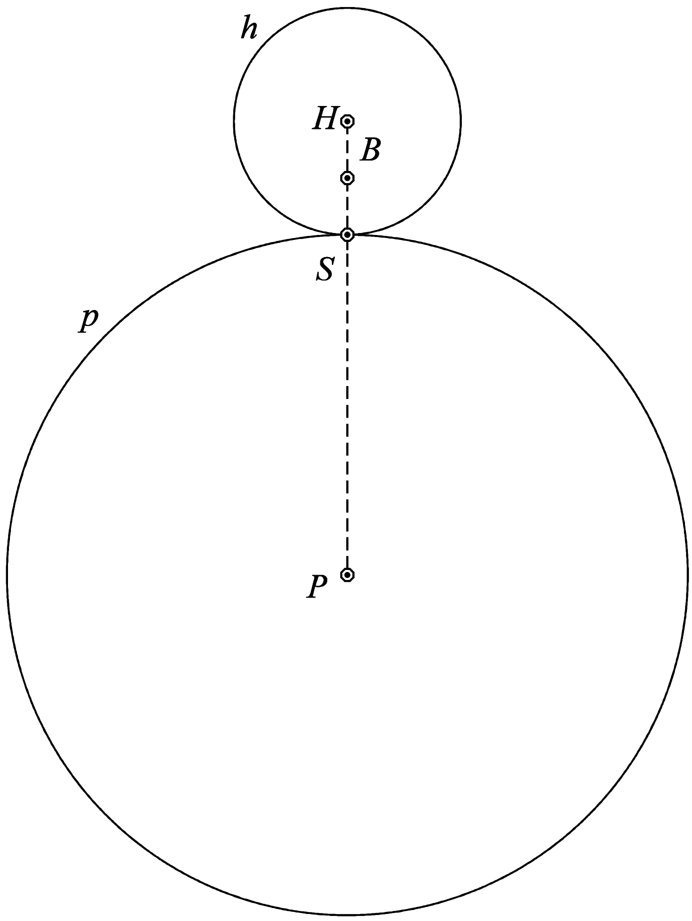


■ **Exercise 3.23.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point A. Construct tangent lines to the trajectory τ^A at each position. Sketch the part of trajectory τ^A determined by all positions of point A and the corresponding tangent lines.

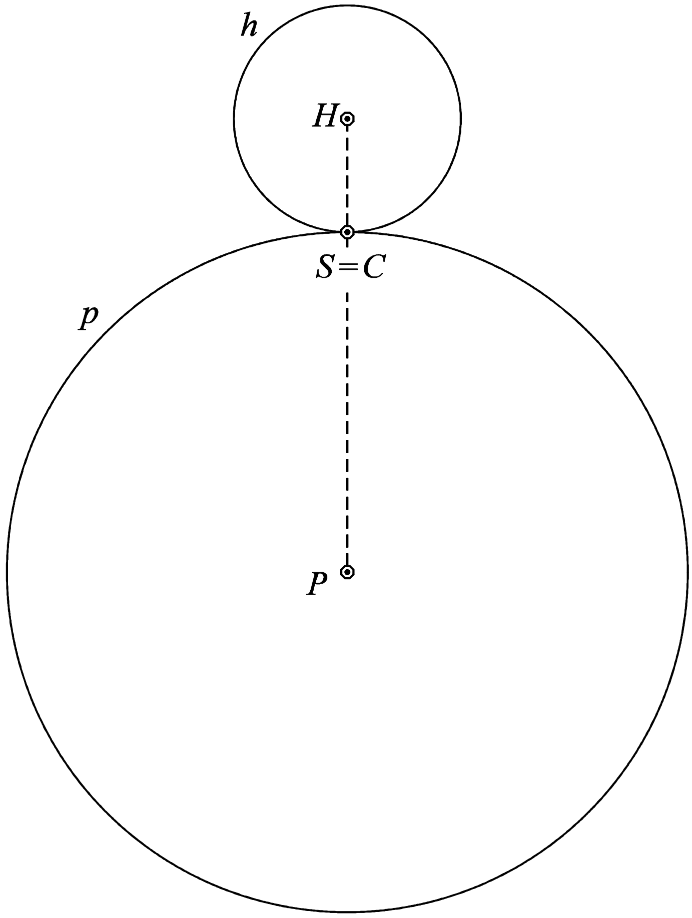
Consider the continuous part of fixed centrode p only.



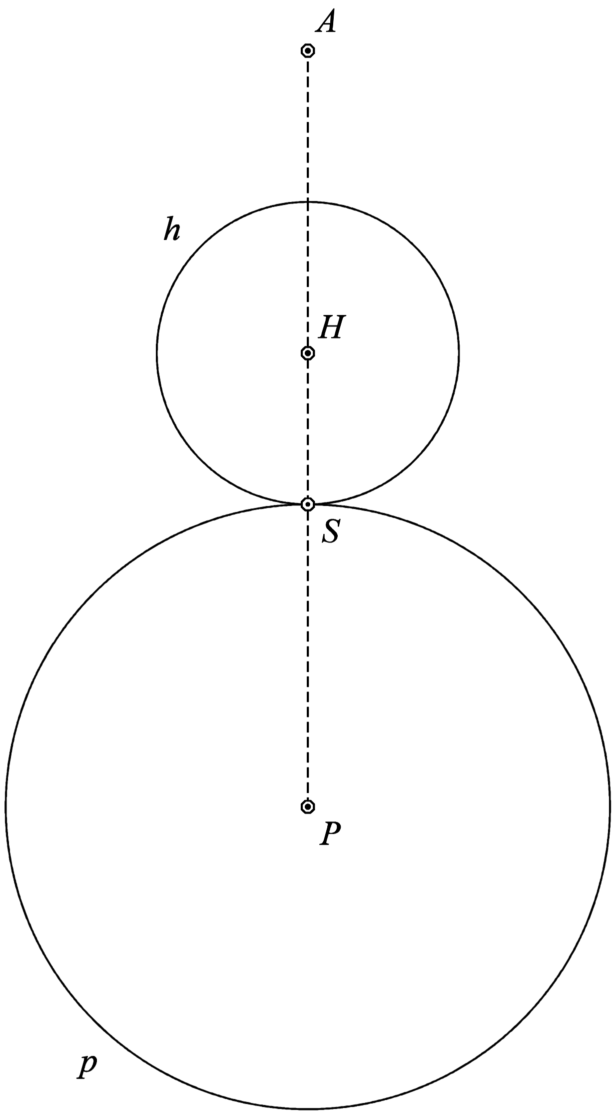
■ **Exercise 3.24.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point B. Construct tangent lines to the trajectory τ^B at each position. Sketch the part of trajectory τ^B determined by all positions of point B and the corresponding tangent lines.



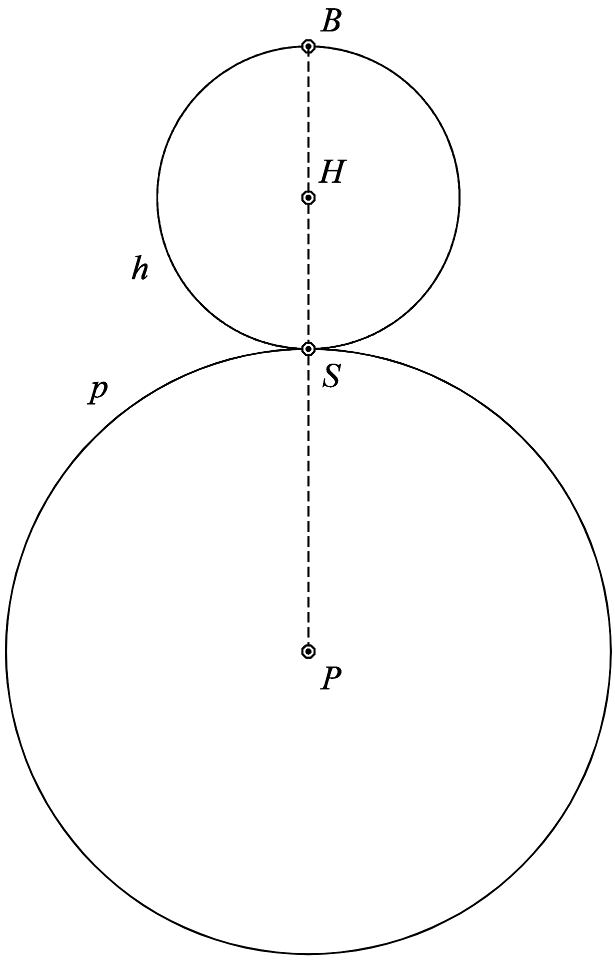
■ **Exercise 3.25.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.



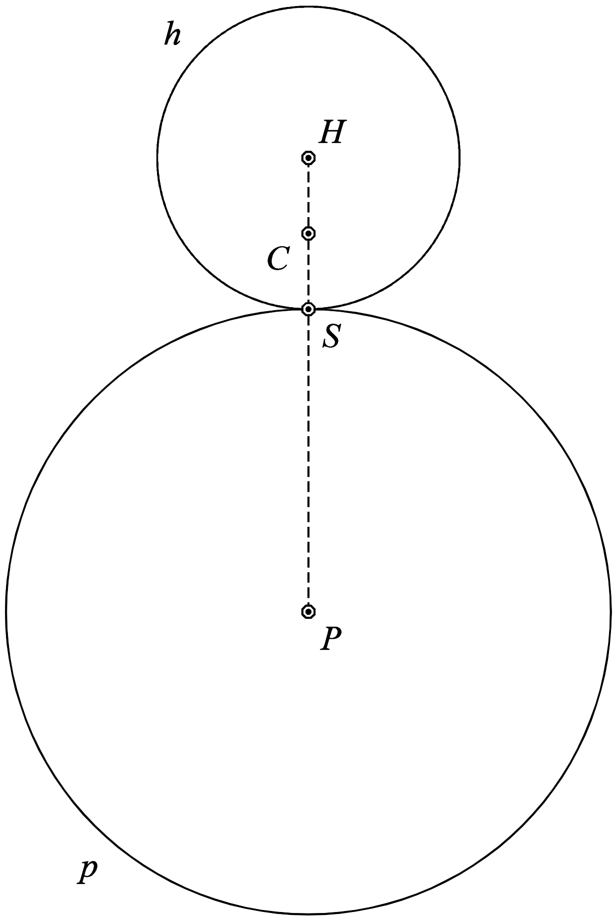
■ **Exercise 3.26.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point A. Construct tangent lines to the trajectory τ^A at each position. Sketch the part of trajectory τ^A determined by all positions of point A and the corresponding tangent lines.



■ **Exercise 3.27.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point B. Construct tangent lines to the trajectory τ^B at each position. Sketch the part of trajectory τ^B determined by all positions of point B and the corresponding tangent lines.

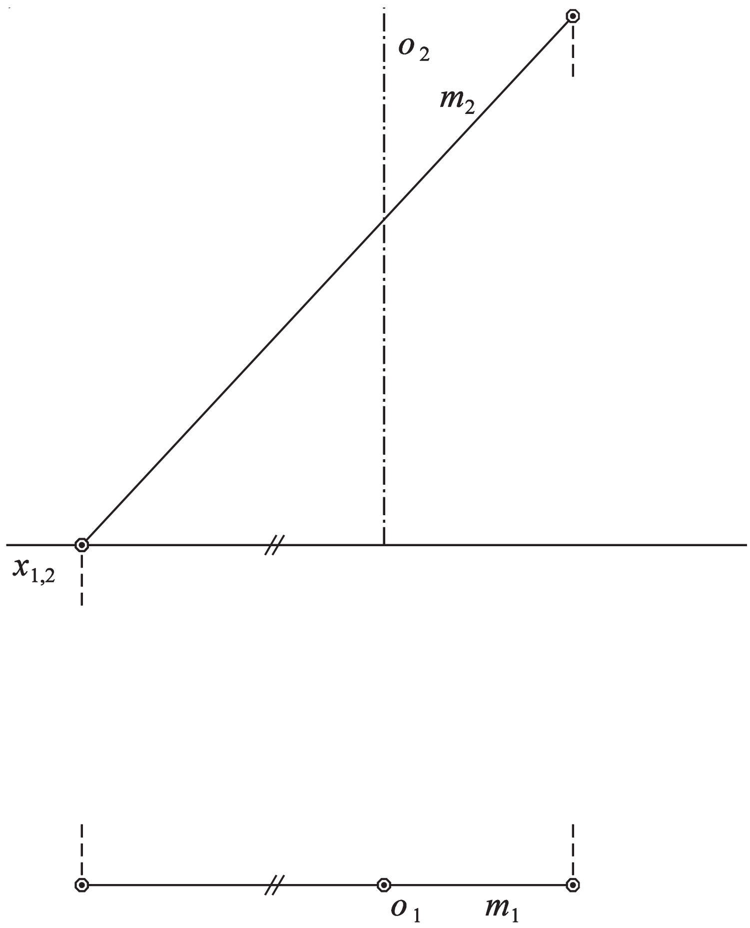


■ **Exercise 3.28.** Motion is given by fixed centrode p and moving centrode h. Construct a sufficient number of new positions of point C. Construct tangent lines to the trajectory τ^C at each position. Sketch the part of trajectory τ^C determined by all positions of point C and the corresponding tangent lines.

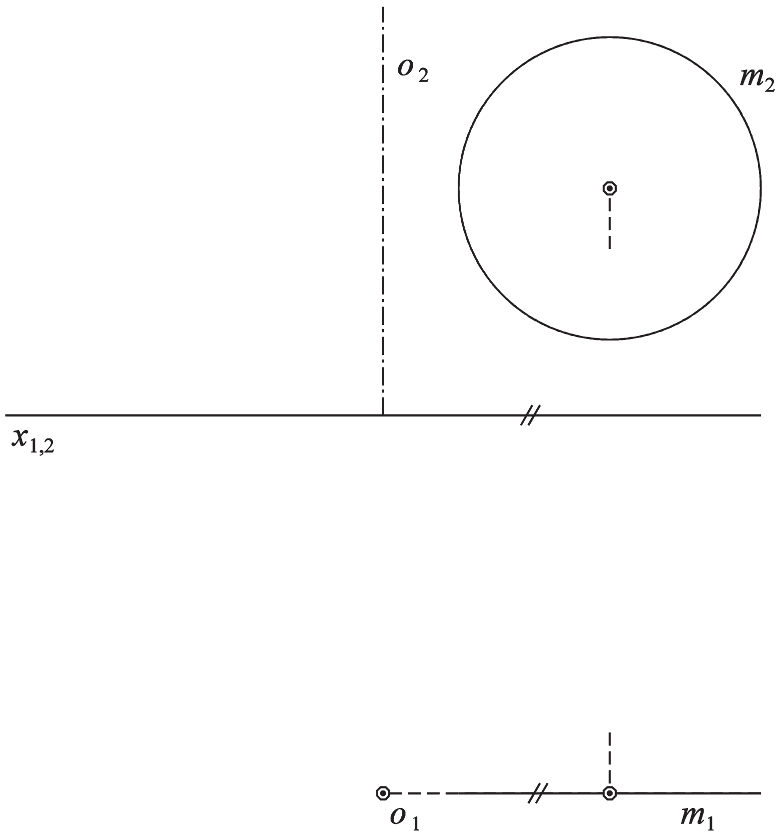


4 Surfaces of revolution

■ **Exercise 4.1.** Surface of revolution (axis o, principal half-meridian m) is given. Using Monge projection, construct the top view and the front view of the surface.



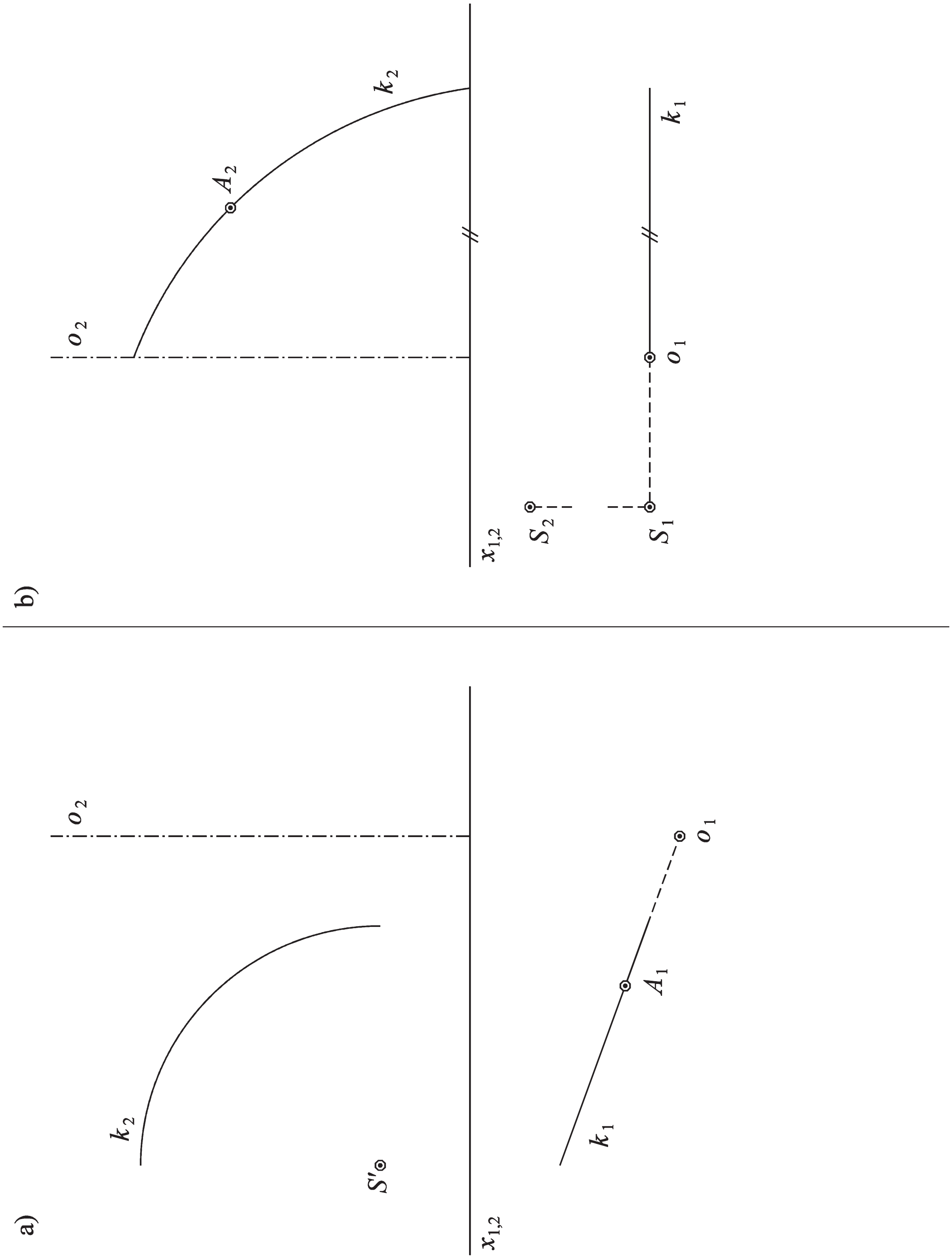
■ **Exercise 4.2.** Surface of revolution (axis o, principal half-meridian m) is given. Using Monge projection, construct the top view and the front view of the surface.



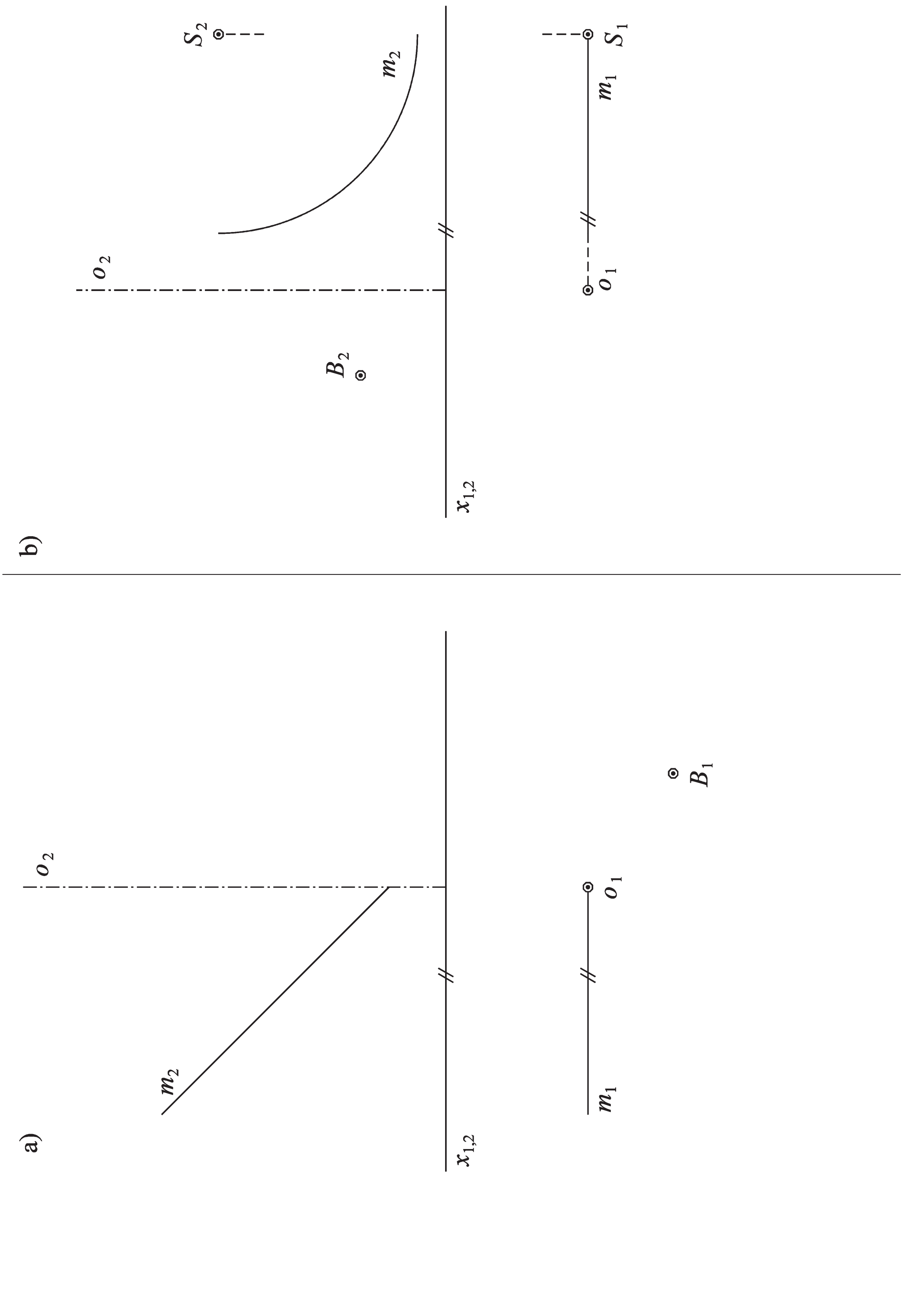
■ **Exercise 4.3.** Surface of revolution (axis o, generating curve k) is given. Using Monge projection,

a) construct tangent plane τ at point $A \in k$,

b) construct tangent plane τ and normal line n at point $A \in k$.

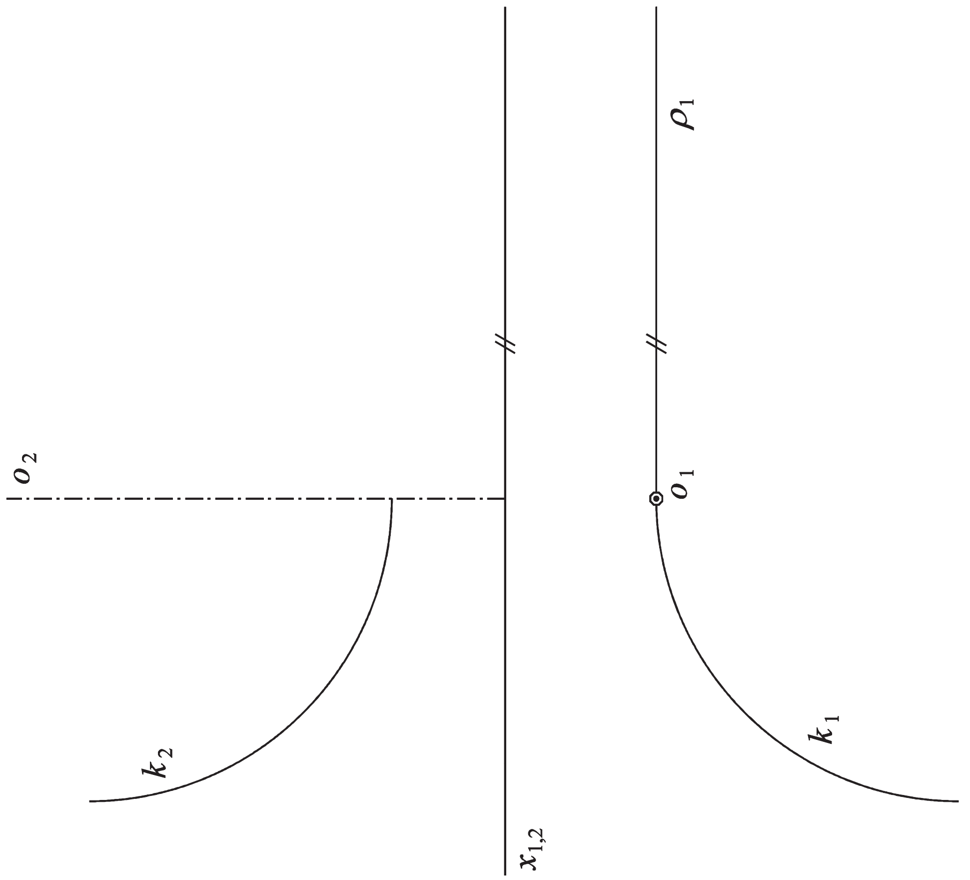


■ **Exercise 4.4.** Surface of revolution (axis o, principal half-meridian m) is given. Using Monge projection, construct a missing view of point B. Construct normal line n at point B.

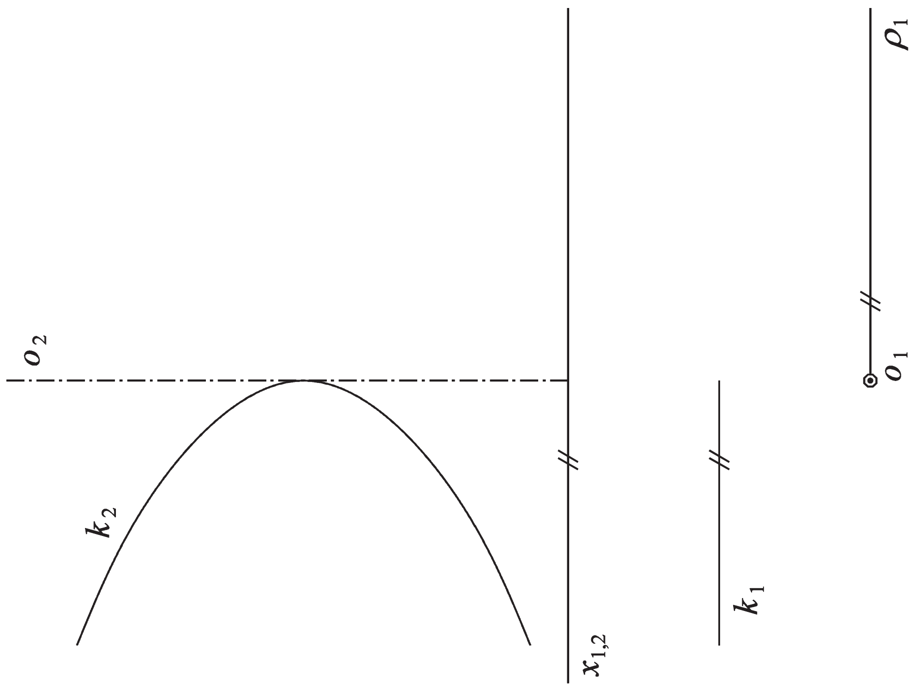


■ **Exercise 4.5.** Surface of revolution (axis o, generating curve k) is given. Using Monge projection, construct its principal half-meridian m in the given half-plane ρ.

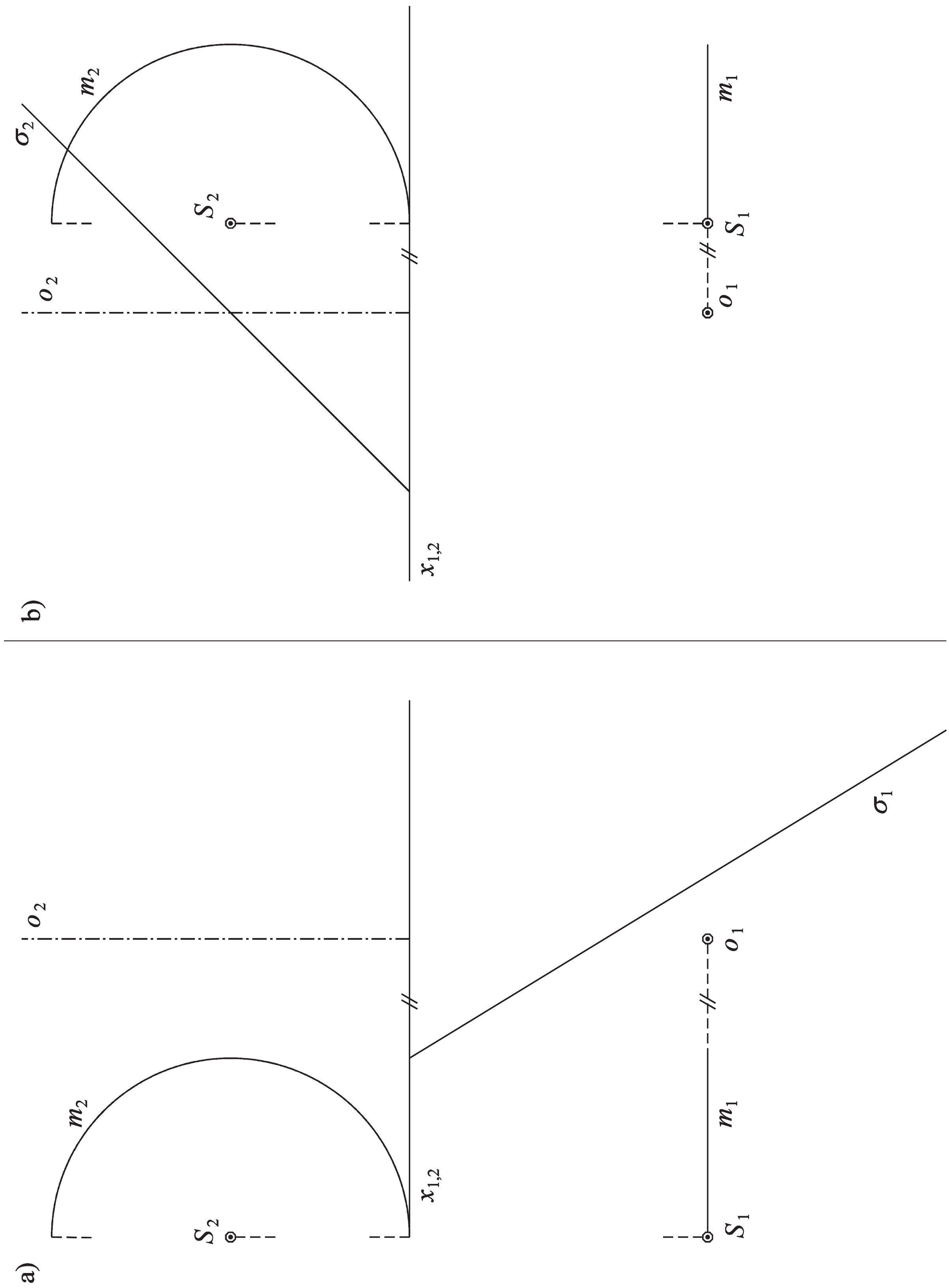
a)



b)



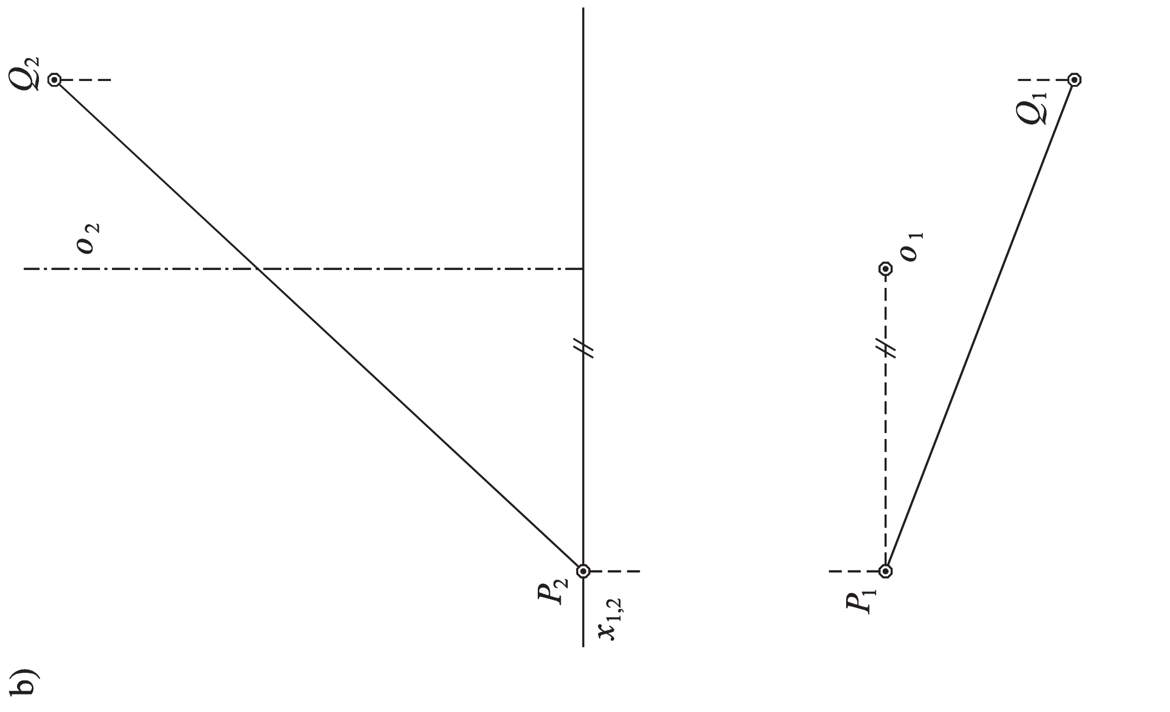
■ **Exercise 4.6.** Surface of revolution (axis o, principal half-meridian m) is given. Using Monge projection, construct intersection curve p of the surface and the given plane σ. Construct normal line n at point $M \in p$, $z_M = 10$.

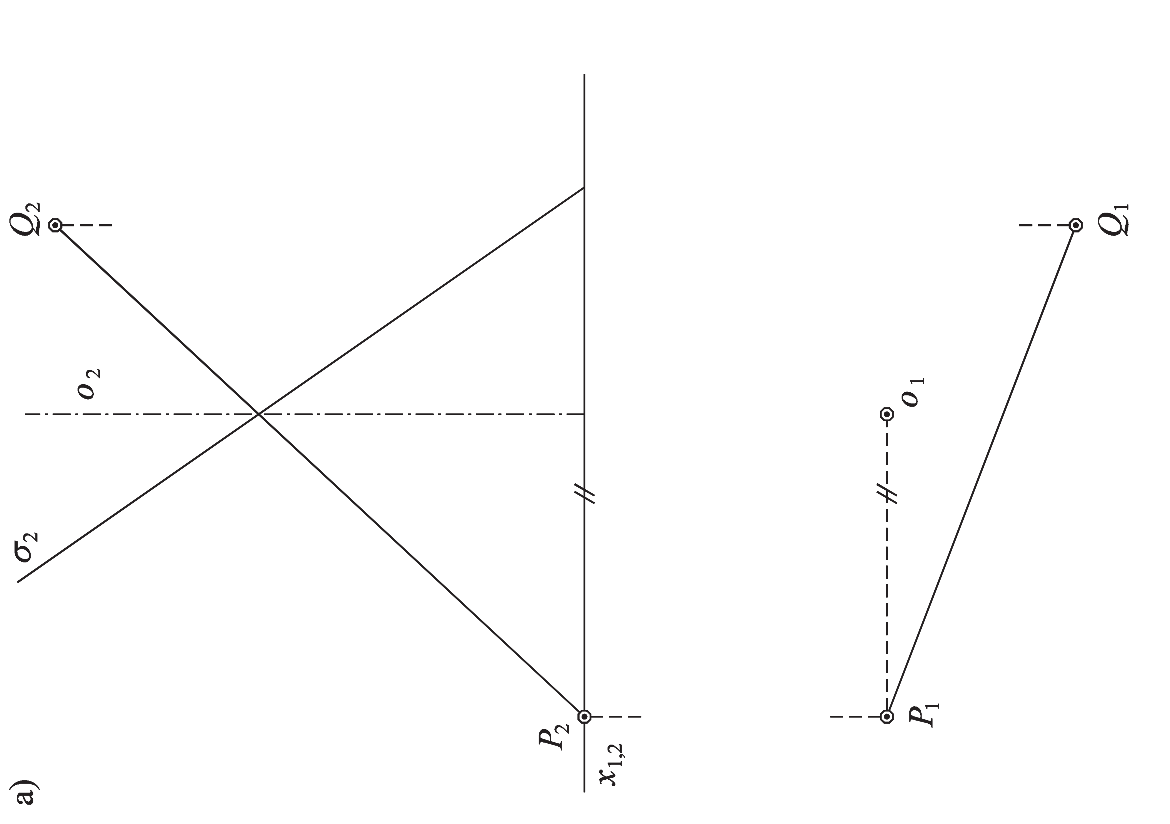


■ **Exercise 4.7.** Surface of revolution (axis o, generating straight line segment PQ) is given. Using Monge projection

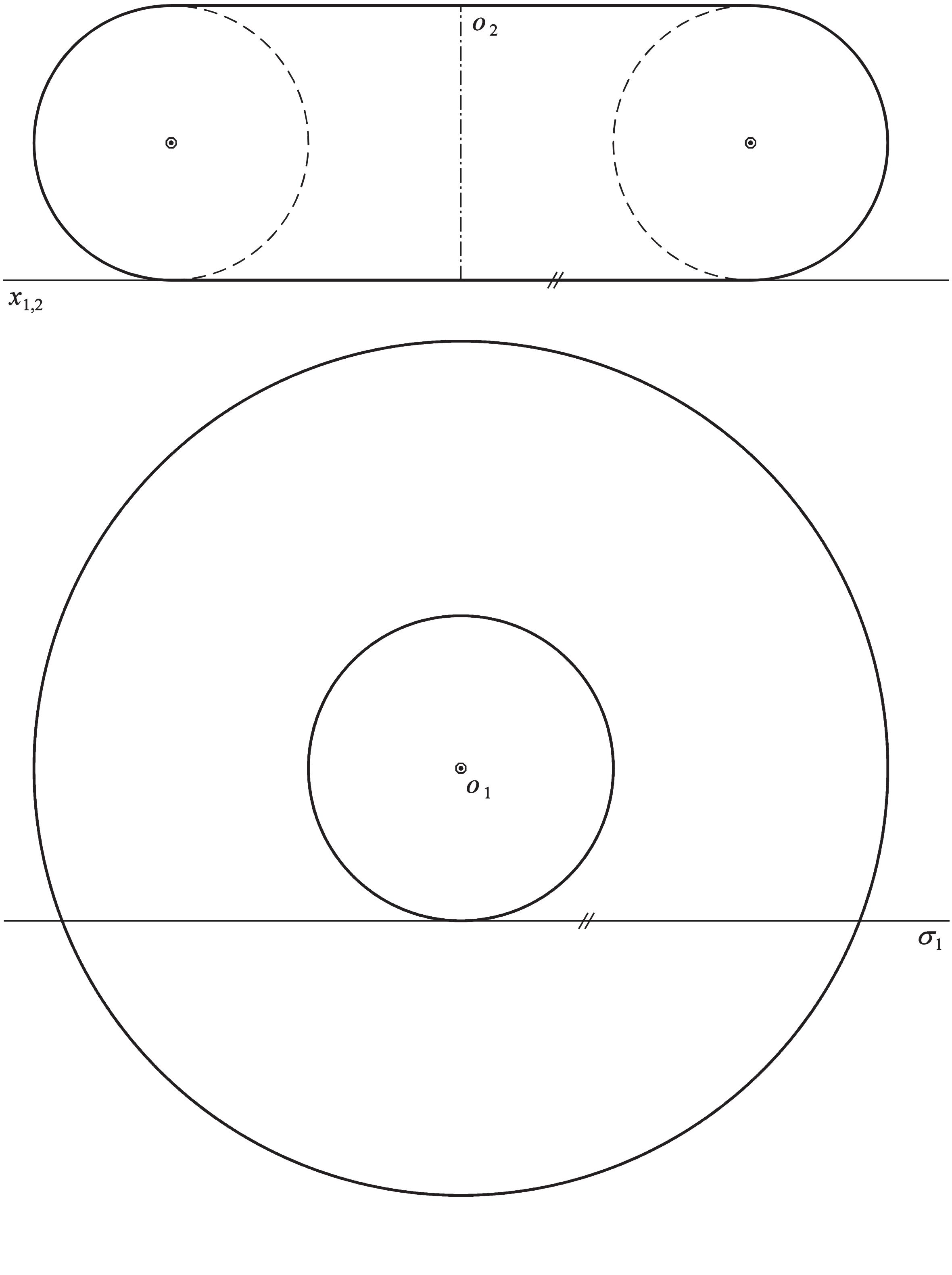
a) construct intersection curve p of the surface of revolution and the given plane σ,

b) construct the top view and the front view of the surface, write the name of the surface and its equation.

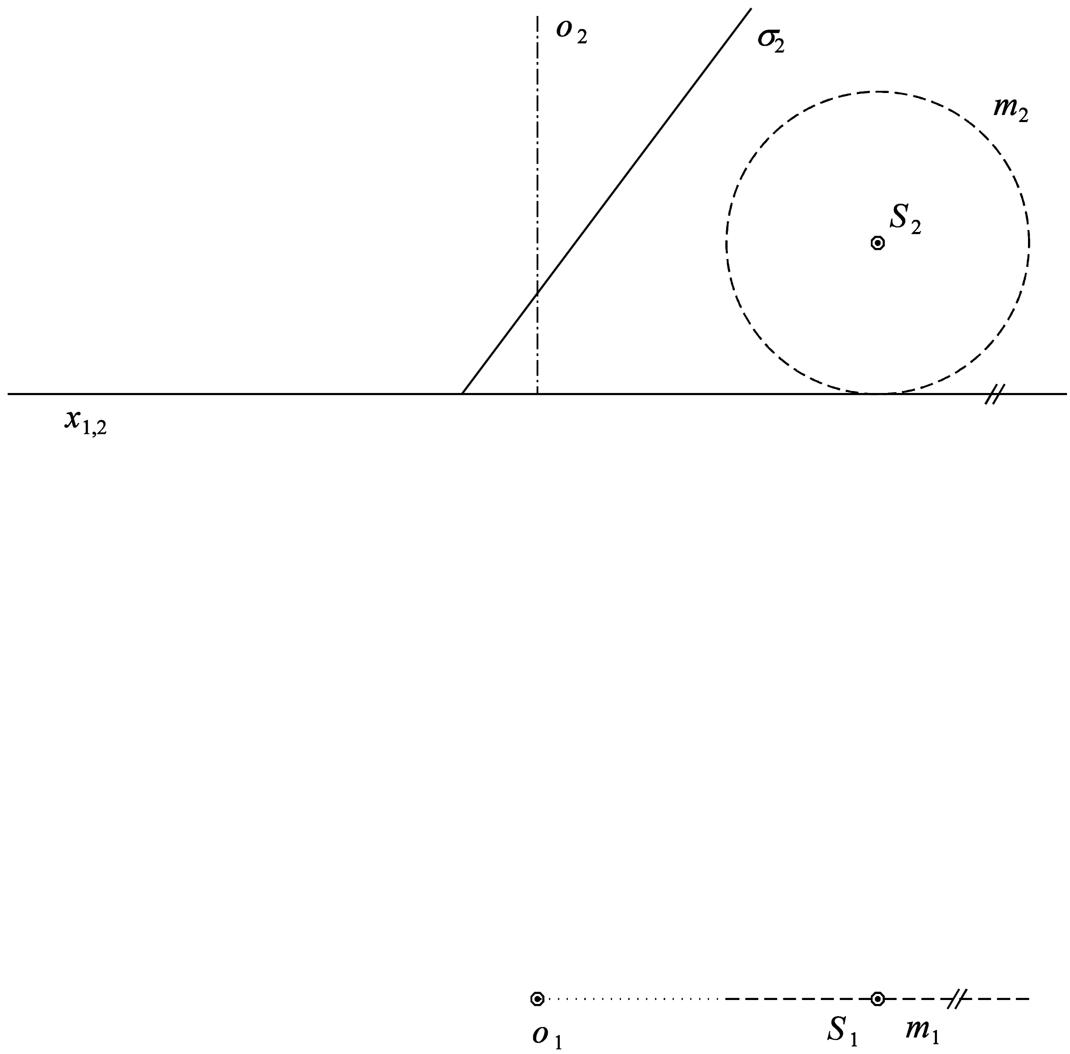




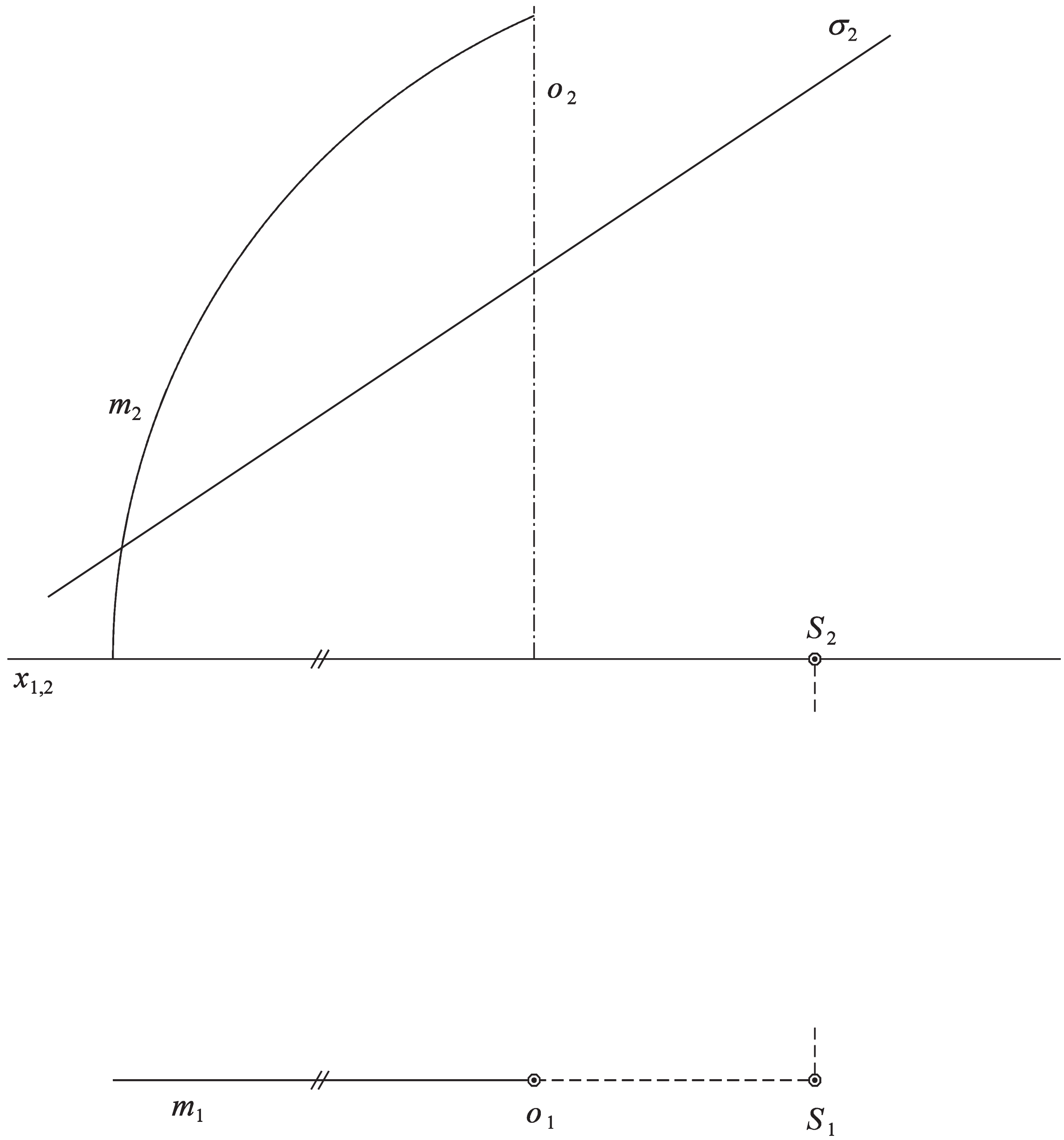
■ **Exercise 4.8.** Using Monge projection, construct intersection curve p of the torus and the given plane σ. Indicate the visibility. Construct normal line n at point of intersection $M \in p$, $z_M = 35$.



■ **Exercise 4.9.** Using Monge projection, construct intersection curve p of the surface of revolution (axis o, principal half-meridian m - circle with centre at point S) and the given plane σ. Indicate the visibility. Write the name of the surface of revolution.

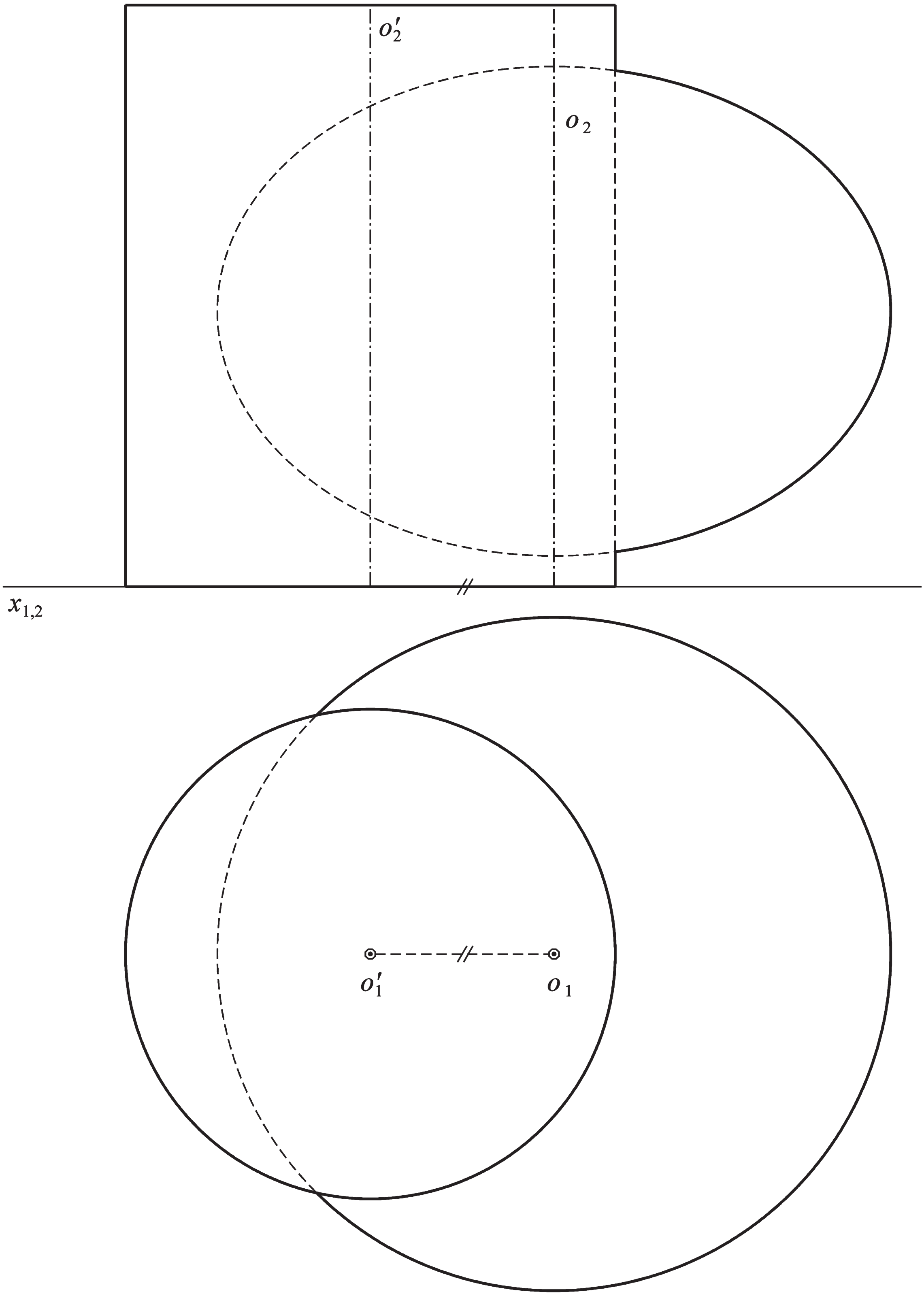


■ **Exercise 4.10.** Surface of revolution (axis o, principal half-meridian m - circular arc with centre at point S) is given. Using Monge projection, construct the top view and the front view of the surface. Construct intersection curve p of the surface of revolution and the given plane σ. Construct normal line n at point of intersection $M \in p$, $z_M = 40$.

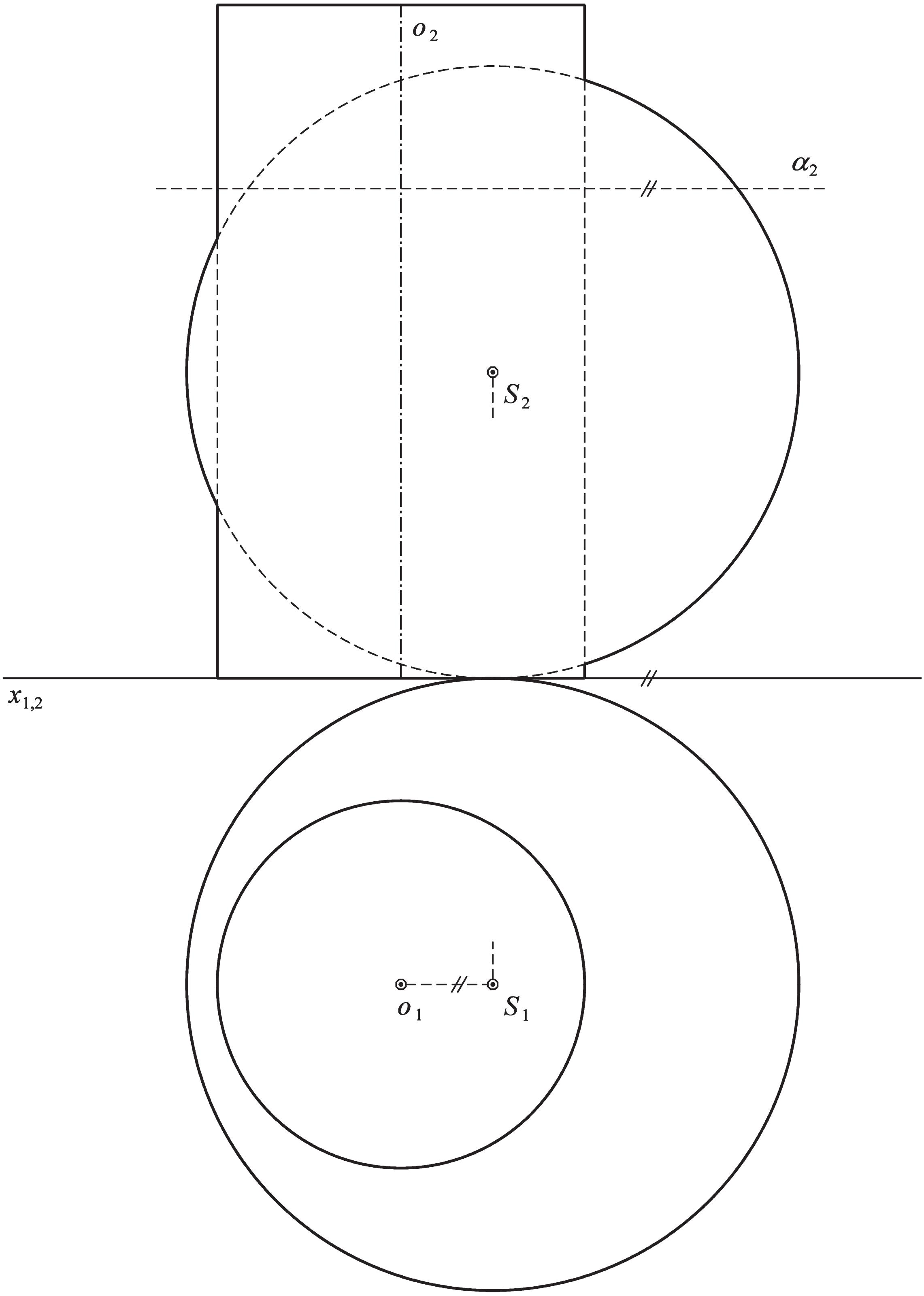


4.1 Intersection of surfaces of revolution

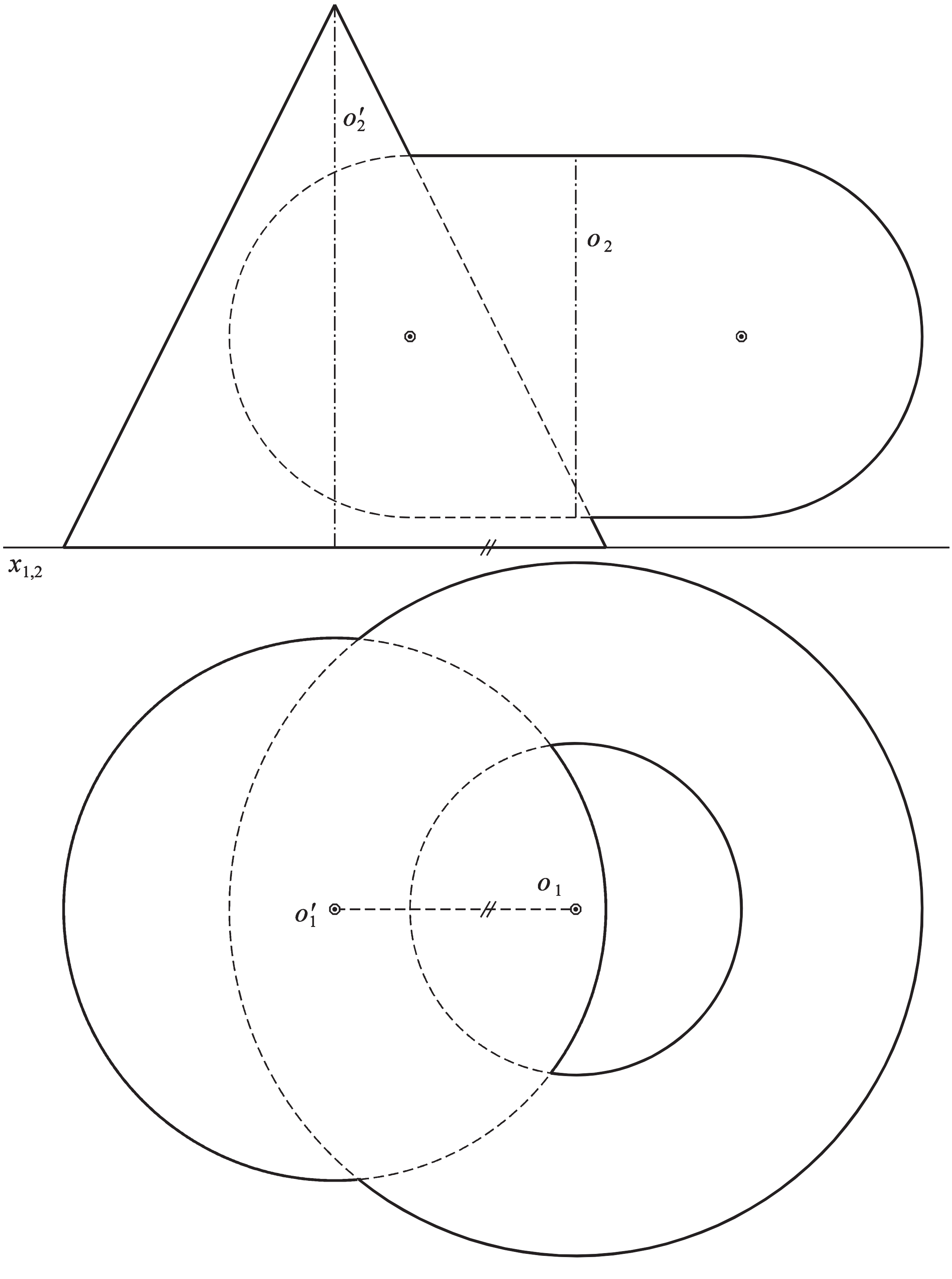
■ **Exercise 4.11.** Ellipsoid of revolution (axis o) and cylinder of revolution (axis o') are given. Using Monge projection, construct intersection curve q of these two surfaces.



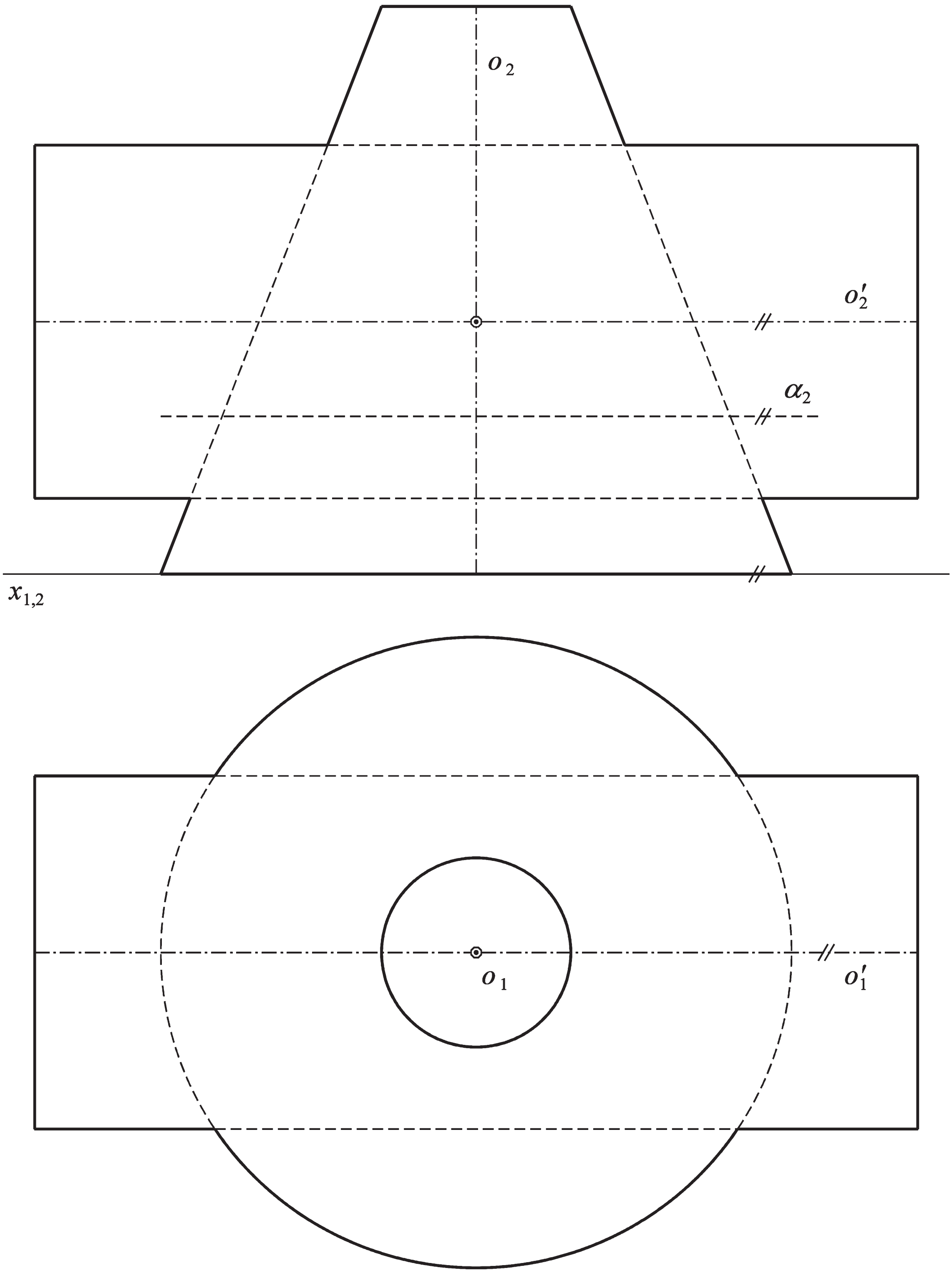
■ **Exercise 4.12.** Cylinder of revolution (axis o) and sphere (centre S) are given. Using Monge projection, construct intersection curve q of these two surfaces. Construct normal lines of both surfaces at point $M \in q$, $M \in \alpha$.



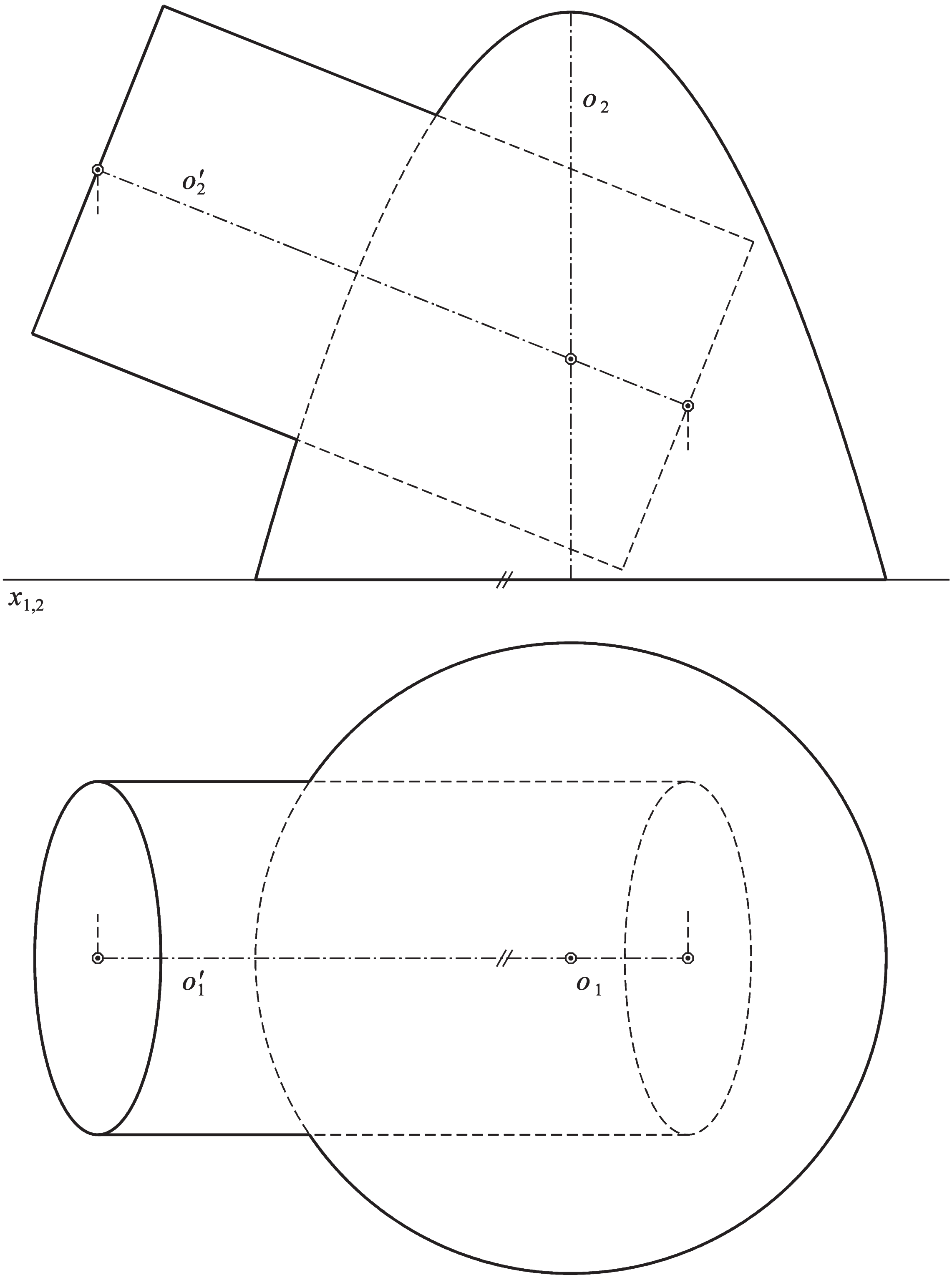
■ **Exercise 4.13.** Surface of revolution (axis o) and cone of revolution (axis o') are given. Using Monge projection, construct intersection curve q of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point $M \in q$, $z_M = 15$.



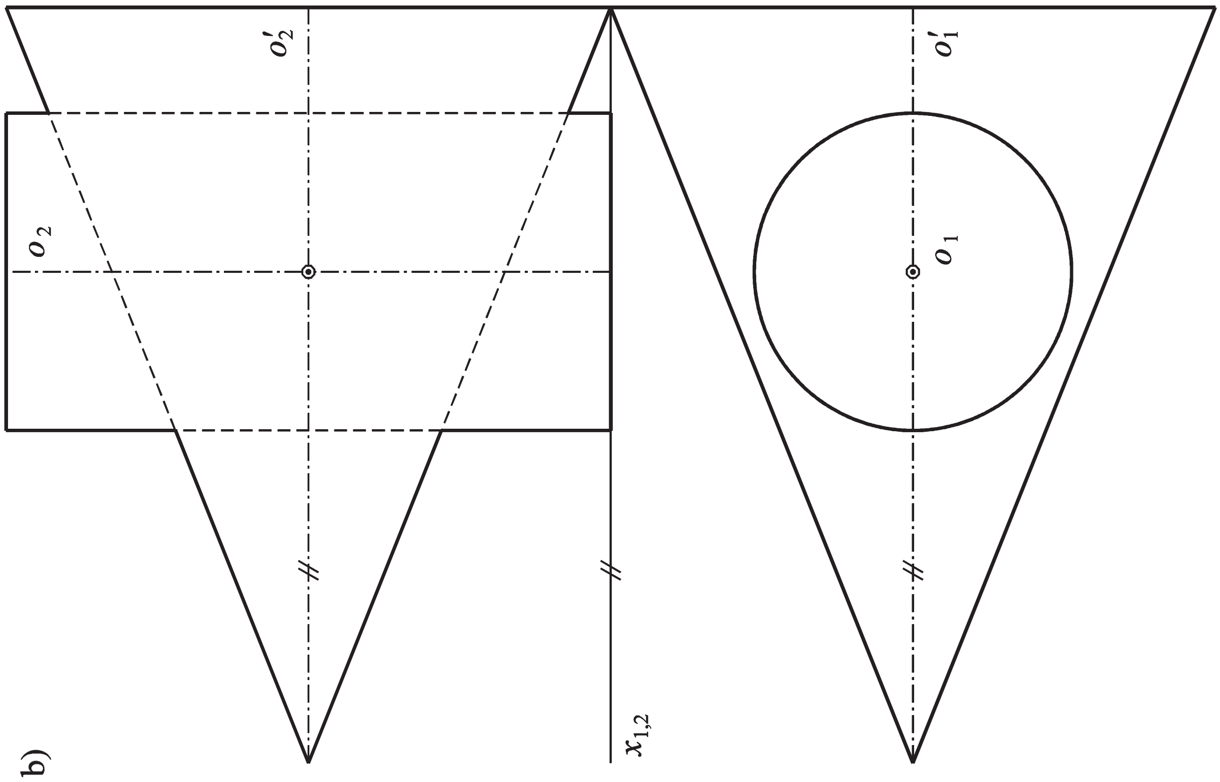
■ **Exercise 4.14.** Truncated cone of revolution (axis o) and cylinder of revolution (axis o') are given. Using Monge projection, construct intersection curve q of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point $M \in q$, $M \in \alpha$.

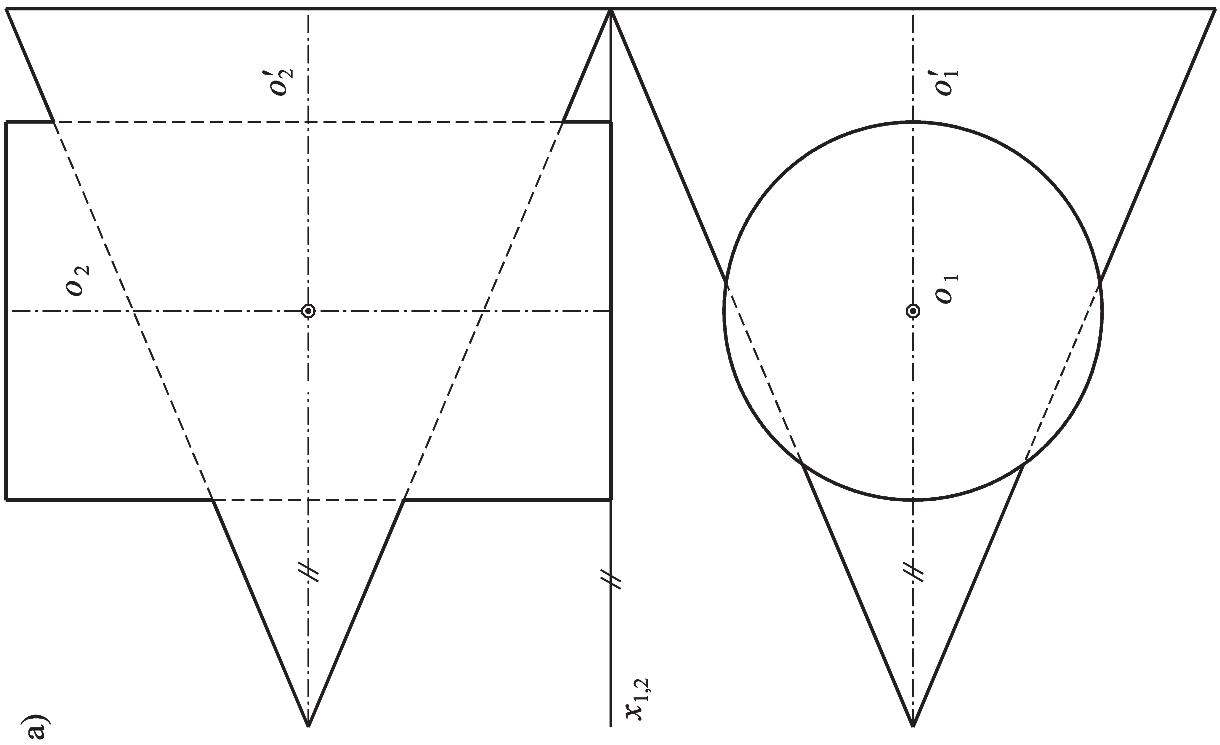


■ **Exercise 4.15.** Paraboloid of revolution (axis o) and cylinder of revolution (axis o') are given. Using Monge projection, construct intersection curve q of these two surfaces. Indicate the visibility. Construct normal lines of both surfaces at point $M \in q$, $z_M = 30$ construct normal lines of both surfaces.



■ **Exercise 4.16.** Cylinder of revolution (axis o) and cone of revolution (axis o') are given. Using Monge projection, construct intersection curve q of these two surfaces. Indicate the visibility.

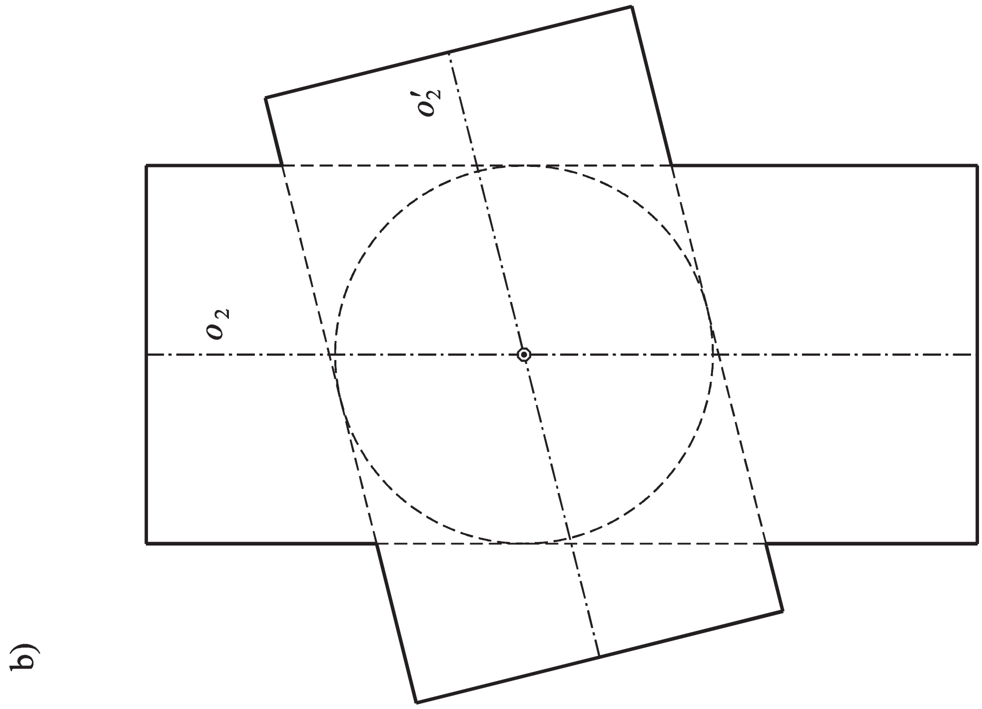


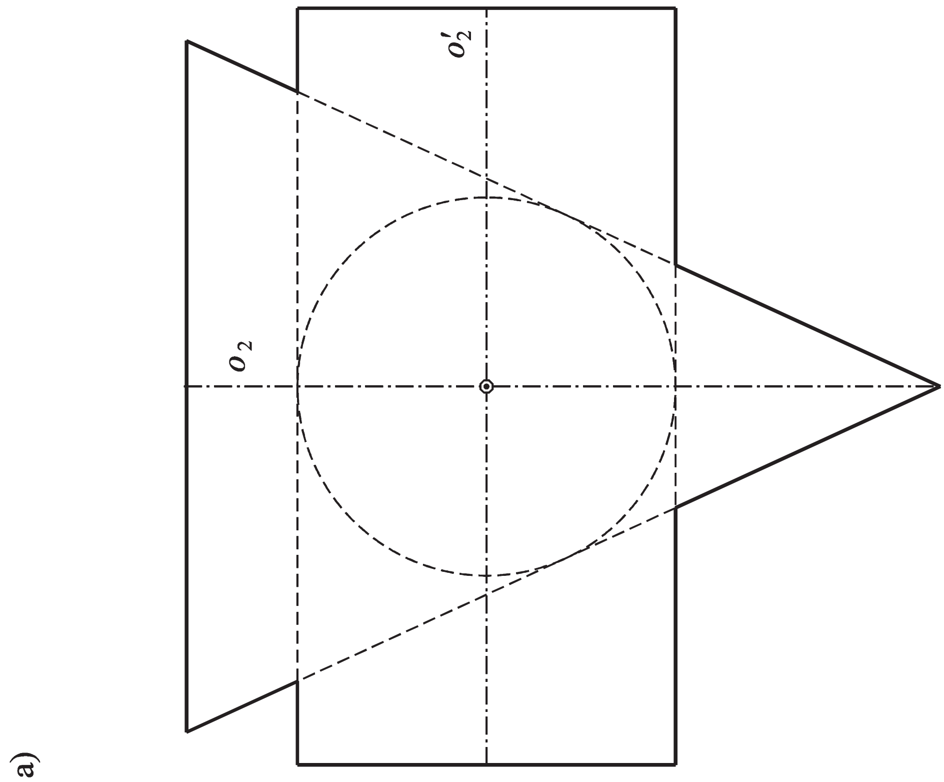


■ **Exercise 4.17.** Construct front view of intersection curve q of two surfaces of revolution.

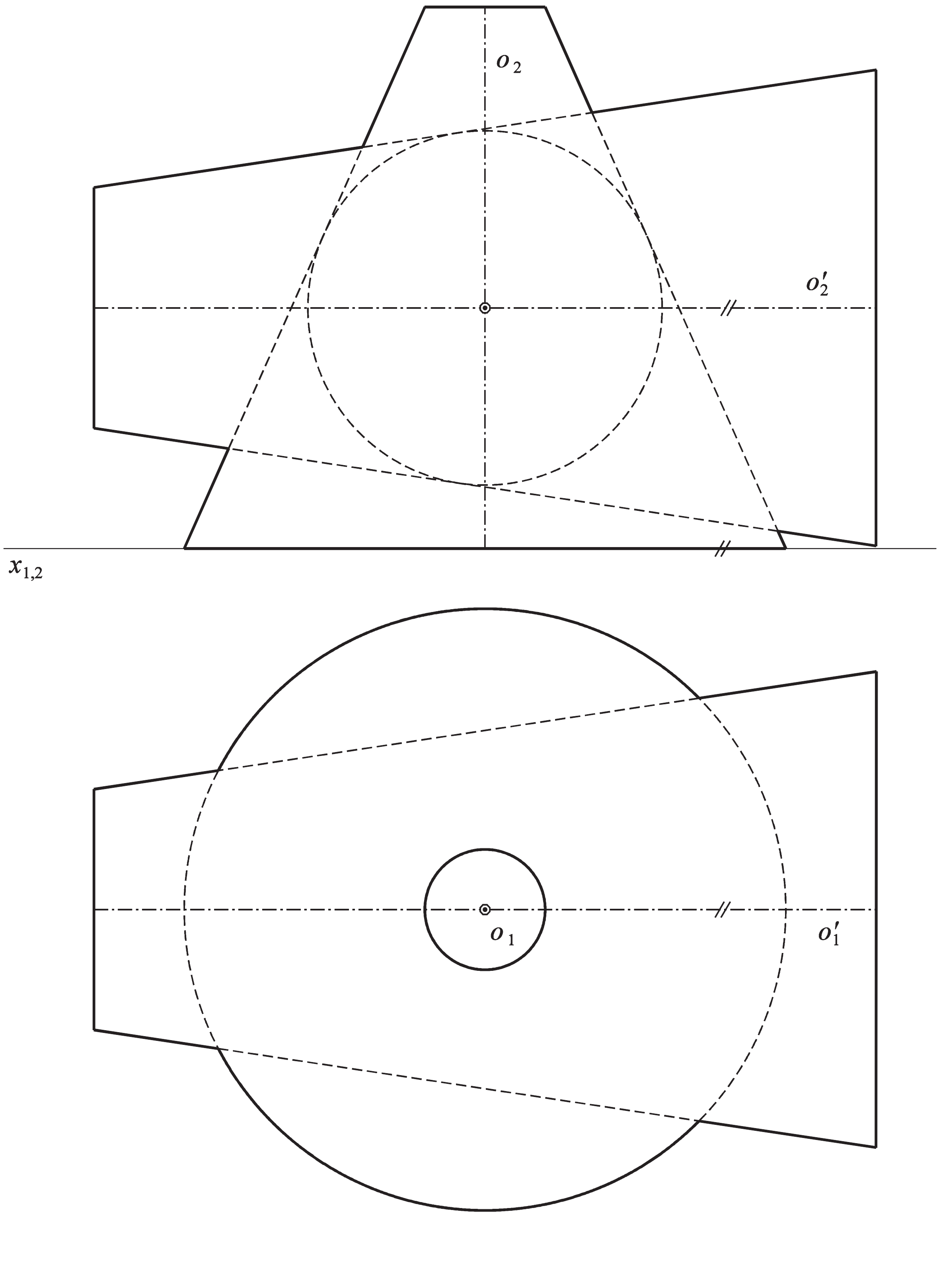
a) Axial section of cone of revolution (axis o) and axial section of cylinder of revolution (axis o') are given.

b) Axial sections of two cylinders of revolution (axes o and o') are given.





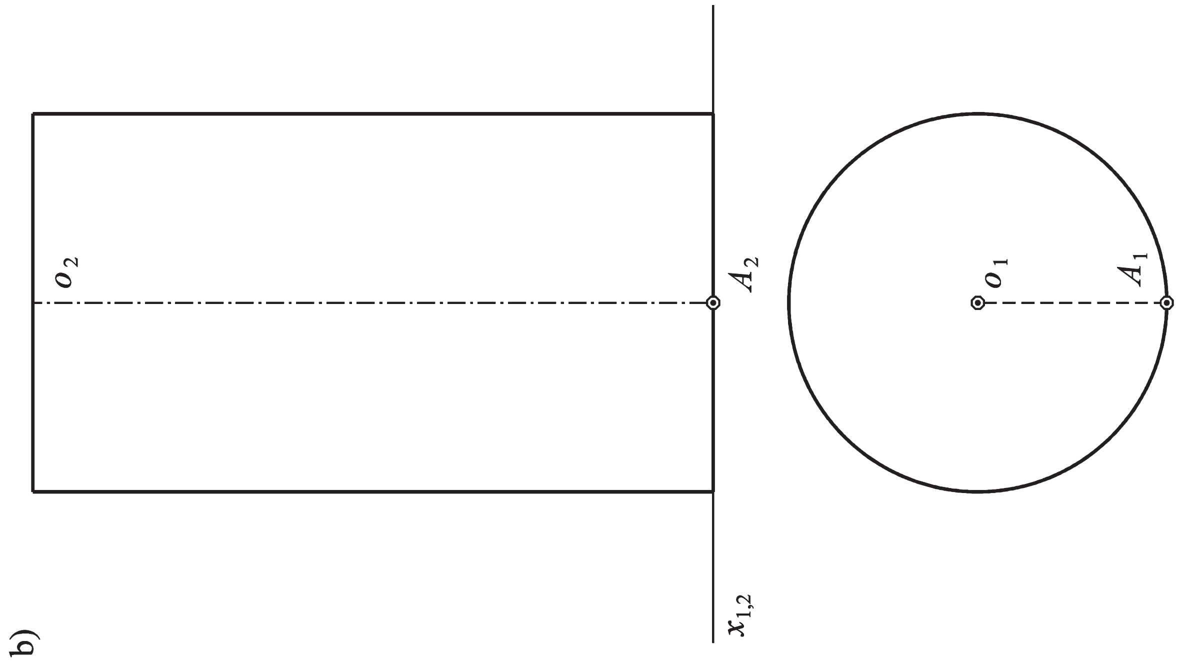
■ **Exercise 4.18.** Two truncated cones of revolution (axes o and o') are given. Using Monge projection, construct intersection curve q of these surfaces. Indicate the visibility.

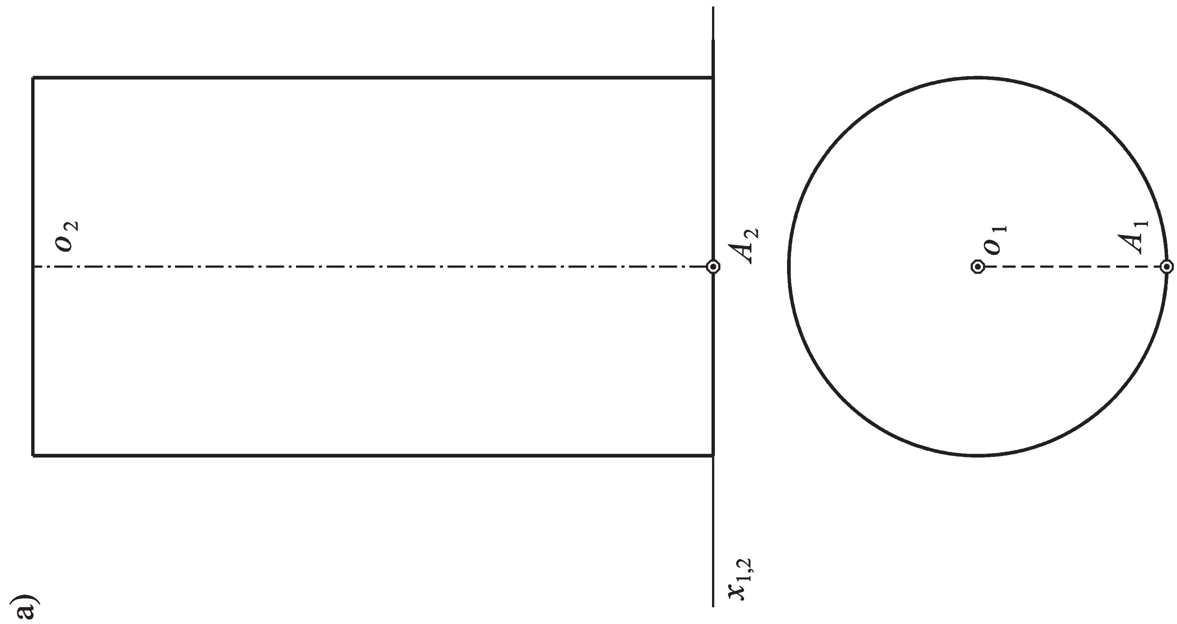


5 Helicoidal surfaces

5.1 Helix

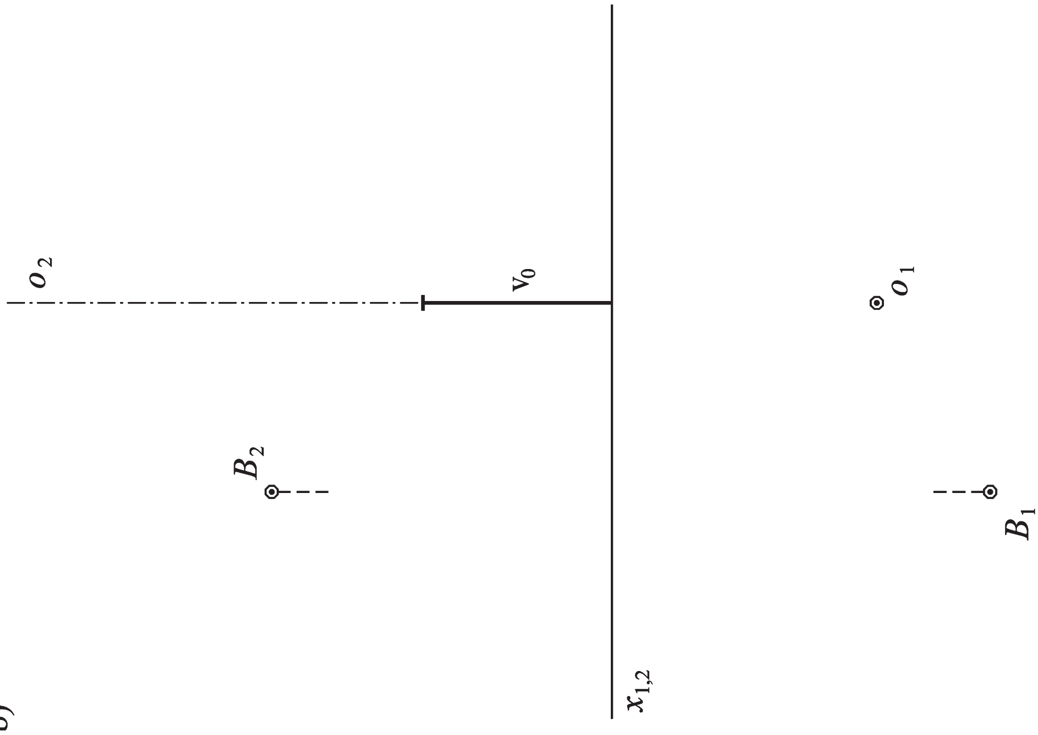
■ **Exercise 5.1.** Considering the given cylinder of revolution (axis o) draw a) right-handed, b) left-handed helix generated by screw motion of point A with lead $v = 120$ mm. Use Monge projection.



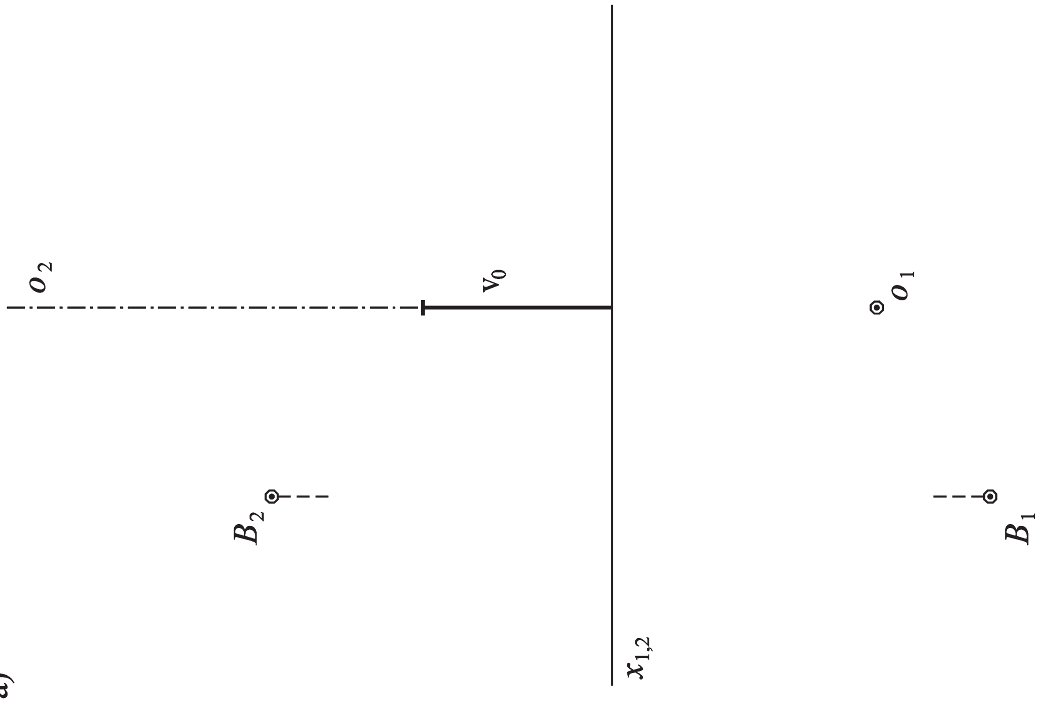


■ **Exercise 5.2.** Helix (B, o, v_0, a) right-handed, b) left-handed) is given. Using Monge projection, construct tangent line to the helix at its generating point B.

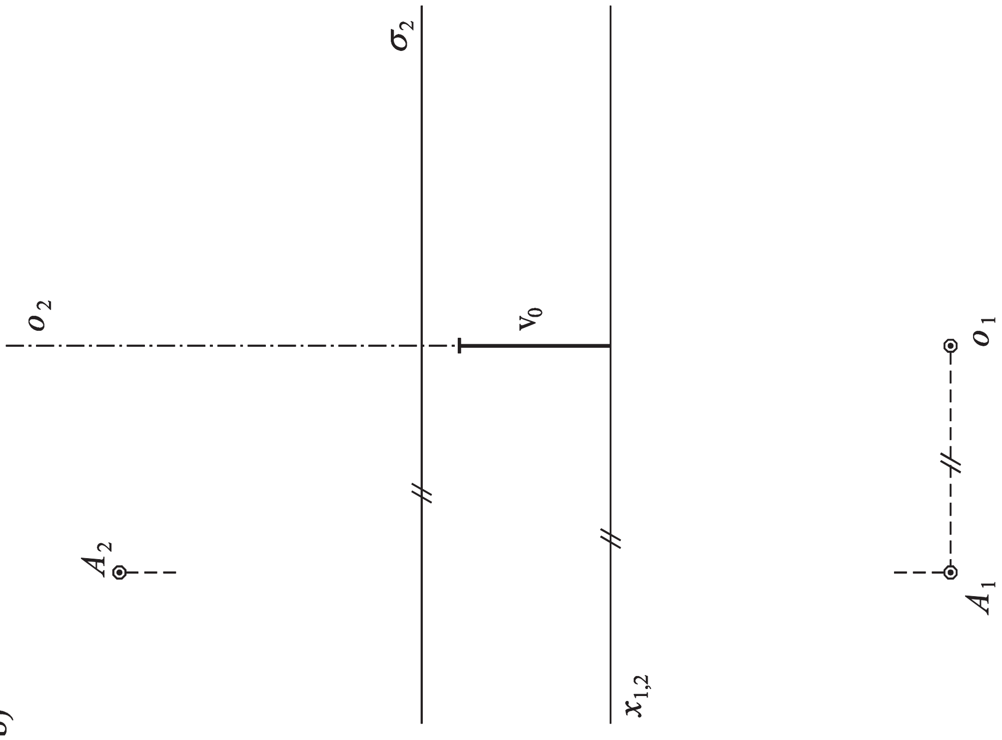
b)



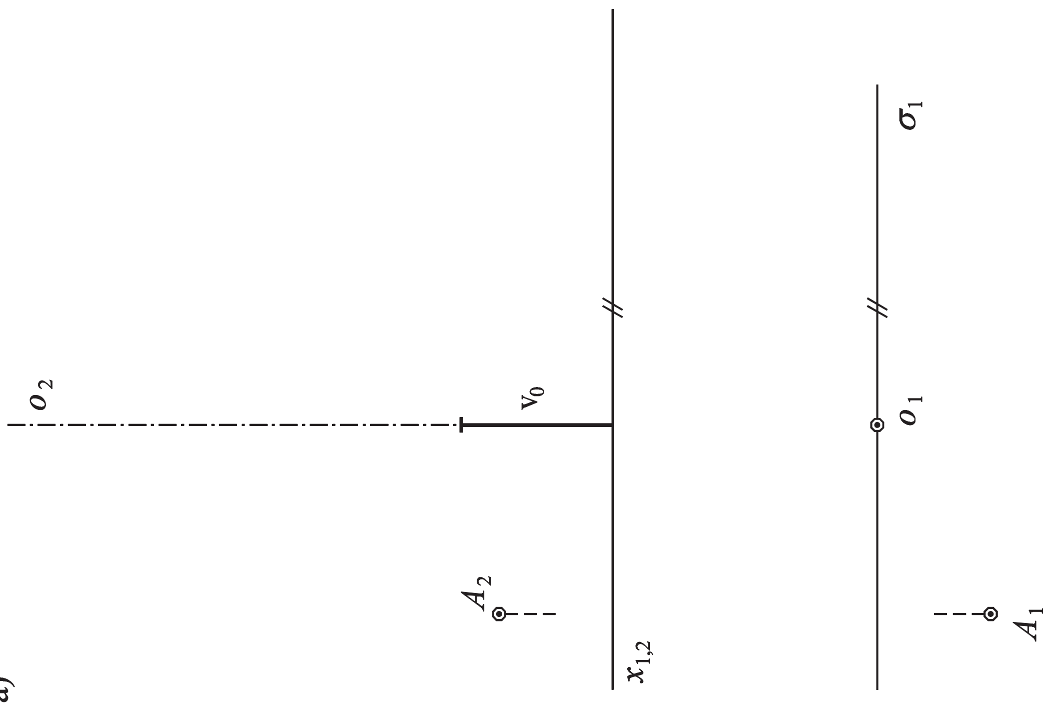
a)



■ **Exercise 5.3.** Helix (A, o, v_0, right-handed) is given. Using Monge projection, construct intersection R of the helix and the given plane σ. Construct tangent line to the helix at point R.

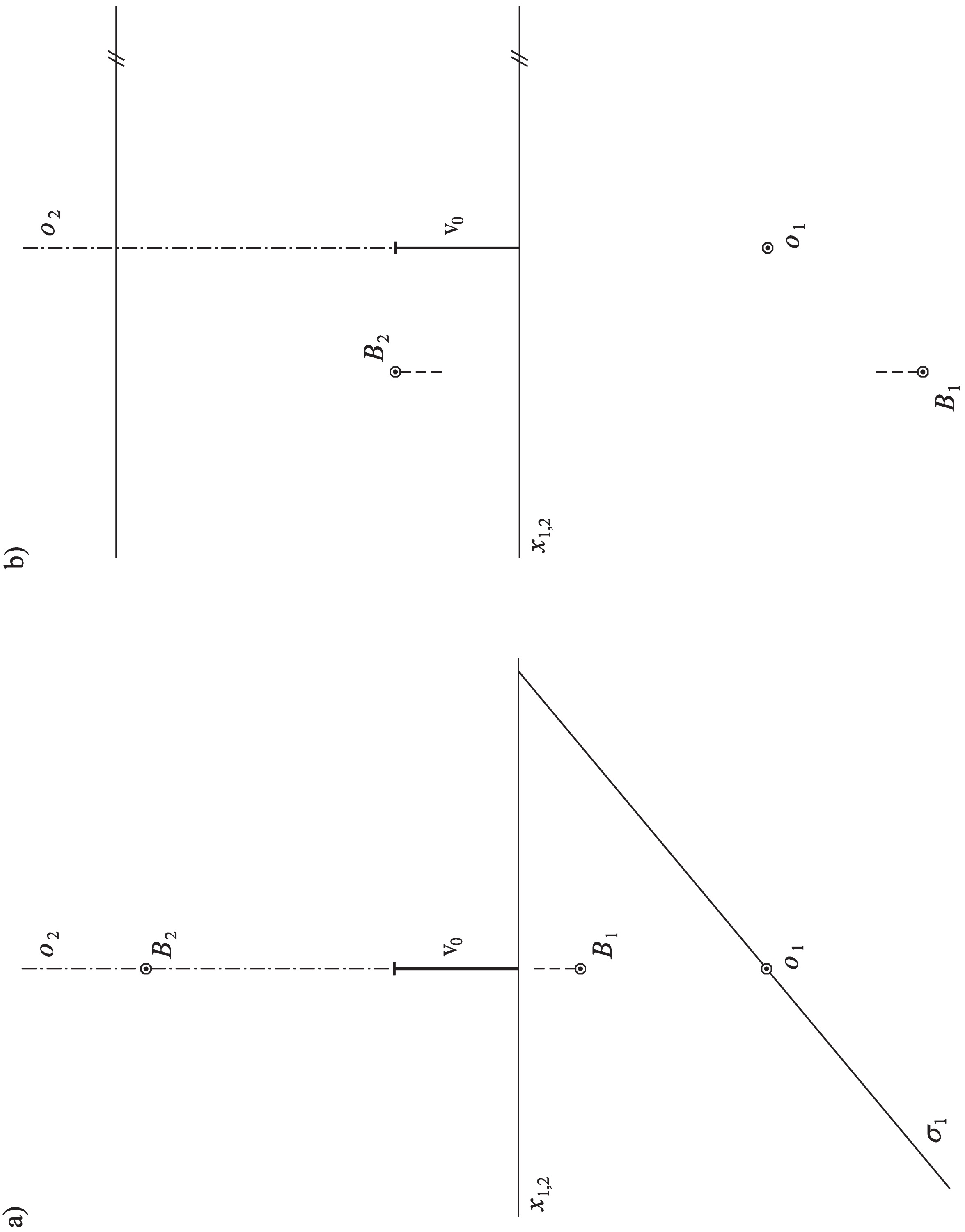


b)



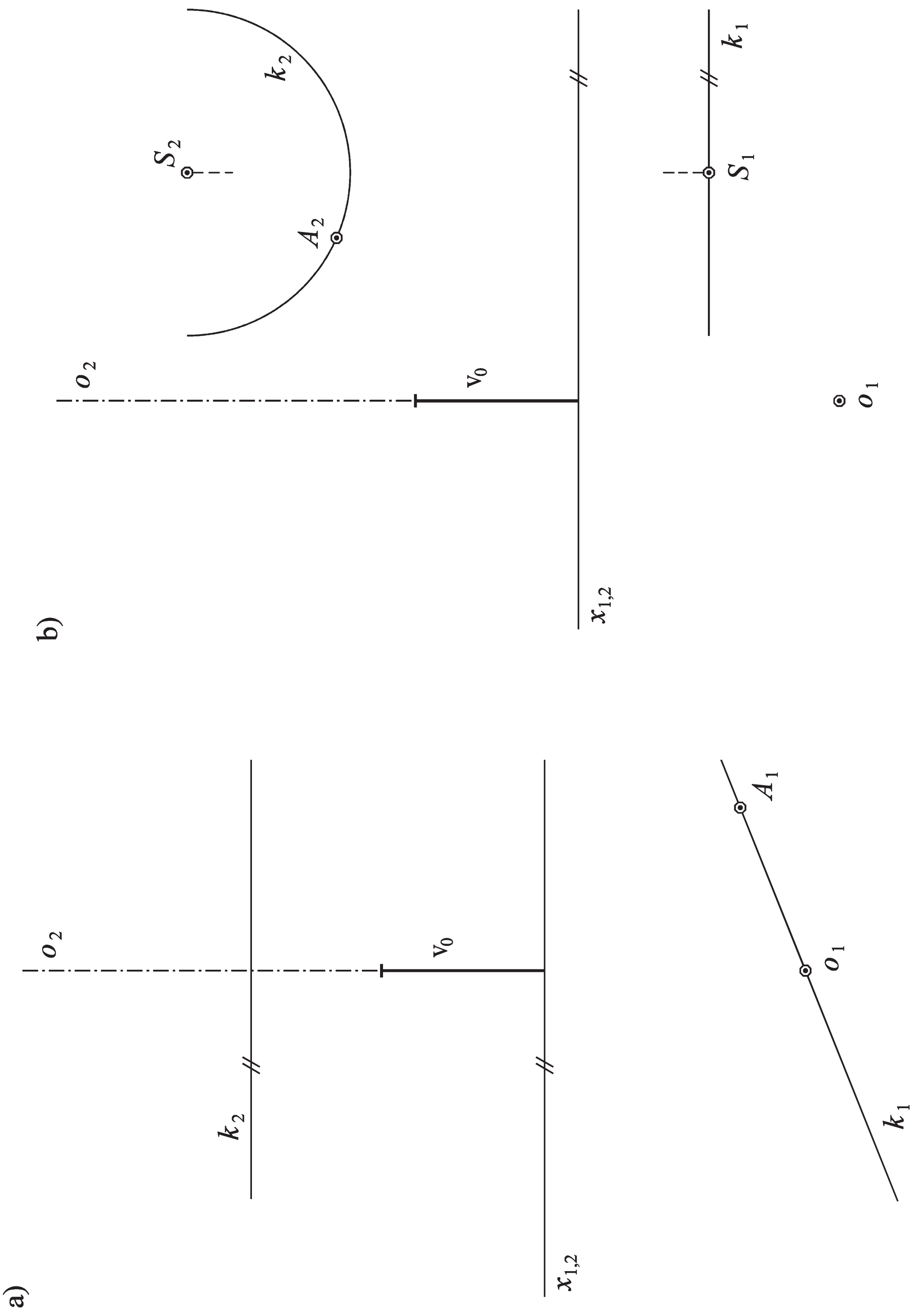
a)

■ **Exercise 5.4.** Helix (B, o, v_0, left-handed) is given. Using Monge projection, construct intersection R of the helix and the given plane σ. Construct tangent line to the helix at point B.

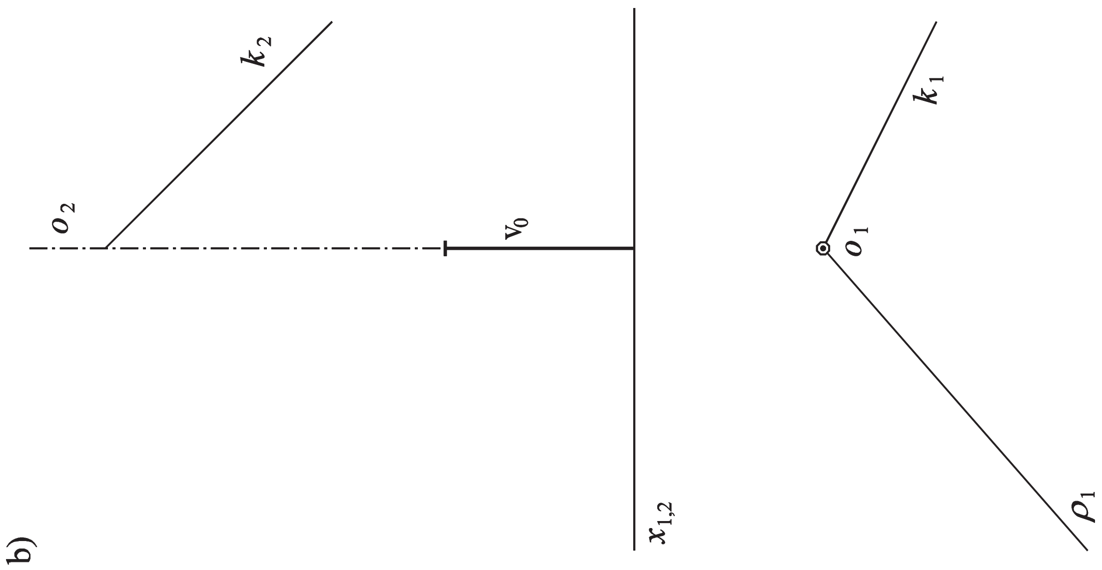


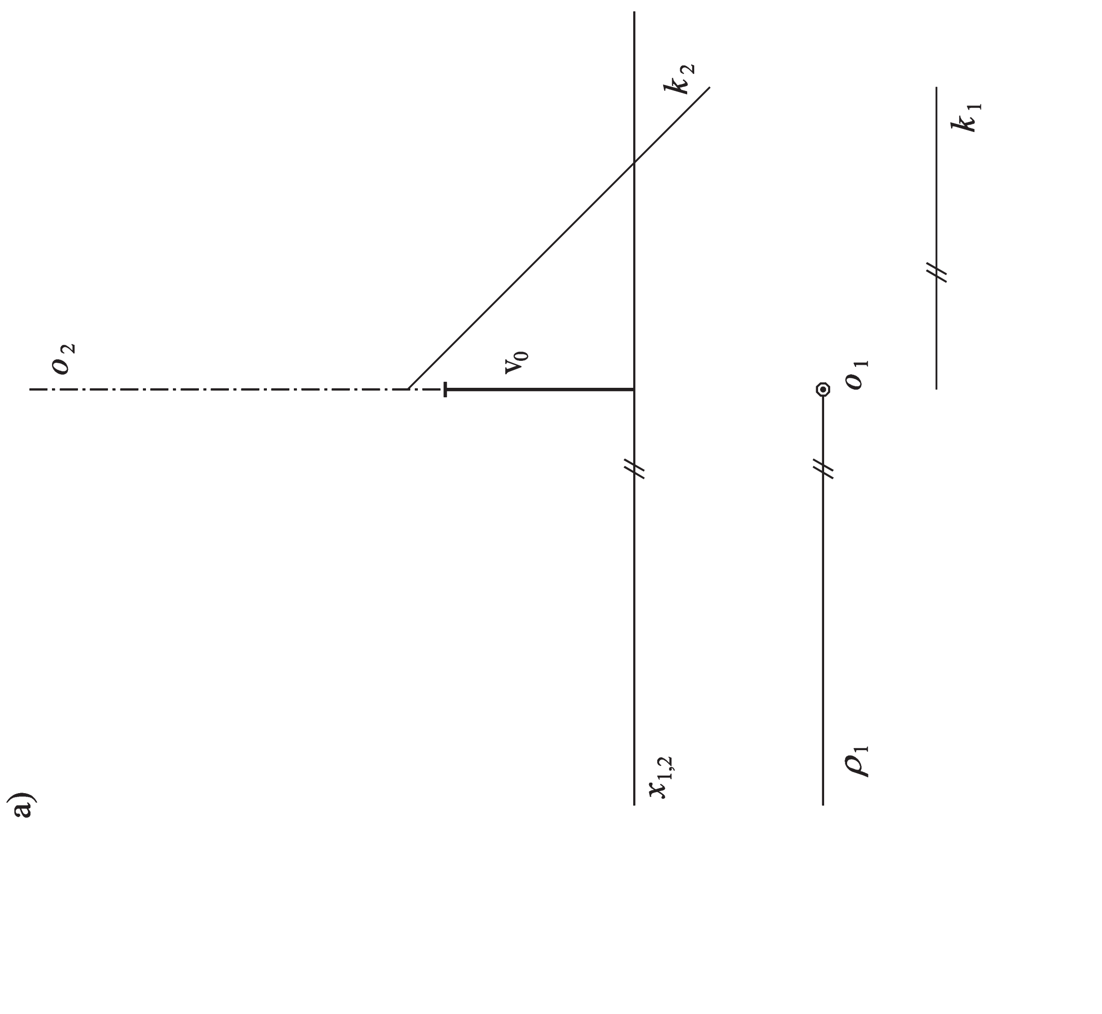
5.2 Helicoidal surfaces

■ **Exercise 5.5.** Helicoidal surface (k, o, v_0, a) right-handed, b) left-handed) is given. Using Monge projection, construct tangent plane τ at point $A \in k$.

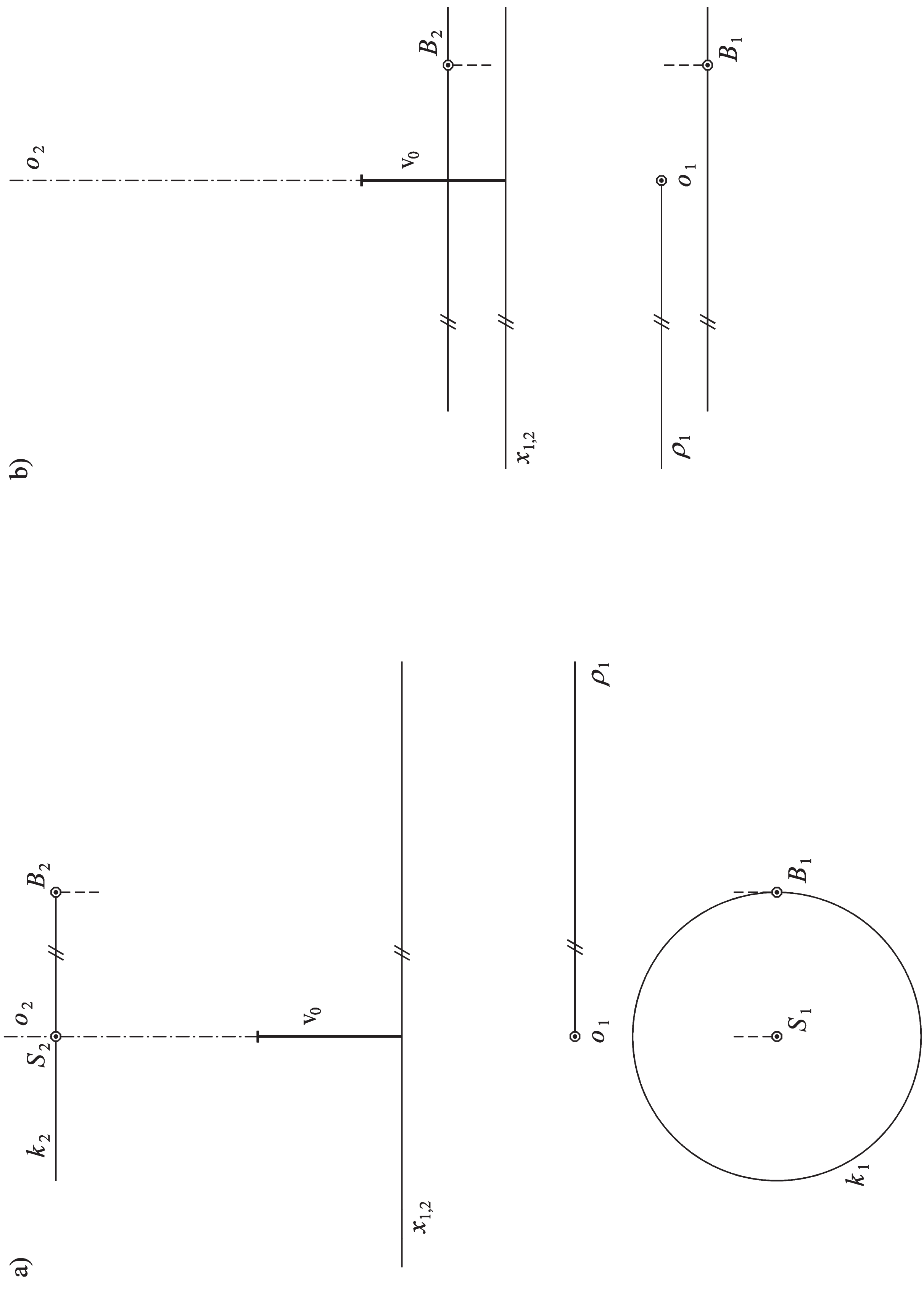


■ **Exercise 5.6.** Helicoidal surface (k, o, v_0, a) left-handed, b) right-handed) is given. Using Monge projection, construct the intersection of the surface and the given plane ρ.

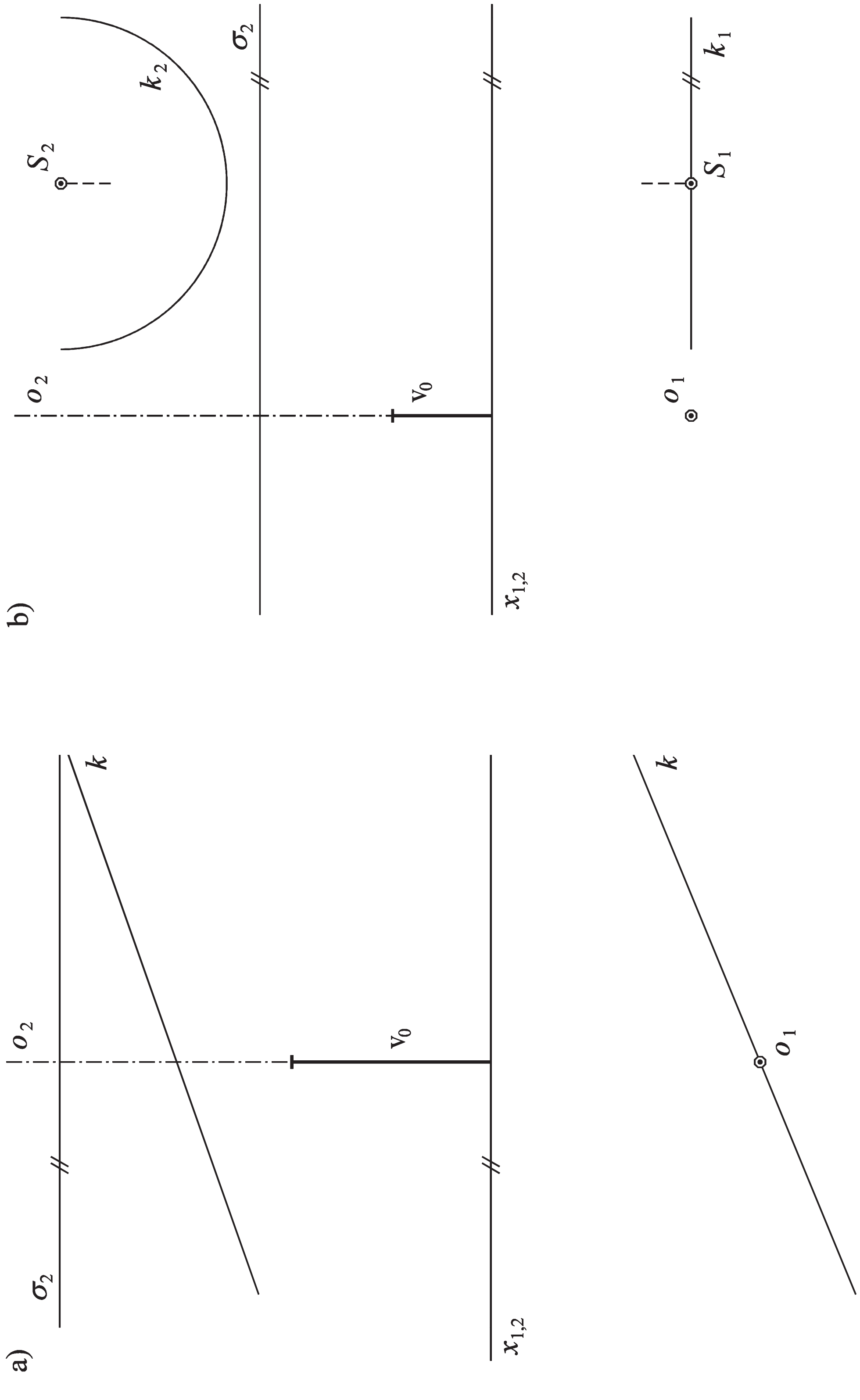




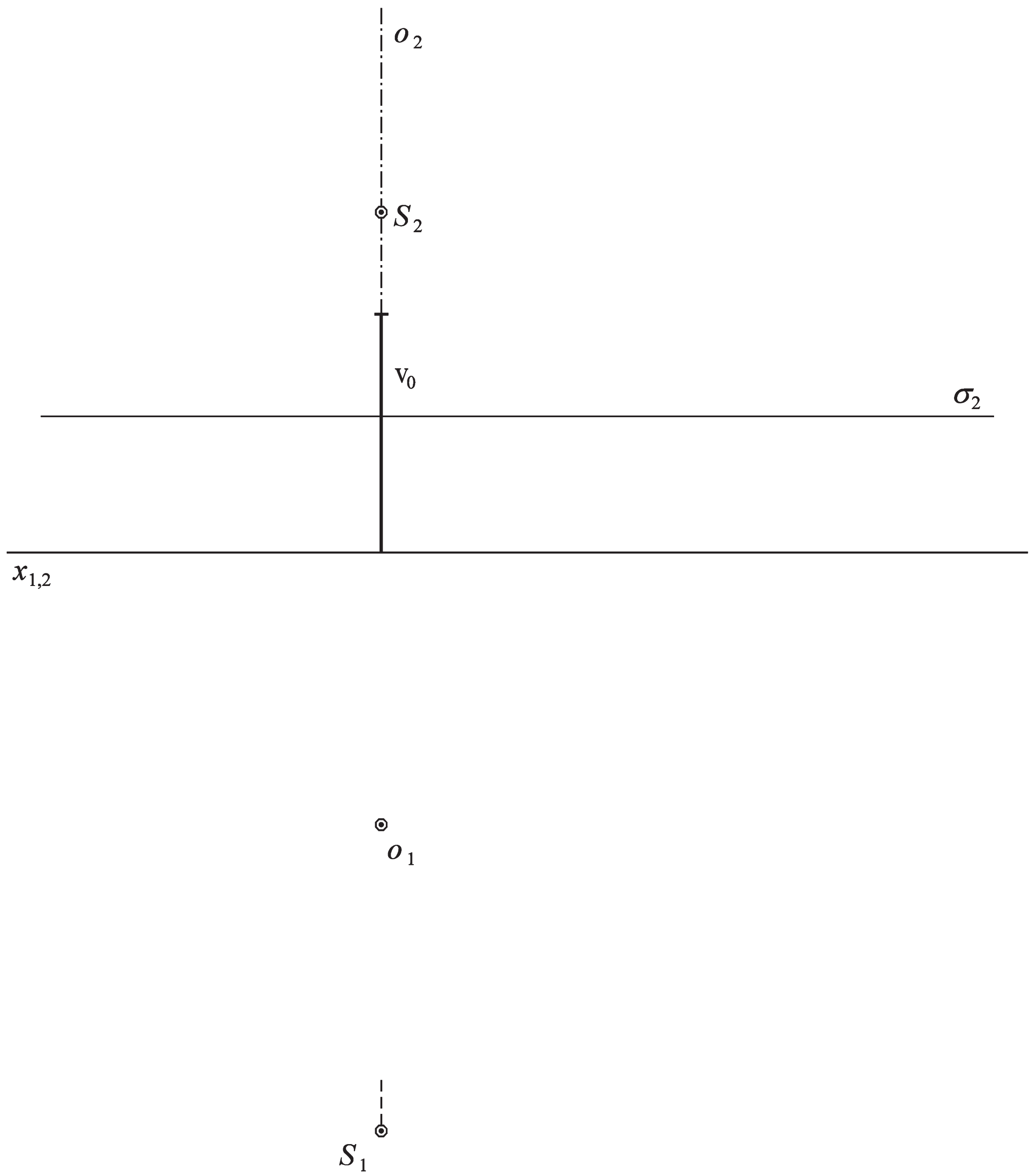
■ **Exercise 5.7.** Helicoidal surface (k, o, v_0, left-handed) is given. Using Monge projection, construct tangent plane τ of the surface at point B. Construct the curve of intersection m of the surface and the given plane ρ (principal half-meridian).



■ **Exercise 5.8.** Helicoidal surface (k, o, v_0, right-handed) is given. Using Monge projection, construct the curve of intersection n of the surface and the given plane σ (normal section).



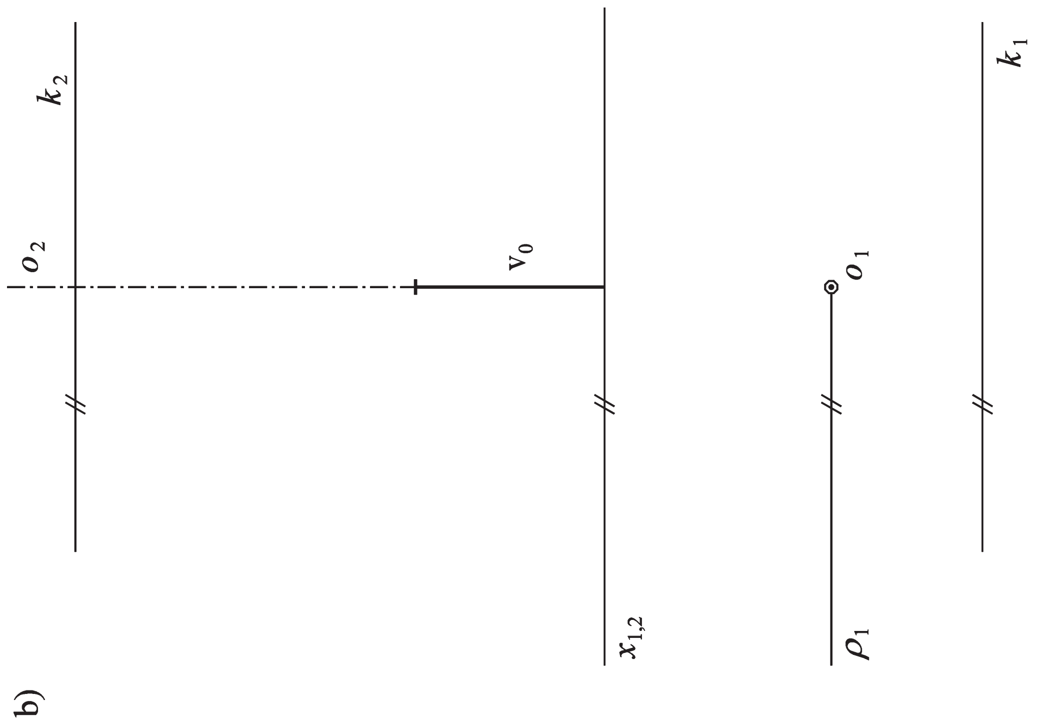
■ **Exercise 5.9.** Serpentine of Archimedes is given by centre S, radius $r = 30$ mm and left-handed screw motion $(o,\ v_0)$. Using Monge projection, construct the generating circle. Construct the normal section n of the helicoidal surface by the given plane σ.

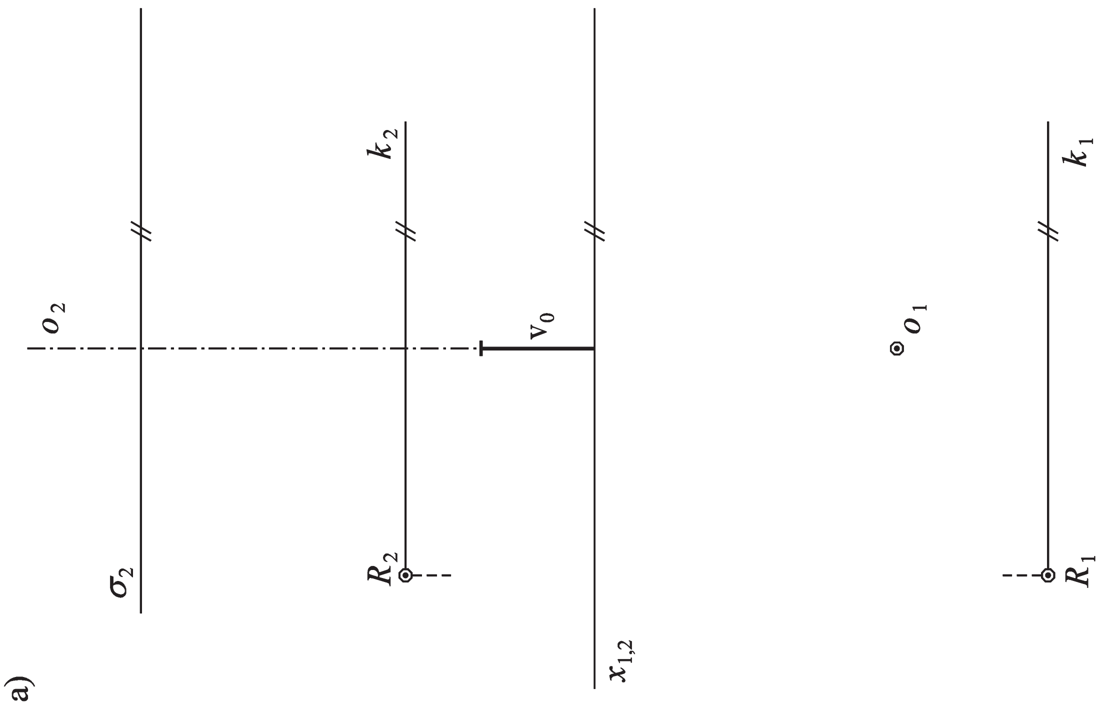


■ **Exercise 5.10.** Helicoidal surface (k, o, v_0, right-handed) is given.

a) Using Monge projection, construct tangent plane τ of the surface at point B. Construct the normal section c of the surface and the given plane σ.

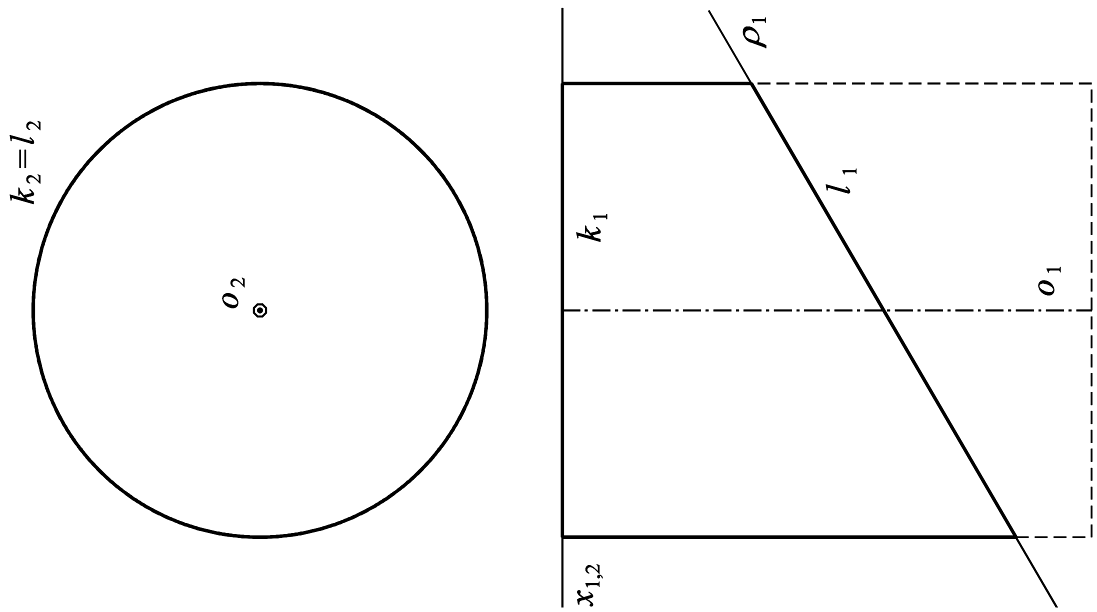
b) Using Monge projection, construct principal half-meridian of the surface in the given plane ρ.



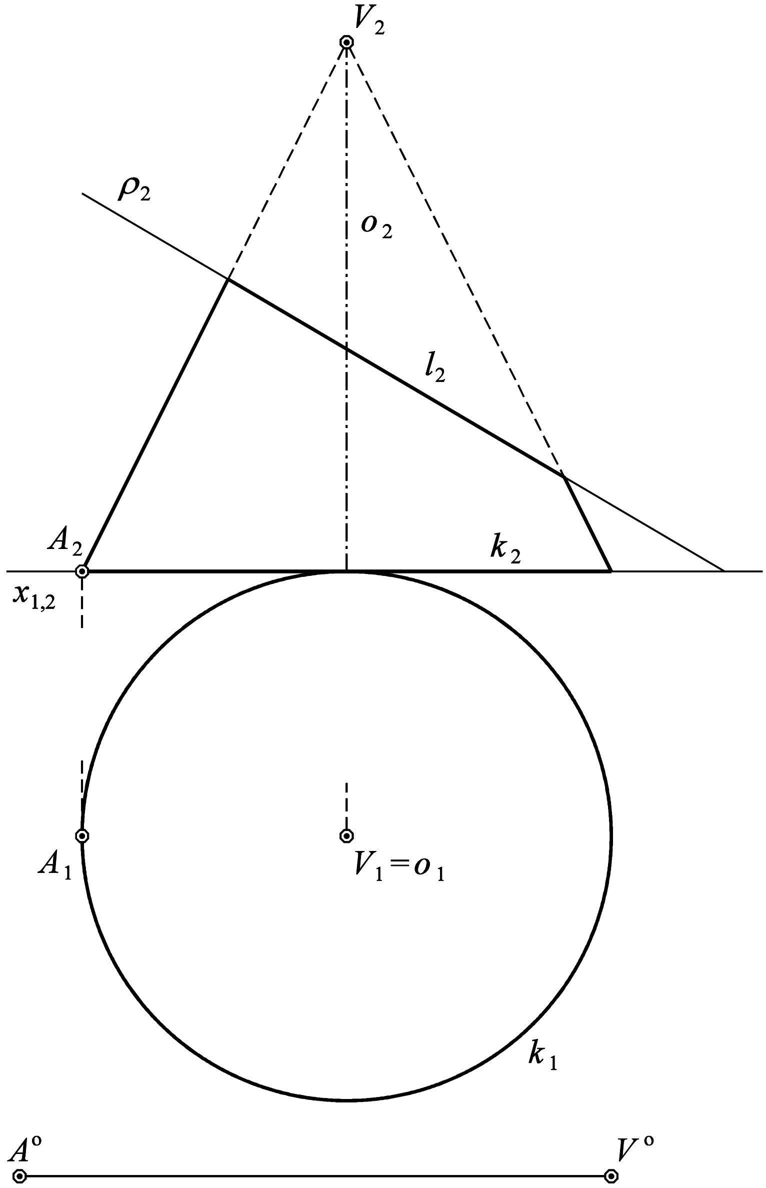


6 Developable surfaces

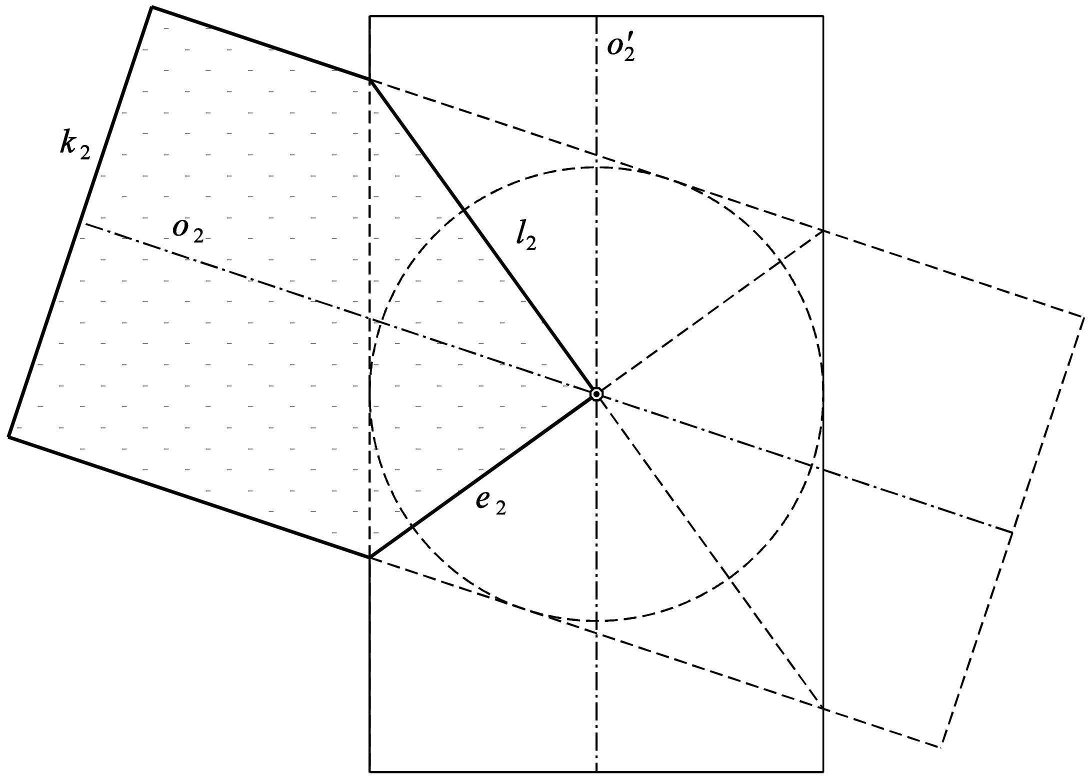
■ **Exercise 6.1.** Develop the part of cylinder of revolution between its base k and ellipse $e \subset \rho$. Use Monge projection.



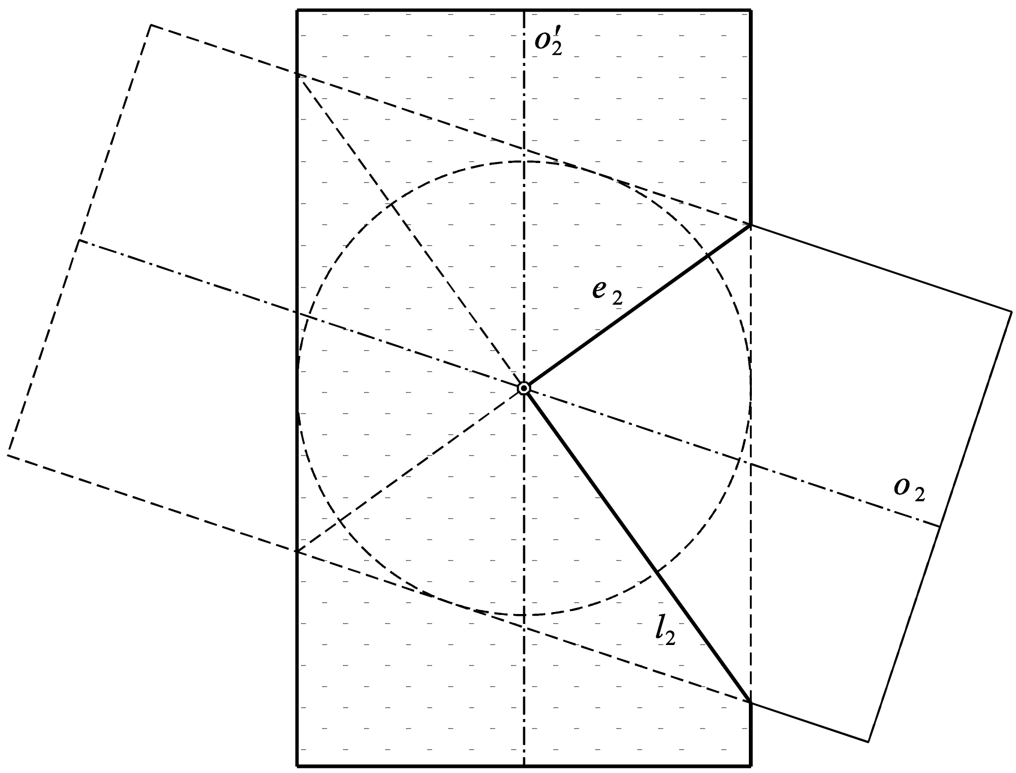
■ **Exercise 6.2.** Develop the part of cone of revolution (V, k) between its base k and ellipse $l \subset \rho$. Use Monge projection.



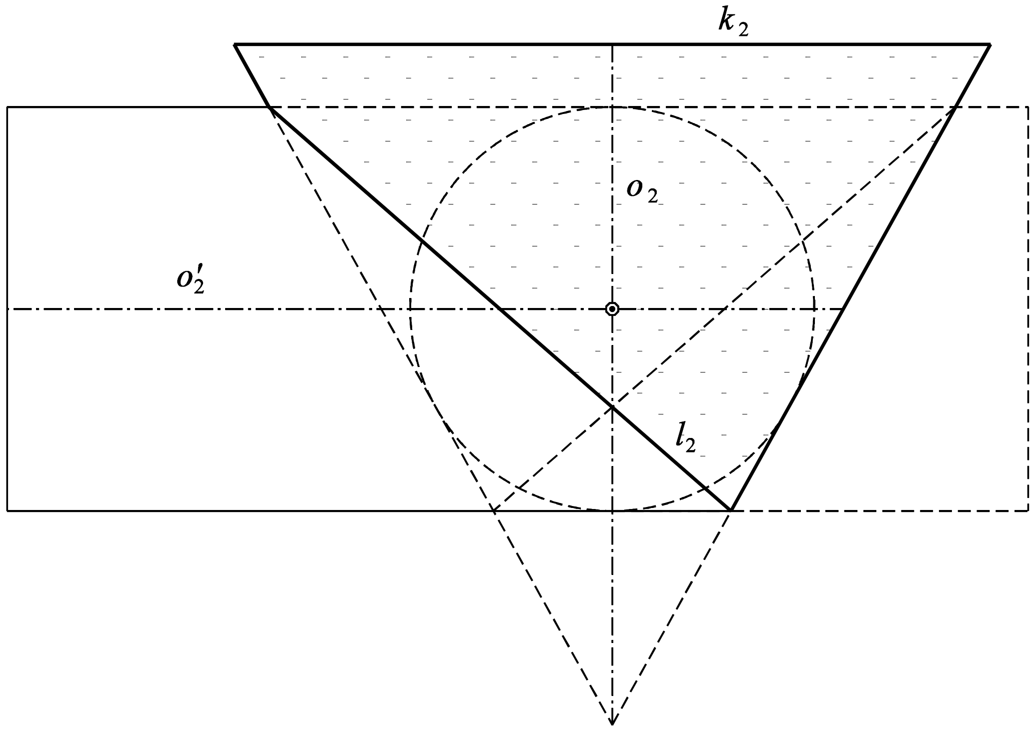
■ **Exercise 6.3.** Degenerated intersection of two cylinders of revolution is given. Develop the dotted part of the cylinder of revolution.



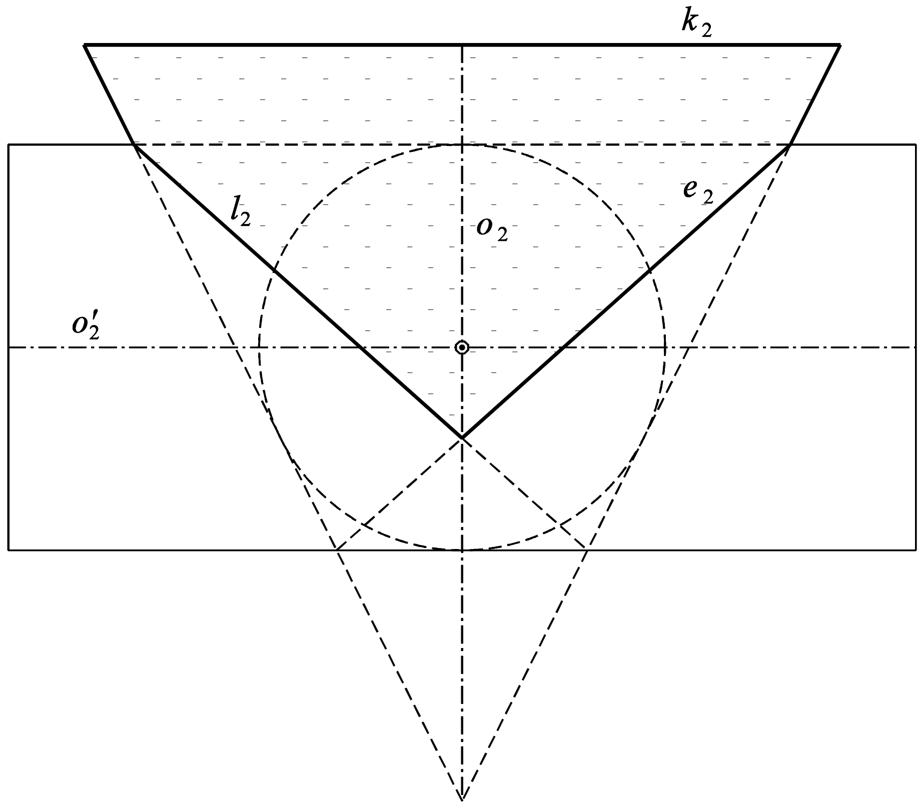
■ **Exercise 6.4.** Degenerated intersection of two cylinders of revolution is given. Develop the dotted part of the cylinder of revolution.



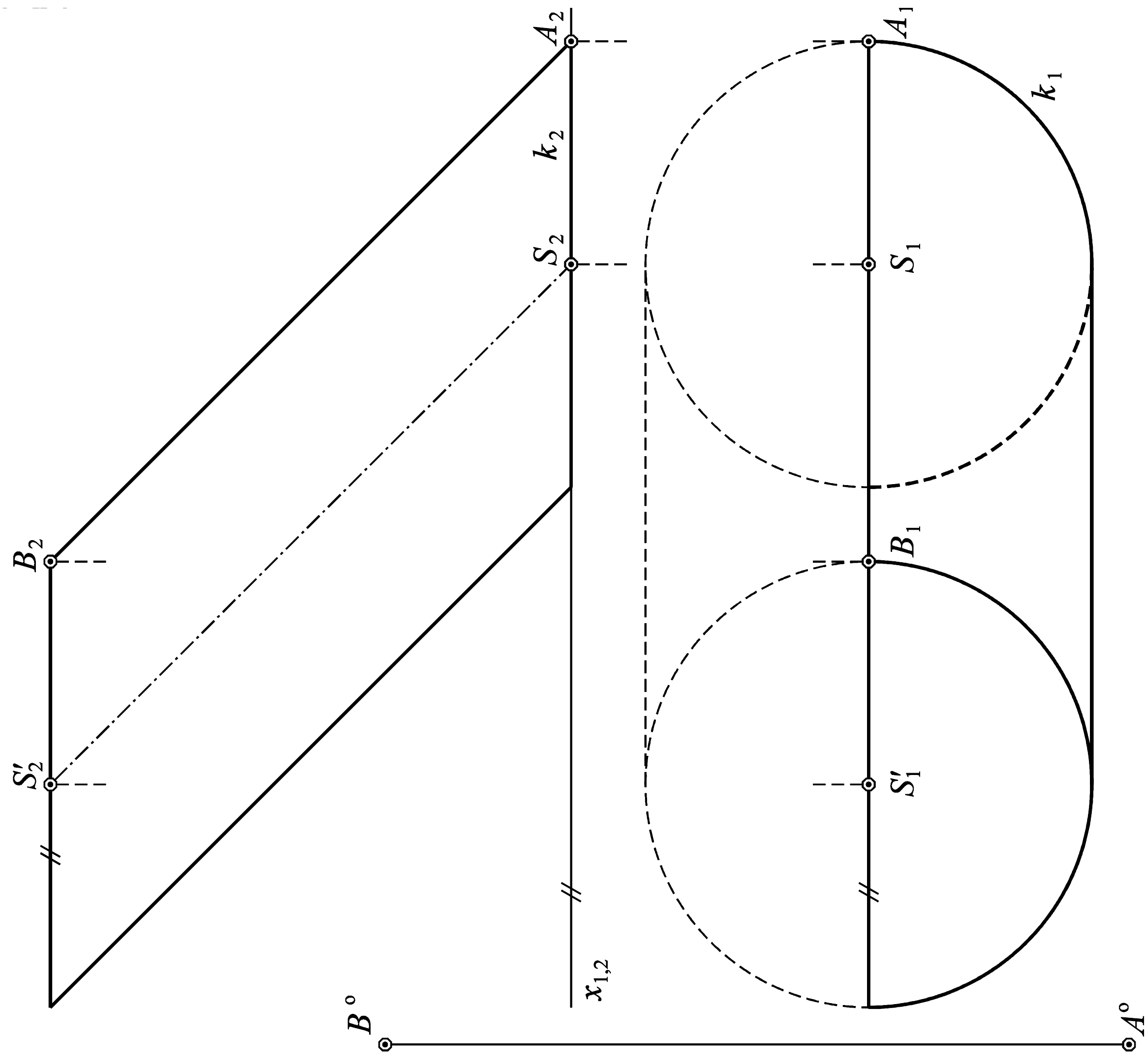
■ **Exercise 6.5.** Degenerated intersection of cylinder of revolution and cone of revolution is given. Develop the dotted part of the cone of revolution.



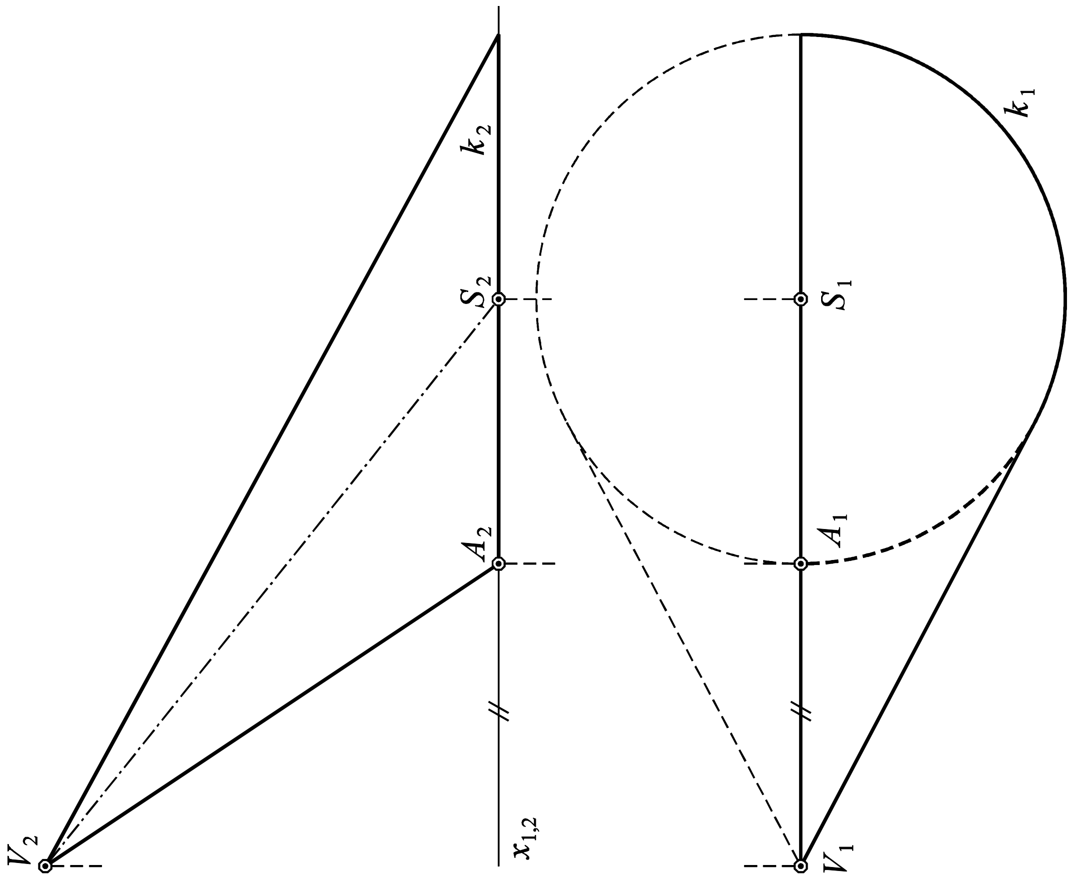
■ **Exercise 6.6.** Degenerated intersection of cylinder of revolution and cone of revolution is given. Develop the dotted part of the cone of revolution.



■ **Exercise 6.7.** Oblique cylinder is given. Develop the front part of the cylinder. Use Monge projection.

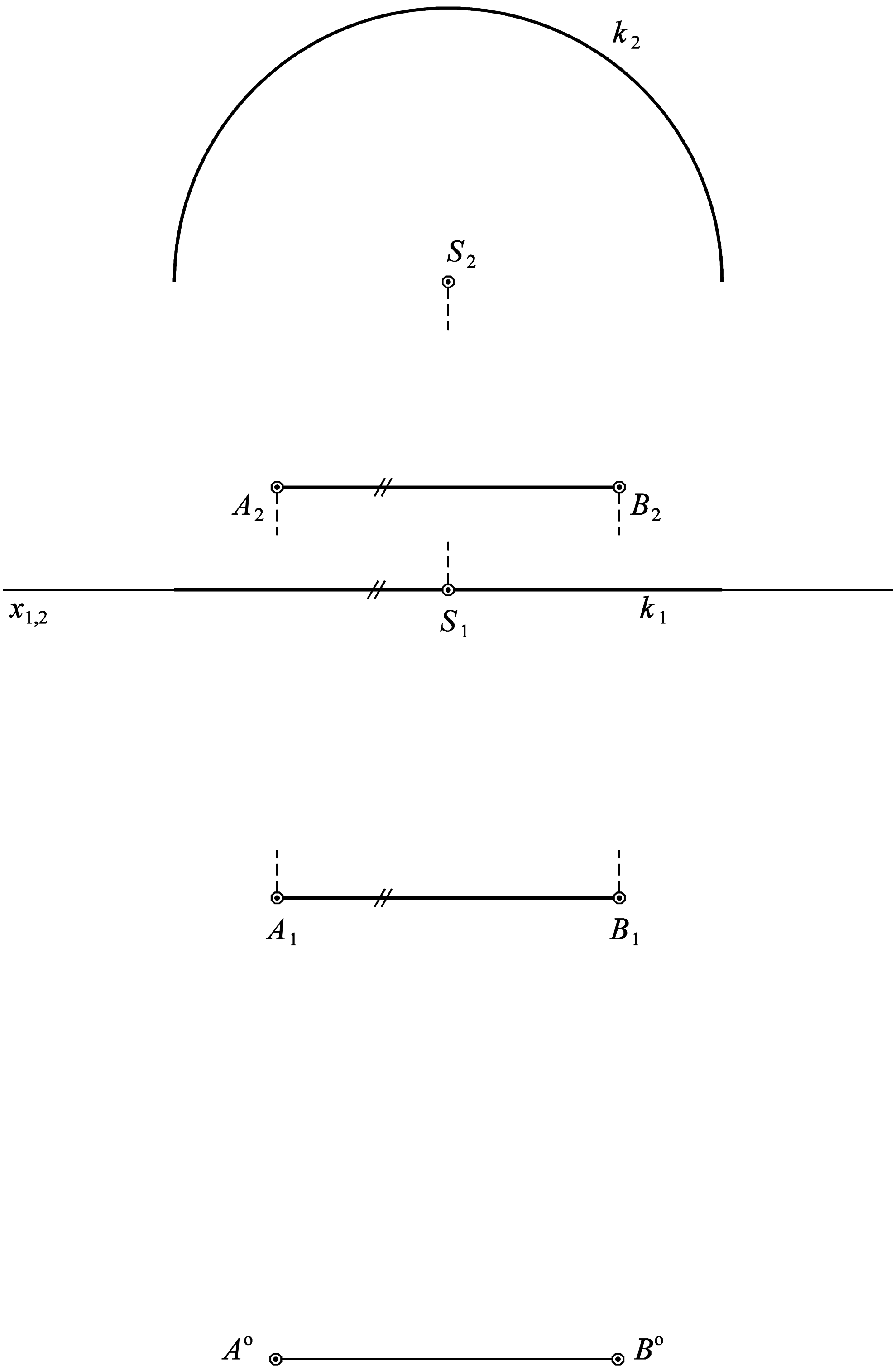


■ **Exercise 6.8.** Oblique cone is given. Develop the front part of the cone. Use Monge projection.

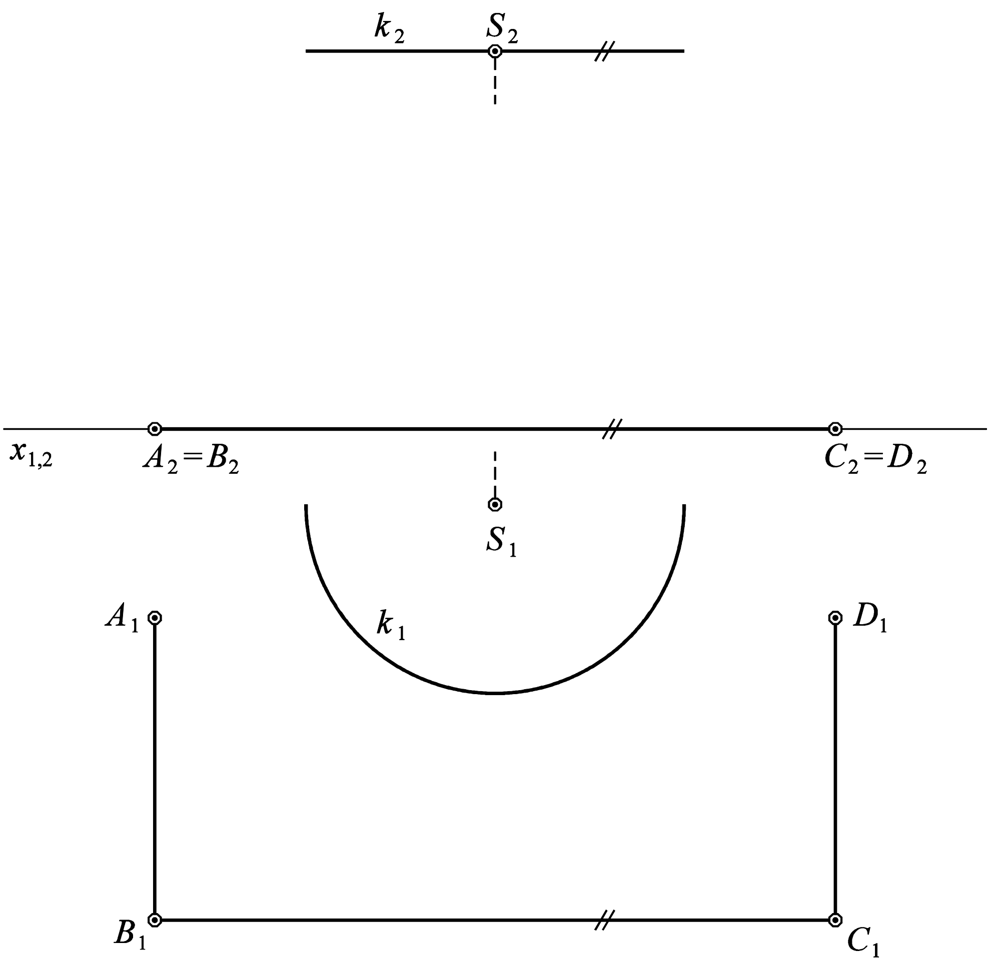


6.1 Transition developable surfaces

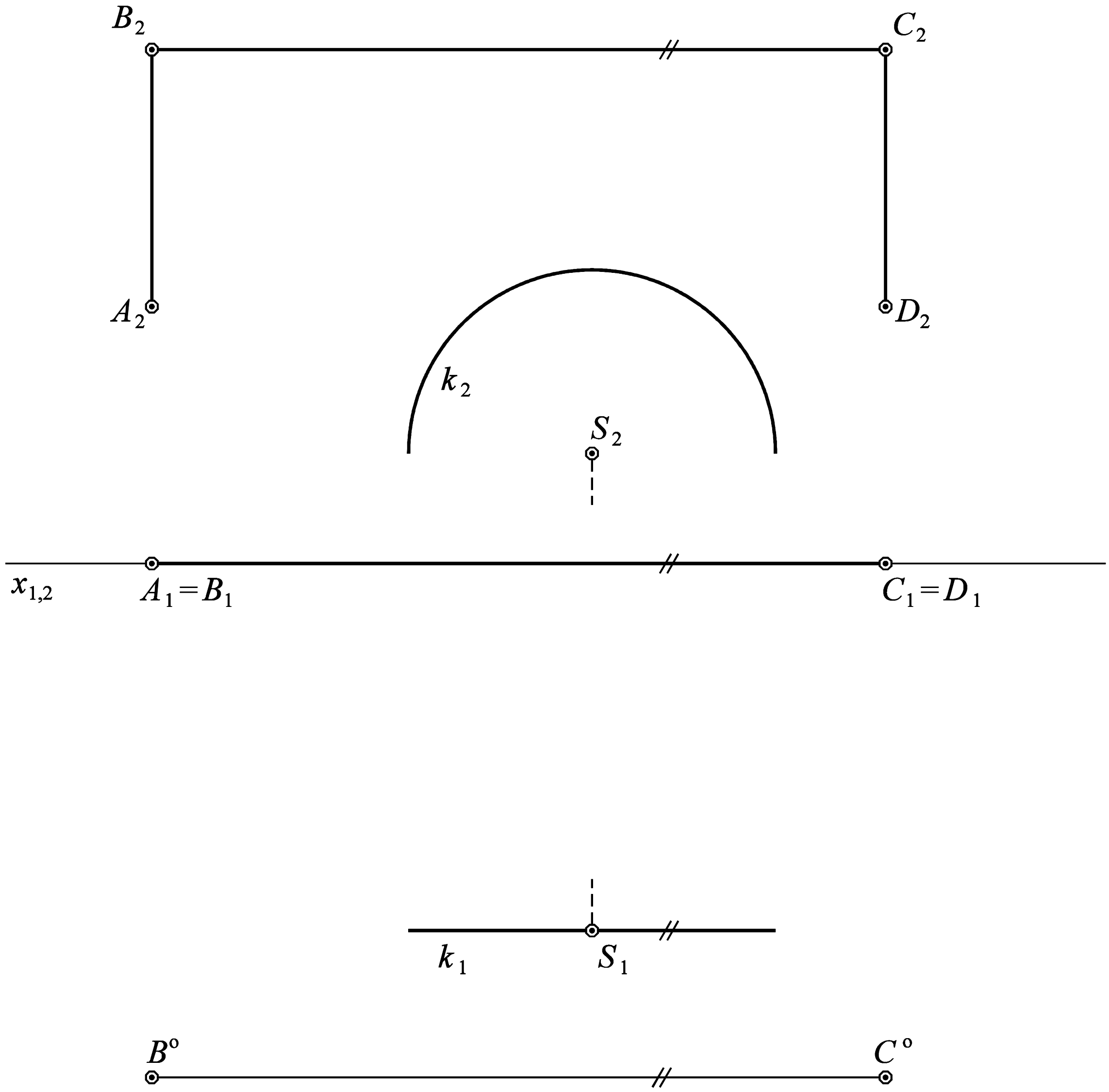
■ **Exercise 6.9.** Circle k and straight line segment AB are given. Construct smooth developable transition surface between the circle k and the straight line segment AB. Use Monge projection.



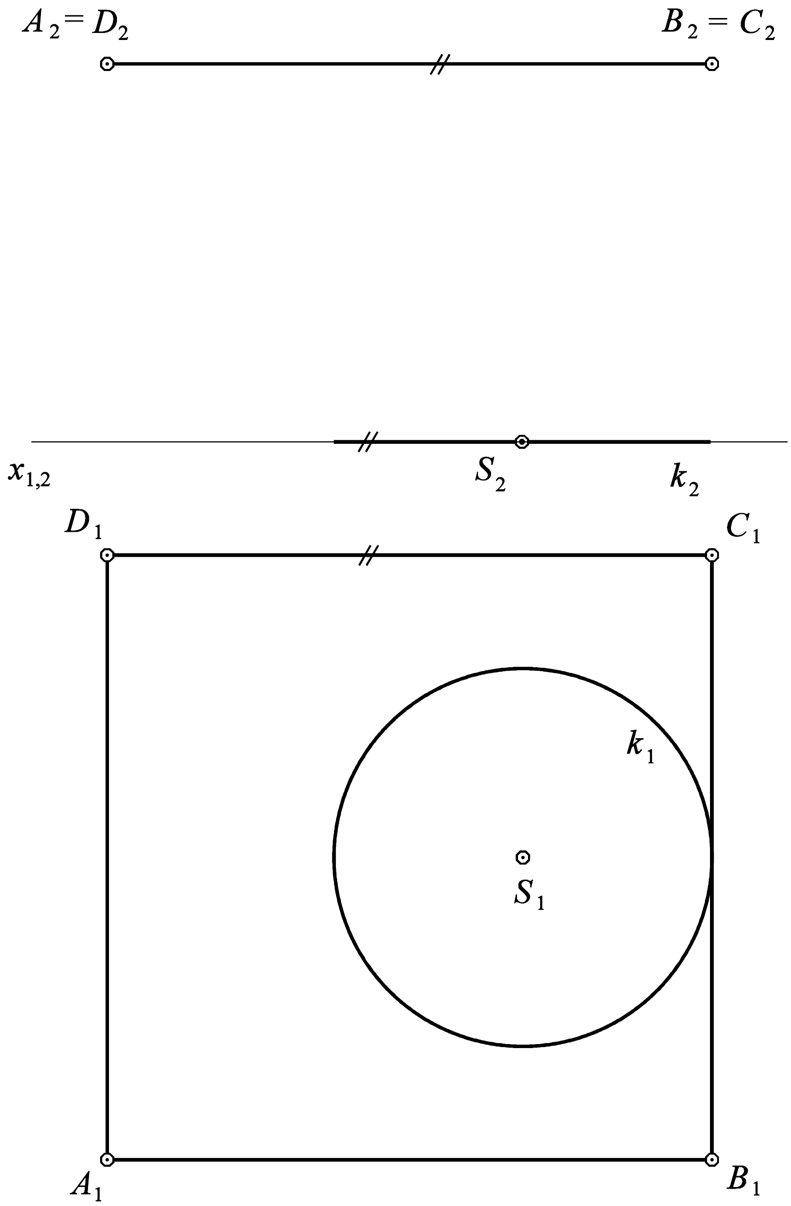
■ **Exercise 6.10.** Circle k and polyline $ABCD$ are given. Construct smooth developable transition surface between the circle k and the polyline $ABCD$. Use Monge projection.

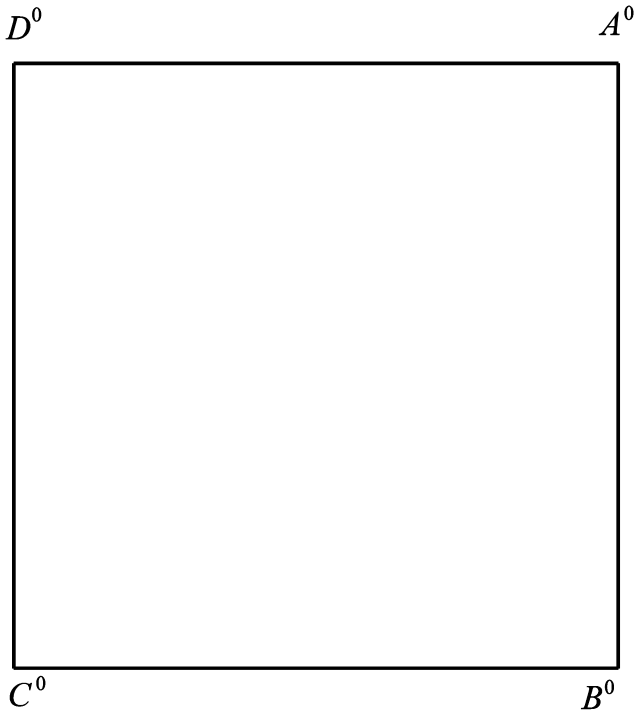


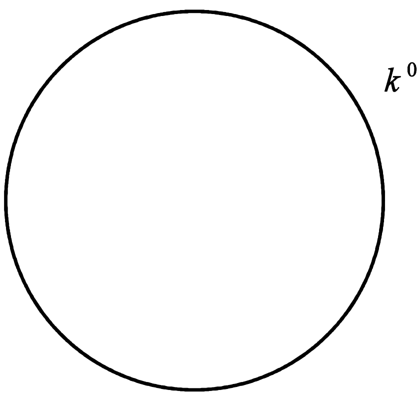
■ **Exercise 6.11.** Circle k and polyline $ABCD$ are given. Construct smooth developable transition surface between the circle k and the polyline $ABCD$. Use Monge projection.



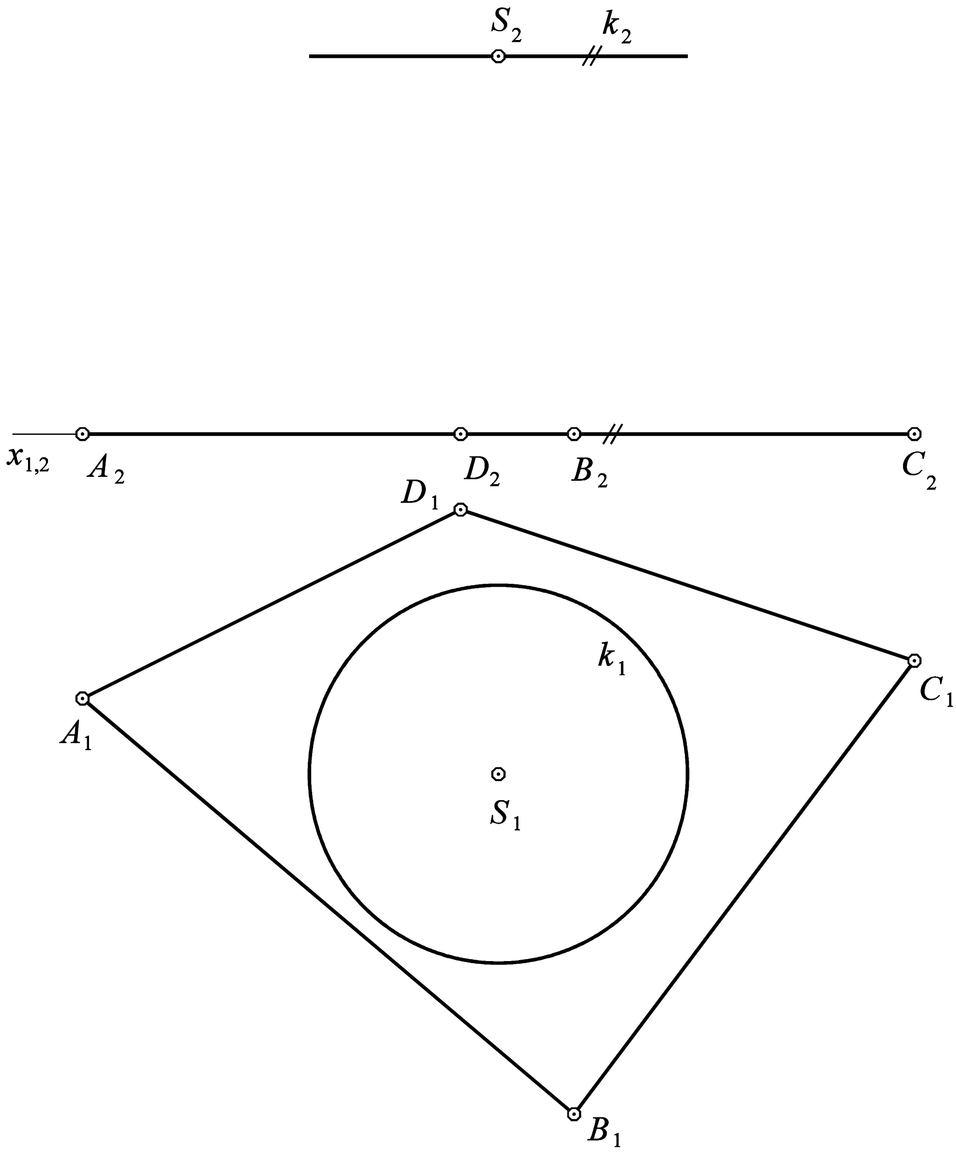
■ **Exercise 6.12.** Circle k and square $ABCD$ are given. Construct smooth developable transition surface between the circle k and the square $ABCD$. Use Monge projection.

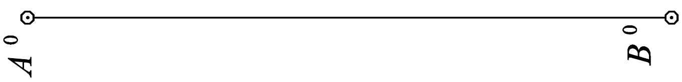



D^0
A^0
C^0
B^0


k^0

■ **Exercise 6.13.** Circle k and quadrilateral $ABCD$ are given. Construct smooth developable transition surface between the circle k and the quadrilateral $ABCD$. Use Monge projection.




A^0
B^0