Probability

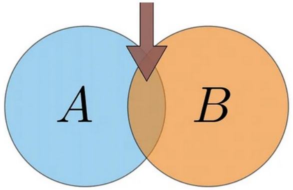
Probability is simply how likely something is to happen.

This is typical in scenario, where you can not determine directly the result, but you can estimate the chance of positive or negative outcome.

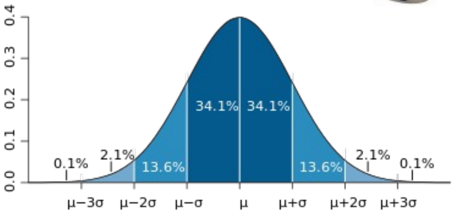
The best example for understanding probability is flipping a coin:
There are two possible outcomes—heads or tails.

Probability of A = (# of ways A can happen) / (total number of outcomes)

Example: What’s the probability of rolling an even number on a dice?



P(even) = 3 / 6 = 1/2



What is the probability of rolling the red and blue dice with result:

a) 6 on both of them
b) odd number on both of them
c) odd at least on one of them
d) sum of points equals 5
e) sum of points smaller than 5?

3 a) $\frac{1}{36}$; b) $\frac{1}{4}$; c) $\frac{3}{4}$; d) $\frac{1}{9}$; e) $\frac{1}{6}$.

We roll the dice three times. What is the probability of:

a) exactly once 6
b) no 6 at all
c) at most one 6
d) at least one 6?

6 a) $\frac{25}{72}$; b) $\frac{125}{216}$; c) $\frac{25}{27}$; d) $\frac{91}{216}$.

What is the probability of flipping with two coins with result:

a) both coins show reverse
b) at least one shows reverse?

What is the probability of flipping with three coins we get a) at least two faces ?

What is the probability of flipping three times with one coins we get b) at least two faces ?

10 a) $\frac{1}{4}$; b) $\frac{3}{4}$. **11** a) $\frac{1}{2}$; b) $\frac{1}{2}$.

How many times do we have to roll the dice so that at least one six is rolled with probability greater than 75%?

14 Alespoň 8×.

How many times do we have to roll the dice pair so that two six is rolled with probability greater than 80%?

15 Alespoň 58×.

We roll the dice 5 times. What is the probability of rolling a six twice?

18 0,161.

We roll the dice 10 times. What is the probability of rolling a six at least three times?

20 0,225.

What is more probable in 10 rolling: a) to get at least ten times 6?
b) to get at most ten times 6?

21 a) 0,0006; b) 0,9999.

What is the probability that Jane and Peter were born in the same month?

24 0,083.

What is the probability that Jane and Peter were born in the same day?

26 0,003.

We pick up randomly 3 students from group of 10 boys and 18 girls. What is the probability of choosing 2 boys and 1 girl?

27 0,247.

There are 30 students in a class. Homework was not done by 5 of them. What is probability of at most two from controlled 6 have not done it?

28 0,959.

The shooter hits the target 92 out of 100 attempts.

a) What is the probability of hitting the target?

b) What is the probability that the target will not be hit?

c) What is the probability of hitting the target twice in two shots?

d) What is the probability that the target will be hit at least once in three shots?

31 a) 0,920; b) 0,080; c) 0,846; d) 0,999.

Two shooters shoot independently at each other at a target. The first hits it with a 60% probability and the second with an 80% probability. If each of them shot one shot, what is the probability of:

(a) neither of them hits the target

(b) one of them hits the target

(c) they both hit the target

(d) what is the sum of the probabilities (a) to (c)?

33 a) 0,08; b) 0,44; c) 0,48; d) 1.

We randomly select a four-digit number. What is the probability that the digit 8 is in its notation:

(a) just once b) right twice c) at least once d) in the second place?

39 a) 0,297; b) 0,051; c) 0,352; d) 0,100.

There are 40 products in the box, and 6 of them are defective. We will pick 5 products. What is the probability that:

(a) there will be 3 defective products among those selected.

(b) at least two of the selected products are defective

(c) there will be at most one defective product among the selected ones?

54 a) 0,017; b) 0,154; c) 0,846.

There are 9 balls in the fate and they are marked with numbers from 1 to 9. We will draw 7 balls. What is the probability that the last two balls are marked with even numbers?

$$\left[v = \binom{9}{2}\right] \cdot \left[p = \binom{4}{2}\right] \quad \left[\frac{p}{v} = \frac{1}{6}\right]$$

A new treatment method is being tested on mice. The probability that treatment with this method is **successful in at least one of two randomly selected mice is 84%**. The results of the treatment of mice are mutually independent. Calculate the probability that one randomly selected mouse will be successfully treated with the new method.

How to calculate P(A)? Through complementary probability.

Treatment unsuccessful for both mice ………………… 1 – 0.84 = 0.16
Formula for it is: P(A') times P(A') (independent phenomena)
P(A') = 0.4

And then P(A) = 1 – P(A') = 60%

Opačný jev: léčení je neúspěšné u obou, pravděpodobnost …………… 1-0,84
Léčení je neúspěšné u jedné myši a zároveň léčení je neúspěšné u jedné myši (nezávislé jevy).
Jev A: léčení je neúspěšné u jedné myši. $P(A) \cdot P(A) =$ …………… [1-0,84=0,16]
P(A) …………… [$\sqrt{0,16}$]
Opačný jev k A: léčení je úspěšné u jedné myši.
Pravděpodobnost $P(A')$ …………… [0,6]

Conditional probability

Until now we assume that phenomena are mutually independent.
This can be described as:

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B).$$

It changes if the events A and B are somehow interconnected / in some relation.

For example, the probability that any given person has a cough on any given day may be only 5%.
But if we know or assume that the person is sick, then they are much more likely to be coughing.
For example, the conditional probability that someone unwell (sick) is coughing might be 75%,
in which case we would have that P(Cough) = 5% and P(Cough|Sick) = 75 %.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

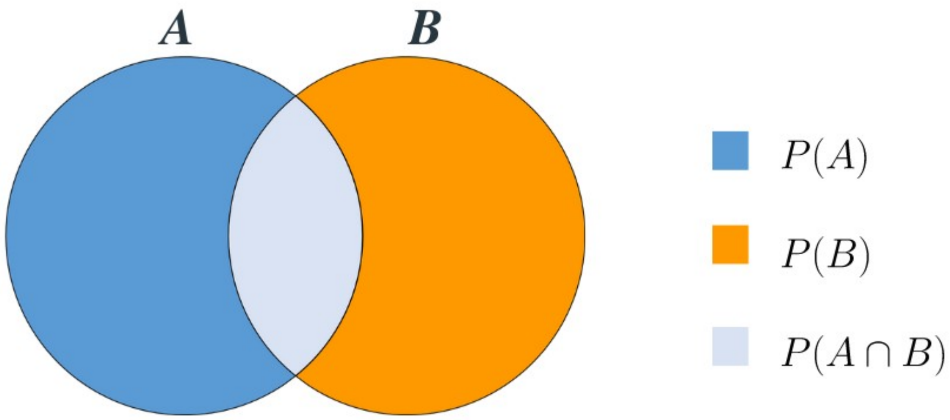
If P(A|B) = P(A), then events A and B are independent.

This can also be understood as the fraction of probability B that intersects with A,
or the ratio of the probabilities of both events happening to the "given" one happening
(how many times A occurs rather than not assuming B has occurred)

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = P(A \mid B)P(B)$$

Conditional Probability

The probability of **event A** given the information that **event B** has already taken place.

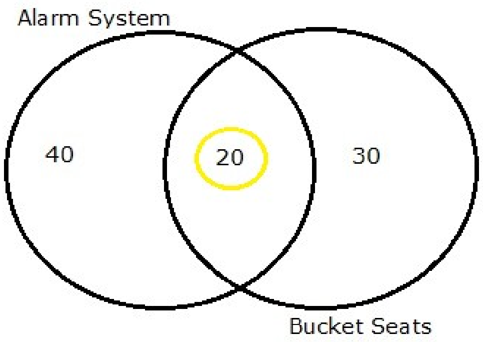


$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

In a group of 100 sports car buyers, 40 bought alarm systems, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random <u>bought an alarm system</u>, what is the probability they **also bought bucket seats**?

Step 1: Figure out P(Alarm). It's given in the question as 40%, or 0.4.
Step 2: Figure out $P(A \cap B)$. This is the intersection of A and B: both happening together. It's 20 out of 100 buyers, or 0.2.

Step 3: Insert your answers into the formula:
$P(B|A) = P(A \cap B) / P(A) = 0.2 / 0.4 = 0.5$, i.e. **50%.**



This question uses the following contingency table: conditional contingency

	Have pets	Do not have pets	Total
Male	0.41	0.08	0.49
Female	0.45	0.06	0.51
Total	0.86	0.14	1

What is the probability a randomly selected person is male, given that they own a pet?

$P(M|Pet) = P(M \cap Pet) / P(Pet) =$
$= 0.41 / 0.86 = 0.477$, or 47.7%.

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We roll the white and black dice. The sum of points are 9. What is the (conditional) probability of rolling a 5 on the white die?

To determine is $P(W5 \mid Sum9)$ according to $P(W5 \cap Sum9) / P(Sum9)$
$P(Sum9) = \{ (3,6)\ (4,5)\ (5,4)\ (6,3) \}$
$P(W5) = \{ (5,1)\ (5,2)\ \dots\ (5,6) \}$, but $P(W5 \cap Sum9) = \{(5,4)\}$
Thus

$$P(W5 \mid Sum9) = P(W5 \cap Sum9) / P(Sum9) = 1/4.$$

There are 9 white balls and 1 red ball. We draw one ball, return it and add ball of the same color. Then we draw a second time. What's the probability of drawing a red in both draws?

We would like to find $P(R1 \cap R2)$ as $P(R1)\, P(R2 \mid R1)$.
$P(R1) = 1/10$ easy.
$P(R2 \mid R1) = 2/11$ as there are 2 red balls of 11 total.
Then

$$P(R1 \cap R2) = 1/10 \times 2/11 = 0.018.$$

In the first seed there are 9 white balls and 1 red ball, in the second seed there are 6 white balls and 3 red balls. We select randomly seed and draw a ball. What's the probability that it is white?

We would like to find $P(W)$ as $P(Seed1)\, P(W| S1) + P(Seed2)\, P(W| S2) = 9/20 + 6/20 =$ **3/4**,
where $P(S1) = P(S2) = 1/2$ easy and $P(W| S1) = 9/10$, $P(W| S2) = 6/10$.

When Morse code is transmitted, there is a certain probability that the "dot" or "dash" that was received is erroneous. This is often taken as interference in the transmission of a message. Therefore, it is important to consider when sending a "dot", for example, the probability that a "dot" was received. This is represented by: $P(\text{dot sent} \mid \text{dot received}) = P(\text{dot received} \mid \text{dot sent}) \frac{P(\text{dot sent})}{P(\text{dot received})}$. In Morse code, the ratio of dots to dashes is 3:4 at the point of sending, so the probability of a "dot" and "dash" are $P(\text{dot sent}) = \frac{3}{7} \; and \; P(\text{dash sent}) = \frac{4}{7}$. If it is assumed that the probability that a dot is transmitted as a dash is 1/10, and that the probability that a dash is transmitted as a dot is likewise 1/10, then Bayes's rule can be used to calculate $P(\text{dot received})$.

$$P(\text{dot received}) = P(\text{dot received} \cap \text{dot sent}) + P(\text{dot received} \cap \text{dash sent})$$

$$P(\text{dot received}) = P(\text{dot received} \mid \text{dot sent}) P(\text{dot sent}) + P(\text{dot received} \mid \text{dash sent}) P(\text{dash sent})$$

$$P(\text{dot received}) = \frac{9}{10} \times \frac{3}{7} + \frac{1}{10} \times \frac{4}{7} = \frac{31}{70}$$

Now, $P(\text{dot sent} \mid \text{dot received})$ can be calculated:

$$P(\text{dot sent} \mid \text{dot received}) = P(\text{dot received} \mid \text{dot sent}) \frac{P(\text{dot sent})}{P(\text{dot received})} = \frac{9}{10} \times \frac{\frac{3}{7}}{\frac{31}{70}} = \frac{27}{31}$$ [15]

There is a rare illness, which occurs in 5% of men and 1% of women. If population consists of 55% women and 45% men, what is average occurrence of it / what is probability of occurrence of this illness for random individual?

[2.8%]

There are two manufacturing lines, which the first one has probability of producing a scrap 4 ‰ and the second one 6 ‰. The production runs from 40% on the first one and from 60% on the second one. What is a probability of scrap production?

[6.4‰]

Imagine that you’re a furniture salesman. The probability of a new customer to your store purchasing a couch on any particular day is 30%. However, if they are entering your store in the month leading up to the Super Bowl, the probability might be 70%. We can represent the conditional probability of selling a couch if it is the month leading up to Super Bowl as P(Selling a couch | Super Bowl month) where the | symbol means “given that”. This conditional probability gives us a way to express probabilities when our beliefs change about the probability of one event happens (a couch sale in this example) given that a certain event has happened (in this case, the advent of the month preceding the Super Bowl).

The probability of a woman between 40 and 50 years of having breast cancer is about 1%. However, this probability changes if a woman has a positive mammogram: the probability a woman has cancer if she has a positive mammogram result rises to about 8.3% [1].

Four candidates A, B, C, and D are running for a political office. Each has an equal chance of winning: 25%. However, if candidate A drops out of the race due to ill health, the probability will change: P(Win | One candidate drops out) = 33.33%.

https://www.statisticshowto.com/probability-and-statistics/statistics-definitions/conditional-probability-definition-examples/