ČVUT

ČESKÉ VYSOKÉ UČENÍ TECHNICKÉ V PRAZE

Preparatory course Mathematics

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Powers and roots

$$a^{-b} = \frac{1}{a^b} \quad \Rightarrow \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

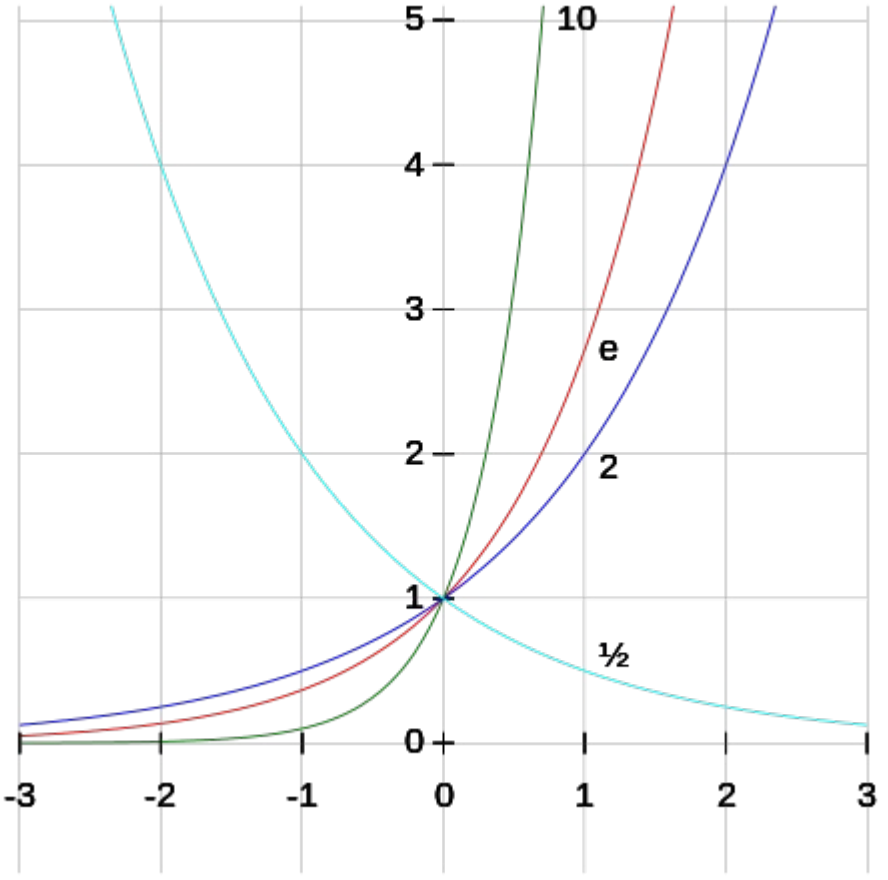
$$a^{\frac{b}{c}} = \sqrt[c]{a^b} \quad \Rightarrow \quad 4^{\frac{3}{2}} = \sqrt[2]{4^3} = 2^3 = 8$$

$$a^b \cdot a^c = a^{b+c} \quad \Rightarrow \quad 2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$$

$$(a^b)^c = a^{b \cdot c} \quad \Rightarrow \quad (2^2)^3 = 2^{2 \cdot 3} = 2^6 = 64$$

$$a^2 - b^2 = (a - b)(a + b) \quad \Rightarrow \quad 9 - x^2 = (3 - x)(3 + x)$$

Plotting exponent function



Ex1: Simplify

$$\left[10^4 - \left(8 \cdot 10^4 - 73 \cdot 10^3\right)\right]^2 \qquad [9 \cdot 10^6]$$

Ex2: Change to the power with base 9

$$81^{90} \cdot 3^{300} \qquad [9^{330}]$$

Ex3: Simplify for a > 0

$$\frac{a^3}{2^2} - \left(\frac{2}{a}\right)^{-3} \qquad \left[\frac{a^3}{8}\right]$$

Ex4: Simplify for d > 0

$$\sqrt{2d^3} \cdot \sqrt{18d} \qquad [6d^2]$$

21 Vypočítejte: **= Compute**

a) $\sqrt[4]{2} \cdot \sqrt[4]{8}$
b) $\sqrt[3]{16} : \sqrt[3]{2}$
c) $(3\sqrt[3]{16} + 2\sqrt[3]{54}) \cdot \sqrt[3]{4}$
d) $(\sqrt[4]{3} + \sqrt[4]{2}) \cdot (\sqrt[4]{27} - \sqrt[4]{8})$
e) $(2\sqrt[3]{18} + 3\sqrt[3]{12}) : \sqrt[3]{6}$
f) $(\sqrt[4]{20} + \sqrt[4]{4}) : \sqrt[4]{4}$

22 Částečně odmocněte: **= Partially remove root**

a) $\sqrt{12} - \sqrt{48} + 2\sqrt{75}$
b) $2\sqrt{8} - \sqrt{8} + 11\sqrt{72}$
c) $8\sqrt{50} + 4\sqrt{32} - 6\sqrt{162}$
d) $\sqrt{98} + \sqrt{200} + \sqrt{128}$
e) $\sqrt{75} - \sqrt{300} + \sqrt{243}$
f) $2\sqrt{108} - 2\sqrt{27} + 12\sqrt{12}$

23 Částečně odmocněte: **= Partially remove root**

a) $\sqrt[3]{16} - \sqrt[3]{54} + 2\sqrt[3]{250}$
b) $5\sqrt[3]{625} - \sqrt[3]{5} - 10\sqrt[3]{40}$
c) $\sqrt[3]{128} + 2\sqrt[3]{16} - \sqrt[3]{1\,024}$
d) $\sqrt[3]{32} + 2\sqrt[3]{4} - \sqrt[3]{500}$

Solutions:

21 a) 2; b) 2; c) 24; d) $1 + \sqrt[4]{54} - \sqrt[4]{24}$; e) $2\sqrt[3]{3} + 3\sqrt[3]{2}$; f) $\sqrt[4]{5} + 1$.

22 a) $8\sqrt{3}$; b) $68\sqrt{2}$; c) $2\sqrt{2}$; d) $25\sqrt{2}$; e) $4\sqrt{3}$; f) $30\sqrt{3}$.

23 a) $9\sqrt[3]{2}$; b) $4\sqrt[3]{5}$; c) 0; d) $-\sqrt[3]{4}$.

24 a) $7\sqrt[4]{3}$; b) $4\sqrt[4]{5}$.

26 Odstraňte odmocninu ze jmenovatele zlomku (zlomky usměrněte): **=Remove root from the denominator**

a) $\frac{1}{\sqrt{3}}$

b) $\frac{12}{\sqrt{6}}$

c) $\frac{\sqrt{3}-\sqrt{14}}{\sqrt{6}}$

d) $\frac{4\sqrt{15}+5\sqrt{12}}{2\sqrt{3}}$

e) $\frac{2}{3-\sqrt{5}}$

f) $\frac{1}{1+\sqrt{7}}$

g) $\frac{15}{2\sqrt{3}-3}$

h) $\frac{2\sqrt{3}-\sqrt{10}}{\sqrt{6}-\sqrt{5}}$

i) $\frac{2+\sqrt{8}}{6-3\sqrt{2}}$

j) $\frac{\sqrt{27}-1}{2+4\sqrt{3}}$

k) $\frac{3\sqrt{5}-5\sqrt{3}}{3\sqrt{5}+5\sqrt{3}}$

l) $\frac{\sqrt{6}-2\sqrt{2}}{\sqrt{15}-2\sqrt{5}}$

26 a) $\frac{\sqrt{3}}{3}$; b) $2\sqrt{6}$; c) $\frac{1}{2}\sqrt{2}-\frac{1}{3}\sqrt{21}$; d) $5+2\sqrt{5}$; e) $\frac{3+\sqrt{5}}{2}$; f) $\frac{\sqrt{7}-1}{6}$; g) $10\sqrt{3}+15$;
h) $\sqrt{2}$; i) $\frac{4}{3}+\sqrt{2}$; j) $\frac{19-5\sqrt{3}}{22}$; k) $\sqrt{15}-4$; l) $\frac{\sqrt{10}}{5}$.

31 Vypočítejte:

a) $\sqrt[3]{2\sqrt{6}-4}\cdot\sqrt[3]{2\sqrt{6}+4}$ b) $\sqrt[3]{7+\sqrt{22}}\cdot\sqrt[3]{7-\sqrt{22}}$

32 Umocněte:

a) $(1+\sqrt{2})^2$ d) $(1-2\sqrt{3})^3$ g) $(2-\sqrt{5})^{-2}$
b) $(5\sqrt{3}-\sqrt{6})^2$ e) $(\sqrt{2}+2)^3$ h) $(\sqrt{3}+2)^{-2}$
c) $(3\sqrt{5}+2\sqrt{10})^2$ f) $(\sqrt{6}+\sqrt{2})^3$ i) $(\sqrt{2}-1)^{-3}$

33 Odmocněte ($a, b, x, y, z, m \in \mathsf{R}$):

a) $\sqrt{a^2}$ c) $\sqrt{a^2+2ab+b^2}$ e) $\sqrt{x^2-6x+9}$
b) $\sqrt{100x^6}\cdot\sqrt{y^2z^4}$ d) $\sqrt{a^2+b^2}$ f) $\sqrt{0{,}25m^2+m+1}$

31 a) 2; b) 3.

32 a) $3+2\sqrt{2}$; b) $81-30\sqrt{2}$; c) $85+60\sqrt{2}$; d) $37-30\sqrt{3}$; e) $20+14\sqrt{2}$; f) $20\sqrt{2}+12\sqrt{6}$; g) $9+4\sqrt{5}$; h) $7-4\sqrt{3}$; i) $7+5\sqrt{2}$.

33 a) $|a|$; b) $10z^2|x^3y|$; c) $|a+b|$; d) nelze odmocnit; e) $|x-3|$; f) $|0{,}5m+1|$.

= Simplify

not possible

Simplify and indicate conditions on x and y

45 Zjednodušte následující výrazy a určete, kdy mají smysl:

a) $\dfrac{x^5 \cdot (x^n \cdot 6 \cdot y^{4n})^3}{9 \cdot y^{4n} \cdot (x^5 \cdot 2 \cdot y^6)^2}$ b) $\dfrac{(16b^3a^{-1})^{-3}}{(a^3b^{-2} \cdot 4)^{-2}}$ c) $\dfrac{(27r^3s^4)^{n-1}}{(3rs^{-2})^{3n+1}}$

46 Zjednodušte následující výrazy a určete, kdy mají smysl:

a) $\dfrac{x^{n+1} + 6x^n}{x^{n-1} + 6x^{n-2}}$ b) $\dfrac{y^{a+2} - y^a}{y^{a+1} + y^a}$ c) $\dfrac{a^{2x} + 6a^x + 9}{a^{2x} - 9}$ d) $\dfrac{x^{2n} - y^{2n}}{x^{n+1} - xy^n}$

45 a) $6 \cdot x^{3n-5} \cdot y^{8n-12}$, $x, y \neq 0$; b) $2^{-8}a^9b^{-13}$, $a, b \neq 0$; c) $3^{-4}r^{-4}s^{10n-2}$, $r, s \neq 0$.

46 a) x^2, $x \neq 0$, $x \neq -6$; b) $y - 1$, $y \neq 0$, $y \neq -1$; c) $\frac{a^x+3}{a^x-3}$, $a^x \neq \pm 3$, $y \neq -1$; d) $x^{n-1} + x^{-1}y^n$, $x \neq 0$, $x^n \neq y^n$.

50 Zapište jako mocninu s racionálním exponentem ($x \in \mathbb{R}^+$): **= write as power of x with exponent**

a) $\sqrt{x}$ b) $\sqrt[3]{x^5}$ c) $\sqrt[5]{x^3}$ d) $\frac{1}{\sqrt[4]{x}}$ e) $\frac{2}{\sqrt[5]{x^3}}$

51 Výsledky následujících úloh zapište jako mocninu čísla 2: **= write as power of 2**

a) $2^{\frac{1}{4}} \cdot 2^{\frac{2}{3}}$ c) $\left[\left(2^{\frac{1}{2}}\right)^{\frac{1}{4}}\right]^{\frac{3}{4}}$ e) $2^{\frac{1}{2}} \cdot 4^{\frac{1}{4}} \cdot 8^{\frac{1}{8}} \cdot 16^{\frac{10}{16}}$

b) $2^{\frac{3}{4}} : 2^{\frac{1}{2}}$ d) $\left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} : 2^{\frac{7}{8}}$ f) $(2^{\frac{1}{3}})^4 \cdot 2^{(\frac{1}{3})^3}$

52 Zjednodušte následující výrazy a určete, kdy mají smysl: **= simplify**

a) $\frac{x^{\frac{1}{2}} \cdot x^{\frac{3}{4}}}{x^2}$ c) $\frac{(y^{\frac{1}{2}})^3 \cdot (y^2)^{\frac{1}{3}}}{y \cdot y^{\frac{2}{3}}}$ e) $\frac{(xy)^{\frac{1}{2}} \cdot (x^2y)^{-\frac{1}{3}}}{(xy^2)^{-\frac{2}{3}}}$

b) $\frac{y^2}{(y^{\frac{1}{6}})^2}$ d) $\frac{(x^{\frac{2}{3}})^{\frac{1}{4}} \cdot (x^{\frac{1}{6}})^{\frac{9}{4}}}{(x^{\frac{7}{2}})^{\frac{5}{6}}}$ f) $\left(\frac{x^{\frac{2}{5}}}{y^{\frac{3}{2}}}\right)^{-2} \cdot \frac{(y^{-1}x^{-2})^{-\frac{1}{2}}}{(xy^2)^{\frac{1}{10}}}$

50 a) $x^{\frac{1}{2}}$; b) $x^{\frac{5}{3}}$; c) $x^{\frac{3}{5}}$; d) $x^{-\frac{1}{4}}$; e) $2x^{-\frac{3}{5}}$.

51 a) $2^{\frac{11}{12}}$; b) $2^{\frac{1}{4}}$; c) $2^{\frac{3}{32}}$; d) $2^{-\frac{1}{8}}$; e) $2^{\frac{31}{8}}$; f) $2^{\frac{37}{27}}$.

52 a) $x^{-\frac{3}{4}}$, $x > 0$; b) $y^{\frac{5}{3}}$, $y > 0$; c) $y^{\frac{1}{2}}$, $y > 0$; d) $x^{-\frac{19}{8}}$, $x > 0$; e) $x^{\frac{1}{2}}y^{\frac{3}{2}}$, $x, y > 0$;
f) $x^{\frac{1}{10}}y^{\frac{33}{10}}$, $x, y > 0$.

Ex5: Simplify and add when it makes sense

$$\left(\frac{1}{x}+\frac{3}{x^2-3x}\right)\frac{1}{x^2-9} \qquad [x+3]$$

Ex6: There are two expressions $\frac{x}{x+1}, \frac{-1}{x^2+x}$ with variable $x \in \mathbf{R}$.

Decide if the following statements are true or false.

- For x = -1 the first expression makes sense.
- For x = 1 the second expression makes sense.
- The common denominator can be x^2 + x.
- The sum of both expressions for positive values of x is equal to $\frac{x-1}{x}$

[no, yes, yes, yes]

Ex7: Simplify and add when it makes sense

$\left(\frac{x-1}{x-2} - \frac{x}{x-1}\right)\left(x - \frac{3x}{x+1}\right)$ $\left[\frac{x}{x^2-1}\right]$ $x \neq \pm 1, x \neq 2$

- $(p+q) : \left(\frac{1}{p} + \frac{1}{q}\right)$ $[pq, p \neq 0, q \neq 0, p+q \neq 0]$

Ex8: Solve the equations in field of real numbers

$\frac{x+10}{x} + \frac{100}{10x-x^2} = \frac{x+20}{x-10}$ $[x = -10, x \neq 0, 10]$

$5 + \frac{3}{3u-12} = \frac{5-u}{u-4}$ [no solution]

55 Řešte rovnice s neznámou $x \in \mathbb{R}$: = Solve the equations in field of real numbers.

a) $x^4 = 625$ c) $x^{\frac{1}{3}} = 0{,}4$ e) $\sqrt{x^3} = 8$ g) $\sqrt[5]{\sqrt[4]{x^5}} = 3$

b) $64x^6 - 1 = 0$ d) $\sqrt[4]{x} = 9$ f) $x^{\frac{2}{3}} = 0{,}25$ h) $\sqrt{125x^2} = 5x\sqrt{5}$

56 Zjednodušte následující výrazy a určete, kdy mají smysl: = Simplify

a) $\left(\dfrac{8-\sqrt{x}}{2-\sqrt{x}} - \dfrac{8+\sqrt{x}}{2+\sqrt{x}}\right) \cdot \dfrac{x^2-16}{4\sqrt{x}}$

b) $\dfrac{2}{1+\sqrt{x}} - \dfrac{1+\sqrt{x}}{\sqrt{x}-1} - \dfrac{x+2}{1-x}$

c) $\left(4 - \dfrac{2}{\sqrt{x}+1}\right) \cdot \left(1 + \dfrac{\sqrt{x}}{\sqrt{x}-1}\right) - \dfrac{6}{x-1}$

d) $\dfrac{2\sqrt{x}}{\sqrt{7}+\sqrt{x}} + \left(\dfrac{7\sqrt{7}+x\sqrt{x}}{\sqrt{7}+\sqrt{x}} - \sqrt{7x}\right) : (7-x)$

e) $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}} + \dfrac{\sqrt{x}+1}{\sqrt{x}-1} - \dfrac{\sqrt{x}-1}{\sqrt{x}+1}\right) \cdot \dfrac{\sqrt{x}}{x+1}$

55 a) $x = \pm 5$; b) $x = \pm\frac{1}{2}$; c) $x = 0{,}064$; d) $x = 6\,561$; e) $x = 4$; f) $x = \pm 0{,}125$; g) $x = 27$; h) $x \in \mathbb{R}_0^+$.

56 a) $-3(x+4)$; $x > 0 \wedge x \neq 4$; b) $\frac{1}{1-x}$; $x \geq 0 \wedge x \neq 1$; c) 8; $x \geq 0 \wedge x \neq 1$; d) 1; $x \geq 0 \wedge x \neq 7$; e) $\frac{x+1}{x-1}$; $x > 0 \wedge x \neq 1$.

Ex9: Determine for which real numbers has $\frac{1-x}{x-7}+1$ negative value.

$$[x \in (7, +\infty)]$$

Ex10: We created a solution by dissolving 2 grams of the active substance in water. The weight of the active ingredient is 5% of the weight of the solution. Calculate the mass of water used for the solution.

$$[38g]$$

Ex11: Cleaning of the liquid takes place in three phases. The second phase lasts a third longer than the first phase, and the third phase lasts twice as long as the second phase. Calculate what percentage of the total cleaning time of the first phase.

$$[20\%]$$

Ex12: One side of the rectangle is one-fifth shorter than the side of the square and the contents of both of these shapes are the same. Let's denote the length of square side by a. Express the length of the longer side of the rectangle depending on the quantity a.

$$[\frac{5}{4}a]$$