

PREPARATORY COURSE FROM MATHEMATICS

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Introduction

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Content

- Numbers, logic
- Algebraic expressions
- Equations and system of linear equations
- Inequalities
- Functions and their properties
- Arithmetic and geometric sequences
- Geometry
- Analytic geometry
- Combinatorics, probability, statistics

Course content can be adjusted to your needs.

Basic Sets

Natural numbers - $\mathbb{N}$ 1, 2, 3 ... positive integers, 0 is not included

Integers - $\mathbb{Z}$ -3, -2, -1 ,0 ,1 ,2 ,3

Rational numbers - $\mathbb{Q}$ $\frac{-2}{5}, \frac{123}{68}, \frac{-256452}{23458} \dots$ can be expressed as a fraction

Real numbers - $\mathbb{R}$ $\frac{26}{37}, \sqrt{2} \dots$ all ordinary numbers ($\pm\infty$ not included)

.
(+ complex numbers - $\mathbb{C}$)

Basic Arithmetic Operations

Addition: $1 + 2 = 3$ one plus two equals three

$1 + 2$ is a **sum**

1 and 2 are **addends**

Subtraction: $3 - 2 = 1$ three minus two equals one

$3 - 2$ is a **difference**

3 is the **minuend**, 2 is the **subtrahend**

Multiplication: $2 \cdot 3 = 6$ two times three equals six

$2 \cdot 3$ is a **product**

2 and 3 are **factors**

Division: $6 : 2 = 3$ six divided by two equals three

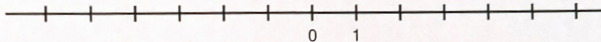
$6 : 2$ is a **quotient**

6 is the **dividend**, 2 is the **divisor**

Basic Sets - Examples I

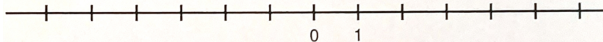
Example 1

Plot interval $\langle 2 - n; n - 3 \rangle$ for $n = 5$.



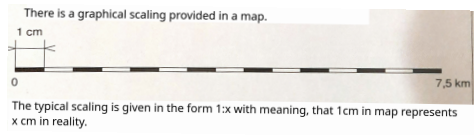
Example 2

Find the smallest natural number n such that interval $\langle 2 - n; n - 3 \rangle$ exists. Plot this interval on numerical axis.



Basic Sets - Examples II

Example 3

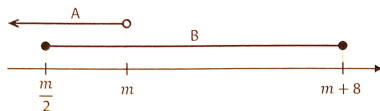


State the scale of the map.

Example 4

There are highlighted intervals A and B.

It holds: $A \cup B = (-\infty; 14)$



Write set B as an interval. Replace endpoints expressed with m by numbers.

Basic Sets - Examples III

Example 5

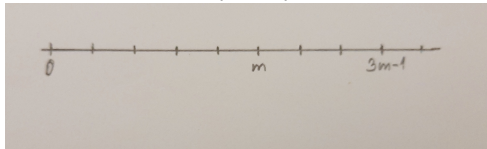
M is the set of all real numbers that simultaneously satisfy two conditions:

- all $x \in M$ are smaller than 4,
- the absolute value of numbers x is greater than or equal to 4.

Write set M as an interval.

Example 6

On the number line, three numbers are represented: 0, m , $3m - 1$. The marked divisions (units) are equally long.



Express the ratio $m : (3m - 1)$

Mark and describe the position of the number 1 on the number line.

Algebraic Expressions and Their Simplification - Basic Identities

- simplifying fractions

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \quad \Rightarrow \quad \frac{\frac{2}{3}}{\frac{6}{4}} = \frac{2 \cdot 4}{3 \cdot 6} = \frac{8}{18} = \frac{4}{9}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}, \quad \Rightarrow \quad \frac{1+3}{4} = \frac{1}{4} + \frac{3}{4} \quad \text{definitely not} \quad \frac{4}{1+3} \neq \frac{4}{1} + \frac{4}{3}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \quad \Rightarrow \quad \frac{1}{4} + \frac{6}{8} = \frac{1 \cdot 8 + 6 \cdot 4}{4 \cdot 8} = \frac{32}{32} = 1$$

- powers

$$a^{-b} = \frac{1}{a^b} \quad \Rightarrow \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$a^{\frac{b}{c}} = \sqrt[c]{a^b} \quad \Rightarrow \quad 4^{\frac{3}{2}} = \sqrt[2]{4^3} = 2^3 = 8$$

$$a^b \cdot a^c = a^{b+c} \quad \Rightarrow \quad 2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$$

$$(a^b)^c = a^{b \cdot c} \quad \Rightarrow \quad (2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$$

$$a^2 - b^2 = (a - b)(a + b) \quad \Rightarrow \quad 9 - x^2 = (3 - x)(3 + x)$$

Algebraic Expressions - Examples I

Example 1

Calculate

$$[10^4 - (8 \cdot 10^4 - 73 \cdot 10^3)]^2 \quad [9 \cdot 10^6]$$

Example 2

Simplify to a power with base 9

$$81^{90} \cdot 3^{300} \quad [9^{330}]$$

Example 3

For $a > 0$, calculate and simplify

$$\frac{a^3}{2^2} - \left(\frac{2}{a}\right)^{-3} \quad \left[\frac{a^3}{8}\right]$$

Example 4

For $d \geq 0$, simplify

$$\sqrt{2d^3} \cdot \sqrt{18d} \quad [6d^2]$$

Algebraic Expressions - Examples II

Example 5

For $x \in \mathbf{R} \setminus \{-2, 0\}$, simplify

$$\frac{(x-2)(x+4)}{x+2} : x^2 + \frac{8}{x+2} \quad \left[\frac{1}{x}\right]$$

Example 6

For $x \in \mathbf{R} \setminus \{-3, 0, 3\}$, simplify

$$\left(\frac{1}{x} + \frac{3}{x^2-3x}\right) \frac{1}{x^2-9} \quad [x+3]$$

Example 7

Two expressions $\frac{x}{x+1}$, $\frac{-1}{x^2+x}$ with variable $x \in \mathbf{R}$ are given. Decide for each of the following statements whether it is true (YES) or not (NO).

- For $x = -1$, the first of the two expressions is defined.
- For $x = 1$, the second of the two expressions is defined.
- A common denominator of the two expressions can be $x^2 + x$.
- For positive values of the variable x , the sum of the two expressions is equal to $\frac{x-1}{x}$

[NO, YES, YES, YES]

Algebraic Expressions - Examples III

Example 8

Express the resulting expression as a single term, without parentheses

$$(-n^4)^{-1} - n^{-4} + (-n)^{-4} \quad \left[\frac{-1}{n^4}\right]$$

Example 9

Simplify and state for which values the expression is defined.

$$\left(\frac{x-1}{x-2} - \frac{x}{x-1}\right) \left(x - \frac{3x}{x+1}\right) \quad \left[\frac{x}{x^2-1}\right] \quad x \neq \pm 1, x \neq 2$$

Example 10

Simplify and state for which values the expression is defined.

$$\left(-\frac{16}{5}\right) \cdot \sqrt{\frac{2}{5}} + \frac{\frac{64}{25}}{2 \cdot \sqrt{\frac{2}{5}}} \quad [0]$$

Example 11

Simplify and state for which values the expression is defined.

$$(p+q) : \left(\frac{1}{p} + \frac{1}{q}\right) \quad [pq, p \neq 0, q \neq 0, p+q \neq 0]$$

Algebraic Expressions - Examples IV

Example 12

For $a \in \mathbf{R}$, the following expression is given

$$\frac{a - \frac{1}{a}}{1 - a^2}$$

- Simplify the expression. $[-\frac{1}{a}]$
- Determine for which values of a the expression is defined. $[a \neq -1, 0, 1]$

Example 13

Solve the equation in the domain $\mathbf{R}$

$$\frac{x+10}{x} + \frac{100}{10x-x^2} = \frac{x+20}{x-10} \quad [x = -10, x \neq 0, 10]$$

Example 14

Solve the equation in the domain $\mathbf{R}$

$$5 + \frac{3}{3u-12} = \frac{5-u}{u-4} \quad [\text{no solution}]$$

Examples with Algebraic Expressions I

Example 1

The expression $\frac{1-x}{x-7} + 1$ is given. Determine all $x \in \mathbf{R}$ for which the value of the given expression is negative.

$$[x \in (7, +\infty)]$$

Example 2

By dissolving 2 grams of an active substance in water, we prepared a solution. The mass of the active substance constitutes 5% of the mass of the solution. Calculate how many grams of water were used to dissolve the active substance.

$$[38g]$$

Example 3

The cleaning of a liquid takes place in three stages. The second stage lasts one third longer than the first stage, and the third stage lasts twice as long as the second stage. Calculate what percentage of the total cleaning time is taken by the first stage.

$$[20\%]$$

Examples with Algebraic Expressions II

Example 4

One side of a rectangle is one fifth shorter than the side of a square, and the areas of both shapes are equal. Let a denote the side length of the square. Express the length of the longer side of the rectangle in terms of a .

$$\left[\frac{5}{4}a\right]$$

Example 5

Only one fifth of the PET bottles produced are not sorted. Of the sorted PET bottles, 70% are recycled. (The unsorted bottles are not recycled.) Calculate what percentage of the PET bottles produced are recycled.

$$[56\%]$$

Examples with Algebraic Expressions III

Example 6

Each horse consumes the same amount of food per day.

A farmer had enough food for his ten horses for 80 days. He sold a neighbouring farmer such an amount of food that her four horses would consume in 25 days. He kept the remaining food. The female farmer paid the farmer 50 Czech crowns for each daily portion of food for one horse.

- How many Czech crowns did she pay the farmer for the food?

[5000]

- How many days will it take the farmer's horses to consume the remaining food? [70]