Binomial theorem

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7

It states how to compute and express all terms in n-power of bracket (x+y):

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

i.e. without sum notation:

$$(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} x^0 y^n,$$

Human examples:

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + nx^{n-1} + x^n.$$

Lets compute:

a) $(1+\sqrt{2})^5$

b) $(1-2\sqrt[3]{3})^6$

c) $(x+\sqrt{x})^4$

d) $\left(y-\frac{1}{2y}\right)^5$

e) $(a\sqrt[3]{a}-3)^3$

f) $\left(\sqrt{\frac{m}{n}}-\sqrt{\frac{n}{m}}\right)^5$

76 a) $41+29\sqrt{2}$; b) $97-708\sqrt[3]{3}-516\sqrt[3]{9}$; c) $x^4+4x^3\sqrt{x}+6x^3+4x^2\sqrt{x}+x^2$; d) $y^5-\frac{5}{2}y^3+\frac{5}{2}y-\frac{5}{4y}+\frac{5}{16y^3}-\frac{1}{32y^5}$; e) $a^4-9a^2\sqrt[3]{a^2}+27a\sqrt[3]{a}-27$; f) $\sqrt{mn}(\frac{m^2}{n^3}-\frac{5m}{n^2}+\frac{10}{n}-\frac{10}{m}+\frac{5n}{m^2}-\frac{n^2}{m^3})$.

Verify that x1= sqrt(2) – 2 is root of:

79 Ověřte, že číslo $x=\sqrt{2}-2$ je kořenem rovnice $x^5-10x^3-24x-16=0$.

Which member of the power does contain y^3?

Který člen binomického rozvoje $(y^2+y^{-1})^9$ obsahuje y^3?

86 6. člen.

Prove:

a) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{n}=2^n$

b) $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\ldots+(-1)^n\binom{n}{n}=0$

c) $\binom{n}{0}-2\binom{n}{1}+2^2\binom{n}{2}-\ldots+(-2)^n\binom{n}{n}=(-1)^n$

d) $4^n+\binom{n}{1}4^{n-1}+\binom{n}{2}4^{n-2}+\ldots+1\binom{n}{n}=5^n$

e) $1+4\binom{n}{1}+4^2\binom{n}{2}+\ldots+4^n\binom{n}{n}=5^n$

99 Návody: a) $2^n=(1+1)^n$; b) $0=(1-1)^n$; c) $(-1)^n=(1-2)^n$; d) $5^n=(4+1)^n$; e) $5^n=(1+4)^n$.

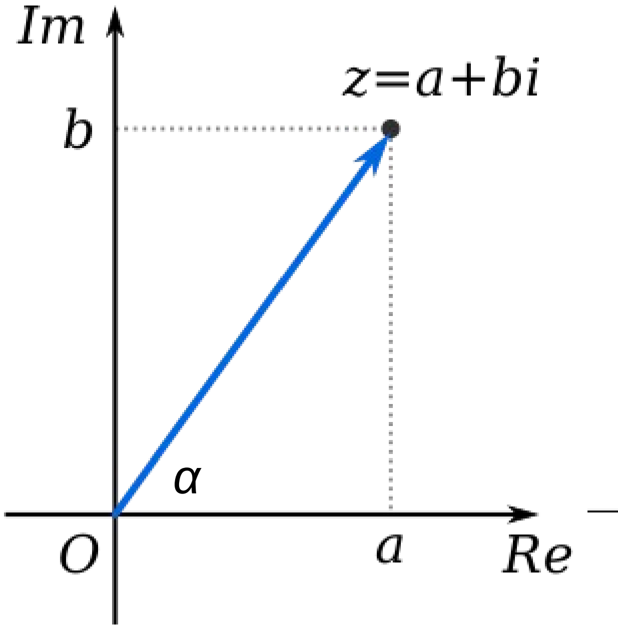
Complex numbers

Based on introduction of imaginary unit *i* such that

$$\mathrm{i}^2 = -1$$

Complex number consists of real and imaginary part, it can plot as point in complex plane

$$\mathrm{Re}(2+3i) = 2 \quad \text{and} \quad \mathrm{Im}(2+3i) = 3\,.$$



Algebraic form of complex number

$$z = a + b\mathrm{i}$$

Complex conjugated number

$$\overline{z} = a - b\mathrm{i}$$

Magnitude of complex number (also absolute value)

$$|z| = \sqrt{a^2 + b^2}$$

Polar form of complex number

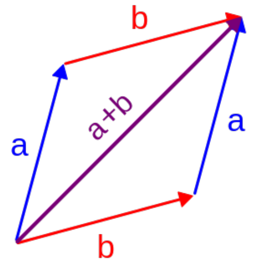
$$z = |z|(\cos\alpha + \mathrm{i}\sin\alpha)$$

Complex numbers

de Moivre's formula

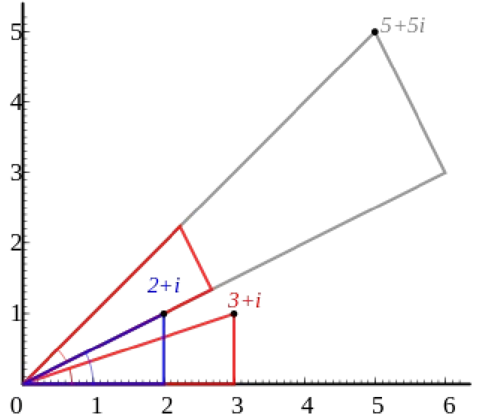
$$(\cos\alpha + \mathrm{i}\sin\alpha)^n = \cos n\alpha + \mathrm{i}\sin n\alpha,\ n \in N$$

Operations:



- addition and subtraction

$$a + b = (x + yi) + (u + vi) = (x + u) + (y + v)i.$$



- multiplication

$$(x + yi)\,(u + vi) = (xu - yv) + (xv + yu)i.$$

$$z_1 z_2 = r_1 r_2 (\cos(\varphi_1 + \varphi_2) + i\sin(\varphi_1 + \varphi_2)).$$

small example:

$$(2 + i)(3 + i) = 5 + 5i.$$

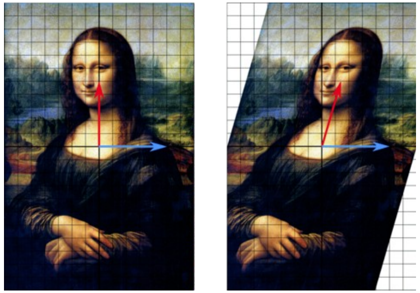
- reciprocal and division

$$\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{x - yi}{x^2 + y^2} = \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2}i.$$

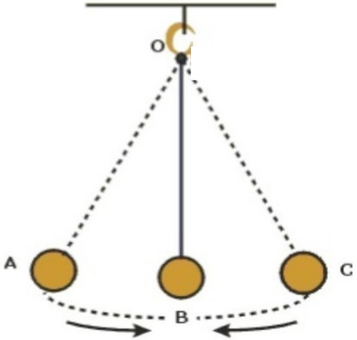
Applications of complex numbers

- Finding roots of polynomials

$$x^2 + 1 = 0$$

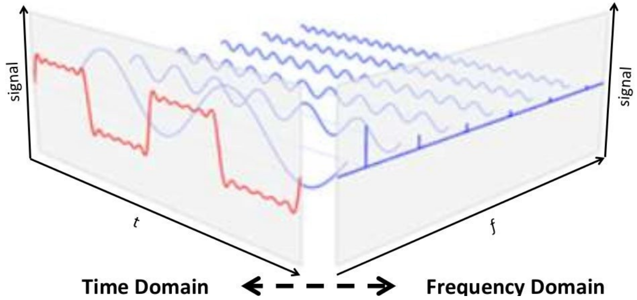


- Eigenvalues of matrices (i.e. characterization of matrix)



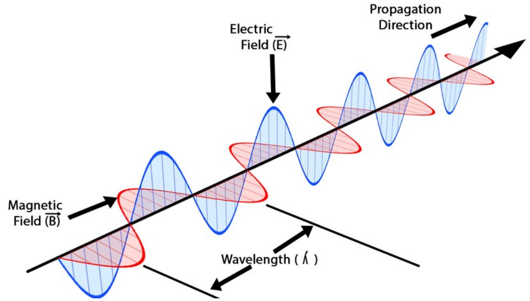
- Characterization of dynamical systems
    - determination of its stability

- Fourier analysis of signal
    - frequency spectrum



- Description of wave propagation

Electromagnetic Wave



Lets compute and verify according to Moivre’s formula:

78 Umocněte podle binomické věty i podle Moivreovy věty:

a) $(1+\mathrm{i})^7$ b) $(\sqrt{2}-\mathrm{i}\sqrt{2})^6$ c) $(-2+2\mathrm{i}\sqrt{3})^5$

78 a) $8-8\mathrm{i}$; b) $64\mathrm{i}$; c) $-512-512\mathrm{i}\sqrt{3}$.

PS: de Moivre's theorem states $(\cos x + i\sin x)^n = \cos nx + i\sin nx,$

Derive formulas by using Moivre’s formula and binomial theorem:

98 Užitím binomické věty a věty Moivreovy odvoďte vzorce pro

a) $\sin 3x$, $\cos 3x$, b) $\sin 5x$, $\cos 5x$.

98 a) $\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$; b) $\sin 5x = 5\sin x - 20\sin^3 x +$

$+ 16\sin^5 x$, $\cos 5x = 5\cos x - 20\cos^3 x + 16\cos^5 x$.

2 Vypočítejte: Compute

a) $\frac{3i+1}{2+i}$ c) $\frac{3i+2}{2i-3}$ e) $\frac{3+i\sqrt{3}}{3-i\sqrt{3}}$

b) $\frac{100}{3+4i}$ d) $\frac{1+4i}{i}$ f) $49(2-i\sqrt{3})^{-2}$

2 a) $1+i$; b) $12-16i$; c) $-i$; d) $4-i$; e) $\frac{1}{2}+\frac{\sqrt{3}}{2}i$; f) $1+4\sqrt{3}i$.

3 Vypočítejte:

a) $(2+i)\cdot i+\frac{3+i}{2-i}$ d) $(5i-1):\left(2-\frac{i+3}{2+i}\right)$

b) $\frac{2+i}{i}+\frac{i}{i+1}-\frac{2i+1}{i-1}$ e) $\frac{\frac{i}{2-i}+\frac{1}{i}}{1+\frac{1}{2i+1}}$

c) $-\frac{i-1}{2}-\frac{i}{i-1}\cdot i+1$

3 a) $3i$; b) 1; c) $1-i$; d) $1+8i$; e) $-\frac{1}{2}i$.

5b) Prove equality of number z1 and z2:

$$z_1=\sin 135^\circ - i\cos 270^\circ$$

$$z_2=\cos\frac{7}{4}\pi + i\sin 8\pi$$

b) $z_1=z_2=\frac{\sqrt{2}}{2}+0i$.

7) Prove that number $z=\frac{i}{p-3i}+\frac{i}{p+3i}$ for any real p is purely imaginary.

For what value of p is z = i /3 ?

7 $z=i\cdot\frac{2p}{p^2+9}$, $p=3$.

8) Determine for which real number b is expression $z = \dfrac{8 - 6b - ib}{1 - ib}$

a) real number, b) complex number, c) purely imaginary?

8 a) $b \in \{0; \frac{7}{6}\}$; b) $b \in \mathsf{R} - \{0; \frac{7}{6}\}$; c) $b \in \{2; 4\}$.

11 Vypočítejte: Compute

a) $i^2; i^3; i^4; i^{50}; i^{125}; i^{505}$
b) $5i^{100} - 3i^{10} + 12i^{75}$
c) $2i^9 - i^{12} + 5i^{16} - 3i^{11}$
d) $i^{-1}; i^{-2}; i^{-3}; i^{-4}; i^{-37}; i^{-78}$
e) $i^{-30} + i^{-40} + i^{-50} + i^{-60}$
f) $i^{-1} + 5i^{-6} - 14i^{-7}$

11 a) $-1; -i; 1; -1; i; i$; b) $8 - 12i$; c) $4 + 5i$; d) $-i; -1; i; 1; -i; -1$; e) 0; f) $-5 - 15i$.

12 Vypočítejte: Compute

a) $1 + i + i^2 + i^3 + i^4$
b) $1 + i^2 + i^4 + i^6 + i^8 + i^{10}$
c) $1 + i^3 + i^5 + i^7 + i^9 + i^{11}$
d) $1 + i^{-1} + i^{-2} + i^{-3} + i^{-4}$
e) $i \cdot i^2 \cdot i^3 \cdot i^4 \cdot i^5 \cdot i^6 \cdot i^7 \cdot i^8 \cdot i^9 \cdot i^{10}$
f) $i^2 \cdot i^4 \cdot i^6 \cdot i^8 \cdot i^{10} \cdot i^{12} \cdot i^{14} \cdot i^{16} \cdot i^{18} \cdot i^{20}$

Sum of geometric sequence ;-)

13 Vypočítejte mocniny následujících závorek: Compute

a) $(1+i)^2$; $(1-i)^2$; $(1+i)^{-2}$; $(1-i)^{-2}$
b) $(1+i)^3$; $(1-i)^3$; $(1+i)^{-3}$; $(1-i)^{-3}$
c) $(1+i)^4$; $(1-i)^4$; $(1+i)^{-4}$; $(1-i)^{-4}$

12 a) 1; b) 0; c) $1 - i$; d) 1; e) $-i$; f) -1. **13** a) $2i; -2i; -\frac{1}{2}i; \frac{1}{2}i$; b) $-2 \mp 2i$; $-2 - 2i; -\frac{1}{4} - \frac{1}{4}i; -\frac{1}{4} + \frac{1}{4}i$; c) $-4; -4; -\frac{1}{4}; -\frac{1}{4}$.

First plot complex numbers z1 and z2. Then graphically determine: a) and b) or c).

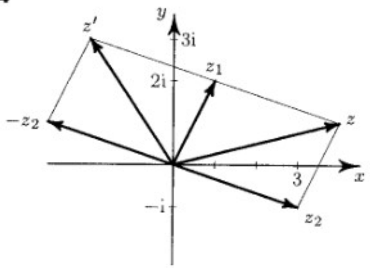
14 Nakreslete obrazy komplexních čísel $z_1 = 1 + 2\mathrm{i}$, $z_2 = 3 - \mathrm{i}$.
Potom graficky určete: a) $z = z_1 + z_2$ b) $z' = z_1 - z_2$

15 Nakreslete obrazy komplexních čísel $z_1 = 2 - \mathrm{i}$, $z_2 = -2 - 4\mathrm{i}$.
Potom graficky určete: a) $2z_1$ b) $\frac{1}{2}z_2$ c) $z = 2z_1 + \frac{1}{2}z_2$

16 Nakreslete obrazy komplexních čísel $z_3 = 1 + 2\mathrm{i}$, $z_4 = 2 - \mathrm{i}$.
Potom graficky určete: a) $z = z_3 \cdot z_4$ b) $z = z_3 : z_4$

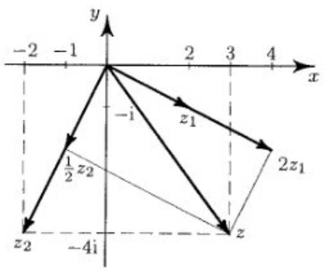
17 Nakreslete obraz komplexního čísla $z_5 = 2 + 3\mathrm{i}$.
Potom graficky určete: a) $\mathrm{i} \cdot z_5$ b) $-\mathrm{i} \cdot z_5$ c) $z_5 : \mathrm{i}$

14



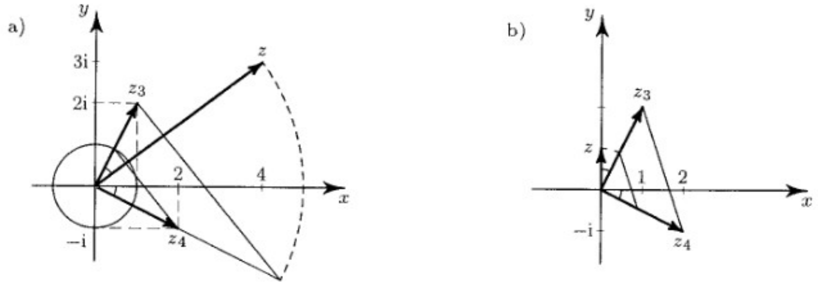
K řešení úlohy 14

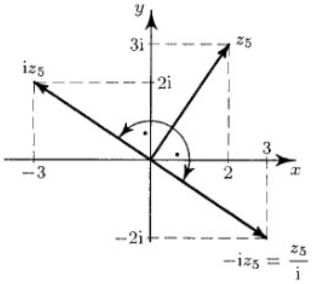
15



K řešení úlohy 15

16





K řešení úlohy 17

Find complex conjugated numbers:

19 Vypočítejte čísla komplexně sdružená k daným číslům:

$$w_1 = (2+i)(3-i) \qquad w_2 = \frac{3+4i}{1-2i} \qquad w_3 = \frac{4-2i}{i}$$

19 $\overline{w_1} = 7 - i$, $\overline{w_2} = -1 - 2i$, $\overline{w_3} = -2 + 4i$.

23 Compute

a) $|(7+i)(4-3i)|$

b) $\left|\frac{4-2i}{3+i}\right|$

c) $\left|\frac{10i}{2\sqrt{6}-2\sqrt{3}i}\right|$

d) $\left|\frac{|4-3i|+i}{3-2i}\right|$

e) $\dfrac{\left|\frac{3-4i}{5i}\right| \cdot \left|\frac{1+i}{3-i}\right|}{|2i-1| + |-i|}$

f) $\left|\frac{|\sqrt{3}-i| \cdot (i-1)}{|i(i-1)| - 2i}\right|$

23 a) $25\sqrt{2}$; b) $\sqrt{2}$; c) $\frac{5}{3}$; d) $\sqrt{2}$; e) $\frac{5-\sqrt{5}}{20}$; f) $\frac{2}{3}\sqrt{3}$.

25 Plot all numbers in complex plane for which hold:

a) $|z| = 3$

b) $|z-i| = 1$

c) $|z-1+i| = 2$

d) $|z-2| \leqq 3$

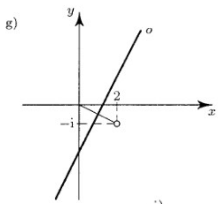
e) $|z-2-i| > 4$

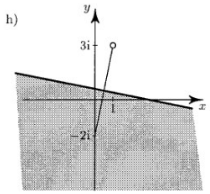
f) $1 < |z+3i-2| \leqq 4$

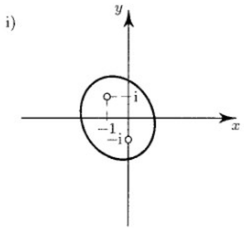
g) $|z| = |z-2+i|$

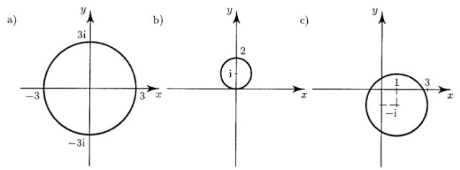
h) $|z-1-3i| \geqq |z+2i|$

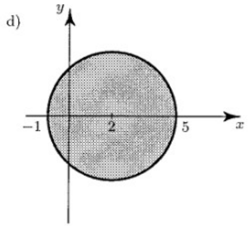
i) $|z+i| + |z+1-i| = 4$

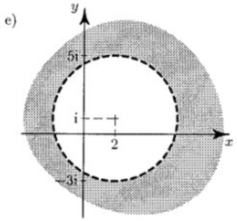


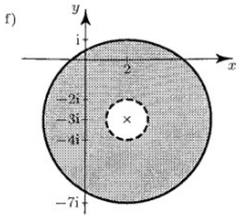












Find polar form of z:

30 Převeďte na goniometrický tvar následující komplexní čísla:

$z_1 = 1 + i$ $\quad z_4 = -2 + 2i\sqrt{3}$ $\quad z_7 = -7 - 7i$

$z_2 = 3$ $\quad z_5 = -\sqrt{3} + i$ $\quad z_8 = \sin 30° + i\cos 30°$

$z_3 = 5i$ $\quad z_6 = 10 - 10i$ $\quad z_9 = 1 + \cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi$

30 $z_1 = \sqrt{2}(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi)$, $z_2 = 3(\cos 0 + i\sin 0)$, $z_3 = 5(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$, $z_4 = 4(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi)$, $z_5 = 2(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi)$, $z_6 = 10\sqrt{2}(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi)$, $z_7 = 7\sqrt{2}(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi)$, $z_8 = 1(\cos\frac{1}{3}\pi + i\sin\frac{1}{3}\pi)$, $z_9 = \sqrt{2-\sqrt{2}}(\cos\frac{3}{8}\pi + i\sin\frac{3}{8}\pi)$.

35 Find absolute value of z and plot it in complex plane:

$z_1 = 5(\cos 120° + i\sin 120°)$ $\qquad z_2 = 2\left(\cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi\right)$

35 $|z_1| = 5 \wedge \varphi = 120°$, $|z_2| = 2 \wedge \varphi = \frac{4}{3}\pi$.

Compute product and division of numbers z1 and z2.

38 Vypočítejte součin a podíl komplexních čísel z_1, z_2. Výsledek vyjádřete v goniometrickém i v algebraickém tvaru.

a) $z_1 = 2(\cos 105° + i\sin 105°)$, $z_2 = 4(\cos 225° + i\sin 225°)$

b) $z_1 = 2(\cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi)$, $z_2 = 4(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi)$

38 a) $z_1 \cdot z_2 = 8(\cos 330° + i\sin 330°) = 4\sqrt{3} - 4i$, $z_1 : z_2 = 0{,}5[\cos(-120°) + i\sin(-120°)] = -\frac{1}{4} - \frac{\sqrt{3}}{4}i$; b) $z_1 \cdot z_2 = 8(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi) = 4\sqrt{3} - 4i$, $z_1 : z_2 = 0{,}5(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi) = -0{,}5i$. **39** a) $z_1 = \frac{8}{5}\sqrt{10} - \frac{6}{5}\sqrt{10} \cdot i$; b) $z_2 = -2\sqrt{2} - \sqrt{2} \cdot i$;

With the help of Moivre's theorem compute:

42 Užitím Moivreovy věty umocněte a výsledek převeďte do algebraického tvaru:

a) $\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)^6$ c) $(1+i)^6$ e) $(-2\sqrt{3}-2i)^{12}$

b) $\left(\cos\frac{3\pi}{32} + i\sin\frac{3\pi}{32}\right)^8$ d) $(1-i\sqrt{3})^5$ f) $(5\sqrt{3}-5i)^7$

42 a) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$; b) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$; c) $-8i$; d) $16 + 16\sqrt{3}i$; e) $2^{24} + 0i$; f) $-5 \cdot 10^6 \cdot \sqrt{3} + 5 \cdot 10^6 i$. **43** $n = 12k + 1 \wedge k \in \mathrm{N}$ nebo $k = 0$.

46 Vypočítejte všechny druhé komplexní odmocniny

a) z čísla 4, b) z čísla −4.

47 Vypočítejte všechny čtvrté komplexní odmocniny

a) z čísla i, b) z čísla 1 − i.

48 Vypočítejte všechny páté komplexní odmocniny z čísla 32.

49 Vypočítejte součet všech třetích komplexních odmocnin z čísla −2.

50 Vypočítejte součet třetích mocnin všech čtvrtých odmocnin z čísla 1.

= Square root of

= 4-th root of

= 5-th root of

= sum of all third roots of

= sum of all third roots of

46 a) $z_{1,2} = 2(\cos k\pi + i\sin k\pi)$, $k \in \{0;1\}$; b) $z_{1,2} = 2[\cos(\frac{\pi}{2} + k\pi) + i\sin(\frac{\pi}{2} + k\pi)]$, $k \in \{0;1\}$.

47 a) $z_{1,2,3,4} = \cos(\frac{\pi}{8} + \frac{k\pi}{2}) + i\sin(\frac{\pi}{8} + \frac{k\pi}{2})$, $k \in \{0;1;2;3\}$;

b) $z_{1,2,3,4} = \sqrt[8]{2}[\cos(\frac{7}{16}\pi + \frac{k\pi}{2}) + i\sin(\frac{7}{16}\pi + \frac{k\pi}{2})]$, $k \in \{0;1;2;3\}$.

48 $z_{1,2,3,4,5} = 2(\cos\frac{2}{5}k\pi + +i\sin\frac{2}{5}k\pi)$, $k \in \{0;1;2;3;4\}$. **49** 0. **50** 0.

53 Řešte rovnice s neznámou $z \in \mathbb{C}$:

a) $2z + 3\overline{z} = 5 + \mathrm{i}$

b) $\left(2 - \frac{1}{\mathrm{i}}\right)\overline{z} - 13 = 2(6{,}5\mathrm{i} - z)$

c) $z\overline{z} - z = \overline{6 - 2\mathrm{i}}$

d) $z(\overline{z} - 4) - 1 = 8\mathrm{i}$

= Solve for complex z:

53 a) $z = 1 - \mathrm{i}$; b) $z = 13 - 39\mathrm{i}$; c) $z_1 = -1 - 2\mathrm{i}$, $z_2 = 2 - 2\mathrm{i}$; d) $z_1 = 1 - 2\mathrm{i}$, $z_2 = 3 - 2\mathrm{i}$.

55 Řešte rovnice s neznámou $z \in \mathbb{C}$:

a) $|z| = 1 + 2\mathrm{i} + z$

b) $|z + \mathrm{i}| = 2z + \mathrm{i}$

c) $|z + 1| - 4\mathrm{i} = z + 3$

d) $|z + 2 - \mathrm{i}| = 5(z + 3\mathrm{i})$

55 a) $z = \frac{3}{2} - 2\mathrm{i}$; b) $z = \frac{\sqrt{3}}{6} - \frac{1}{2}\mathrm{i}$; c) $z = 2 - 4\mathrm{i}$; d) $\overline{z} = 1 - 3\mathrm{i}$.

Find solutions of equations, plot them:

68 Řešte rovnice s neznámou $x \in \mathbb{C}$. Výsledek zapište nejprve v goniometrickém tvaru, pak ve tvaru algebraickém. Kořeny znázorněte v Gaussově rovině.

a) $x^3 - 27 = 0$

b) $x^4 + 16 = 0$

c) $x^6 - 1 = 0$

d) $x^3 - 64\mathrm{i} = 0$

69 Řešte rovnice s neznámou $x \in \mathbb{C}$. Výsledky zapište v goniometrickém tvaru. Kořeny znázorněte v Gaussově rovině.

a) $x^3 - 1 - \mathrm{i} = 0$

b) $x^6 - 1 + \mathrm{i}\sqrt{3} = 0$

c) $(\mathrm{i}x)^4 + \sqrt{3} - \mathrm{i} = 0$

d) $(2x)^5 - 16 = 16\mathrm{i}\sqrt{3}$

70 Uvedené rovnice s neznámou $x \in \mathbb{C}$ řešte dvěma způsoby. Buď jako rovnice kvadratické, nebo jako rovnice binomické.

a) $x^2 = 1 + \mathrm{i}\sqrt{3}$

b) $x^2 + 2x + 5 = 0$

68 a) $x_{1,2,3} = 3(\cos\frac{2}{3}k\pi + \mathrm{i}\sin\frac{2}{3}k\pi)$, $k \in \{0;1;2\}$, $x_1 = 3$, $x_{2,3} = -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}\mathrm{i}$;
b) $x_{1,2,3,4} = 2[\cos(\frac{1}{4}\pi + \frac{1}{2}k\pi) + \mathrm{i}\sin(\frac{1}{4}\pi + \frac{1}{2}k\pi)]$, $k \in \{0;1;2;3\}$, $x_{1,2} = \pm\sqrt{2} + \sqrt{2}\mathrm{i}$, $x_{3,4} = \pm\sqrt{2} - \sqrt{2}\mathrm{i}$; c) $x_{1,2,3,4,5,6} = 1(\cos\frac{1}{3}k\pi + \mathrm{i}\sin\frac{1}{3}k\pi)$, $k \in \{0;1;2;3;4;5\}$, $x_{1,2} = \pm 1$, $x_{3,4} = \pm\frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$, $x_{5,6} = \pm\frac{1}{2} - \frac{\sqrt{3}}{2}\mathrm{i}$;
d) $x_{1,2,3} = 4[\cos(\frac{1}{6}\pi + \frac{2}{3}k\pi) + \mathrm{i}\sin(\frac{1}{6}\pi + \frac{2}{3}k\pi)]$, $k \in \{0;1;2\}$, $x_{1,2} = \pm 2\sqrt{3} + 2\mathrm{i}$, $x_3 = -4\mathrm{i}$.

69 a) $x_{1,2,3} = \sqrt[6]{2}[\cos(\frac{1}{12}\pi + \frac{2}{3}k\pi) + \mathrm{i}\sin(\frac{1}{12}\pi + \frac{2}{3}k\pi)]$, $k \in \{0;1;2\}$;
b) $x_{1,2,3,4,5,6} = \sqrt[6]{2}[\cos(\frac{5}{18}\pi + \frac{1}{3}k\pi) + \mathrm{i}\sin(\frac{5}{18}\pi + \frac{1}{3}k\pi)]$, $k \in \{0;1;2;3;4;5\}$;
c) $x_{1,2,3,4} = \sqrt[4]{2}[\cos(\frac{5}{24}\pi + \frac{1}{2}k\pi) + \mathrm{i}\sin(\frac{5}{24}\pi + \frac{1}{2}k\pi)]$, $k \in \{0;1;2;3\}$;
d) $x_{1,2,3,4,5} = 1 \cdot [\cos(\frac{1}{15}\pi + \frac{2}{5}k\pi) + \mathrm{i}\sin(\frac{1}{15}\pi + \frac{2}{5}k\pi)]$, $k \in \{0;1;2;3;4\}$.

70 a) $x_{1,2} = \pm\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}\mathrm{i}$; b) $x_{1,2} = -1 \pm 2\mathrm{i}$.