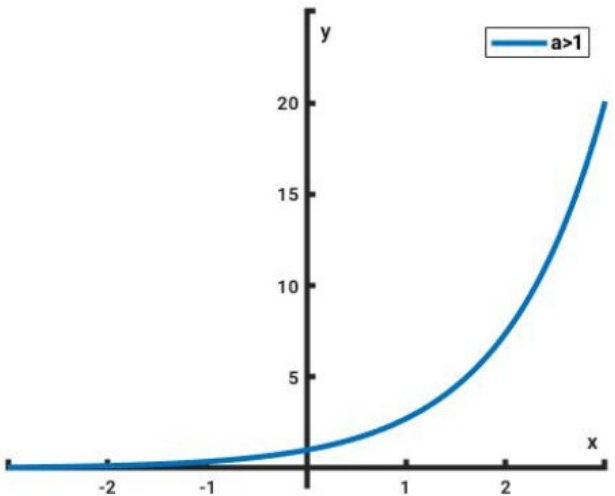
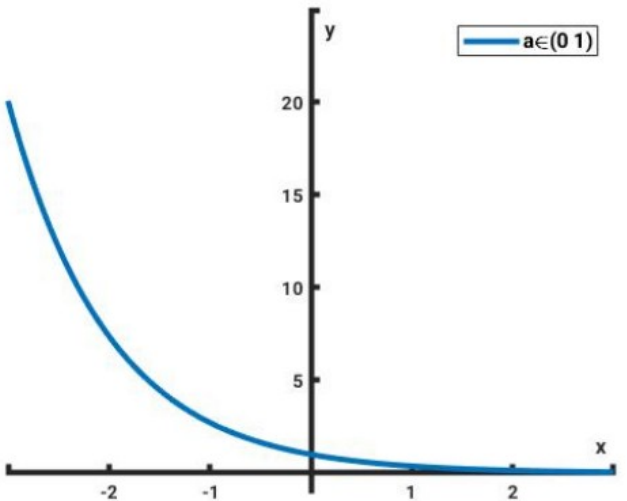
Logarithms and exponentials

Exponential

$$y = f(x) = a^x \qquad a > 0,\ a \neq 1$$

$$D(f) = R, \quad H(f) = (0, \infty)$$

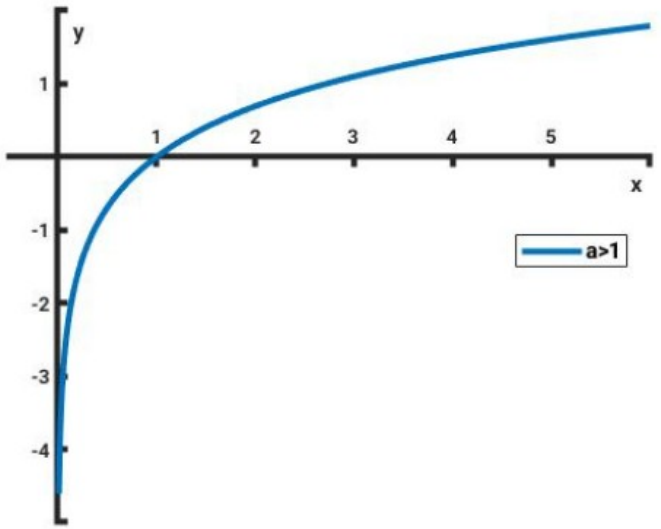


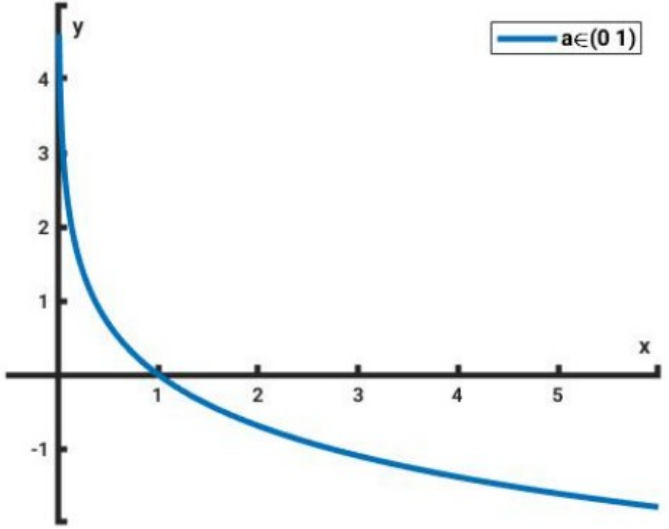


Logarithm

$$y = f(x) = \log_a x \qquad a > 0,\ a \neq 1$$

$$D(f) = (0, \infty), \quad H(f) = R$$





Formulas

$$\log_a 1 = 0, \quad \log_a a = 1, \quad \log_a \frac{1}{a} = -1,$$

$$\log_a b + \log_a c = \log_a (b \cdot c)$$

$$\log_a b - \log_a c = \log_a \left(\frac{b}{c}\right)$$

$$k \cdot \log_a b = \log_a b^k$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$x = a^{\log_a x}$$

What is a logarithm?

Logarithms are another way of thinking about exponents.

For example, we know that 2 raised to the 4^{th} power equals 16. This is expressed by the **exponential** equation $2^4 = 16$.

Now, suppose someone asked us, "2 raised to which power equals 16?" The answer would be 4. This is expressed by the **logarithmic** equation $\log_2(16) = 4$, read as "log base two of sixteen is four".

$$2^4 = 16 \iff \log_2(16) = 4$$

Both equations describe the same relationship between the numbers 2, 4, and 16, where 2 is the **base** and 4 is the **exponent**.

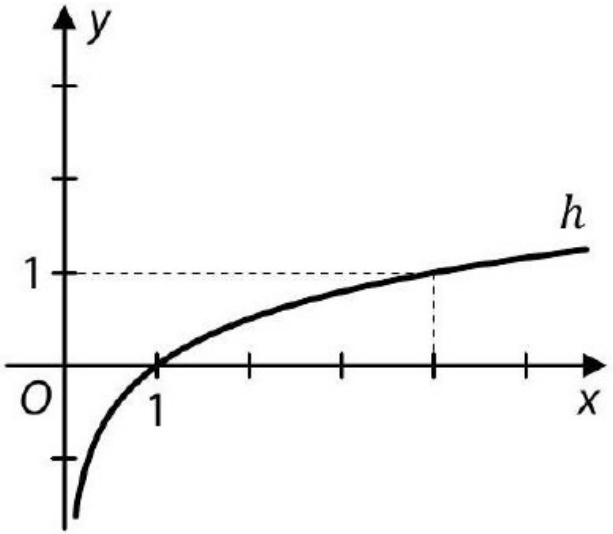
The difference is that while the exponential form isolates the power, 16, the logarithmic form isolates the exponent, 4.

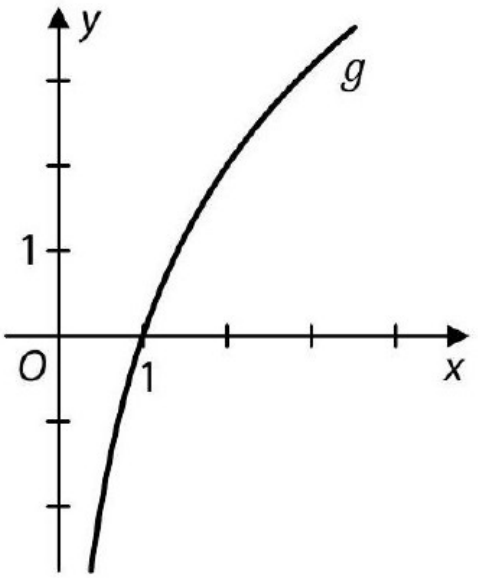
Here are more examples of equivalent logarithmic and exponential equations.

Logarithmic form		Exponential form
$\log_2(8) = 3$	$\iff$	$2^3 = 8$
$\log_3(81) = 4$	$\iff$	$3^4 = 81$
$\log_5(25) = 2$	$\iff$	$5^2 = 25$

source: Khan academy

Ex1: There is a graph of logarithmic function *h: y = log_a x* .
Determine the base *a* and find for which *x* holds:
h(x) = 3.





Ex2: There is a graph of logarithmic function g*: y = log_a x,* where it holds *log_a x = 2.*
Which statement is true?

A) $\log_a \sqrt{2} = \sqrt{2}$

B) $\log_a \sqrt{8} = \sqrt{8}$

C) $\log_a 4 = 4$

D) $\log_a 8 = 8$

E) žádná z uvedených rovností

Ex3: In real number solve equation $\left(\frac{27}{8}\right)^{x+2} \cdot \frac{2^x}{3^x} = \frac{3}{2}$

Ex4: Find all possible x for which function exists: $f: y = \log_9(1 - x)$
For which x holds: *y = 0.5 ?*

Ex5: Solve equations: $2^{5x} - \log_5 \sqrt{5} = 0$

$2^{1000} : 2^{500} + 3 \cdot 2^{500} = 2^x$

$\log_{10}(-2x) = 0$

$\log_{10} 10^x + x \cdot \log_{10} 1 = \log_{10} 1000$

$2^x : 32^{0,5} = \sqrt[3]{32}$

$2^{-x} + 2 = 0$

57 Určete reálná čísla a, b tak, aby graf funkce $y = a \cdot 2^x + b$ procházel body $A[0; \frac{7}{2}]$, $B[-1; 2]$.

= Find a,b in order to fct y pass through points A, B.

58 Určete reálná čísla a, b tak, aby graf funkce $y = 2^{x+a} + b$ procházel body $A[1; 15]$, $B[-2; 1]$.

59 Určete všechny hodnoty reálného parametru q tak, aby daná funkce byla rostoucí.

= Find q in order to fct y be increasing.

a) $y = \left(\frac{2q^2}{q^2+1}\right)^x$ b) $y = \left(\frac{1}{q}\right)^x$ c) $y = \left(\frac{q+3}{q-1}\right)^x$

Solutions:

4.11 Exponenciální funkce

57 $a = 3 \wedge b = \frac{1}{2} \Rightarrow y = 3 \cdot 2^x + \frac{1}{2}$. **58** $a = 3 \wedge b = -1 \Rightarrow y = 2^{x+3} - 1$.

59 a) $q \in (-\infty; -1) \cup (1; \infty)$. b) $q \in (0; 1)$; c) $q \in (1, \infty)$

66 Načrtněte grafy funkcí (určete definiční obor, obor funkčních hodnot, průsečíky grafu s osou x a s osou y):

f_1: $y = 2^x - 4$ $\quad$ f_4: $y = |2^{x+1} - 4|$ $\quad$ f_7: $y = \left(\frac{1}{2}\right)^{x-3} - 1$

f_2: $y = 2^{x+1} - 4$ $\quad$ f_5: $y = 2^{|x+1|} - 4$ $\quad$ f_8: $y = \left(\frac{1}{2}\right)^{x+2} + 4$

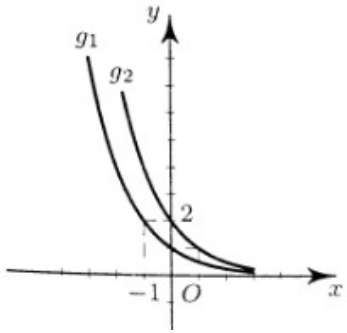
f_3: $y = -(2^{x+1} - 4)$ $\quad$ f_6: $y = 2^{|x|+1} - 4$ $\quad$ f_9: $y = -\left(\frac{1}{2}\right)^{x+2} + 4$

67 Načrtněte grafy funkcí:

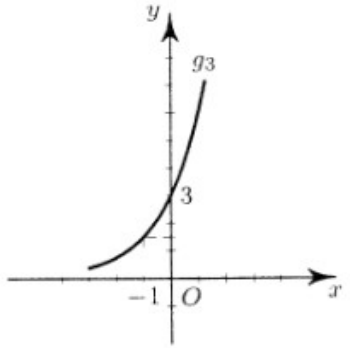
g_1: $y = 2^{-x}$ $\quad$ g_3: $y = 3 \cdot 2^x$ $\quad$ g_5: $y = e^x$

g_2: $y = 2^{-x+1}$ $\quad$ g_4: $y = 2^x + (\frac{1}{2})^x$ $\quad$ g_6: $y = 10^x$

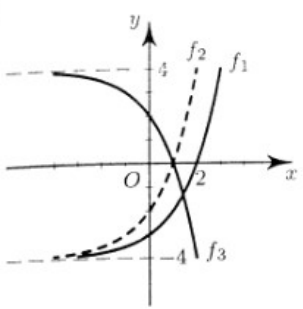
=Plot graphs.

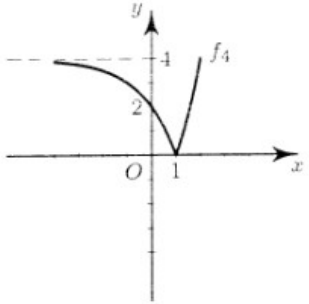


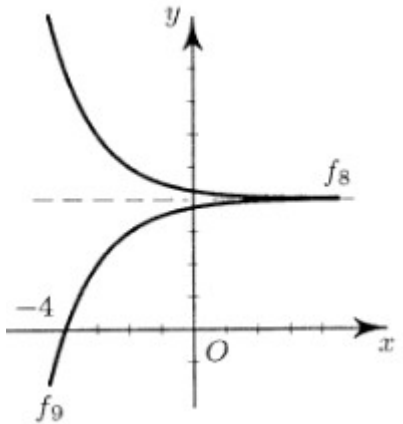
=Plot graphs.

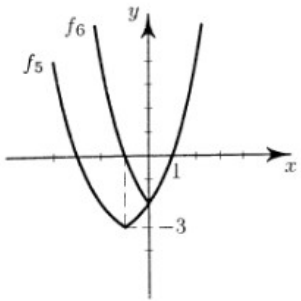


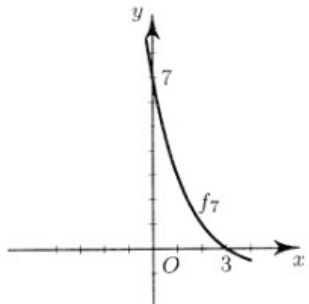
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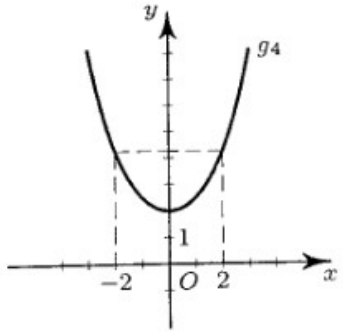


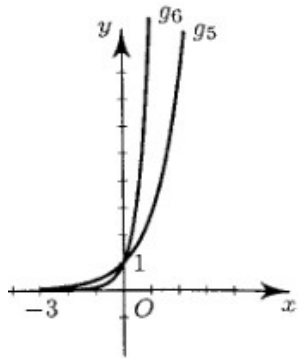












68 Danou rovnost přepište pomocí definice logaritmu: **= Rewrite equations by using logarithm.**

a) $3^2 = 9$ b) $2^{-3} = \frac{1}{8}$ c) $9^{\frac{1}{2}} = 3$

69 Vypočítejte: **= Compute.**

a) $\log_{\frac{1}{3}} 9$ c) $\log_7 \sqrt{7}$ e) $\log_8 \sqrt{2}$ g) $\log_{0,25} 4$

b) $\log_5 125$ d) $\log_{\sqrt{2}} \frac{1}{16}$ f) $\log_5 1$ h) $\log_{0,2} 0{,}04$

70 Najděte všechna $x \in (0; \infty)$, pro něž platí: **= Find all positive x satisfying:**

a) $\log_3 x = 4$ c) $\log_{\frac{1}{5}} x = -1$ e) $\log_5 x = 0$ g) $\log_{17} x = 1$

b) $\log_{\sqrt{2}} x = 4$ d) $\log_2 x = -\frac{1}{3}$ f) $\log_{\frac{1}{4}} x = \frac{3}{2}$ h) $\log x = -\frac{3}{5}$

72: Compute and determine, if x is positive or negative. If x>0, indicate whether x > 1 or not.

a) $x_1 = 2\log 4 + \log 3 - \log 6$ c) $x_3 = 2\log_{0,2} 4 + \log_{0,2} 3 - \log_{0,2} 6$

b) $x_2 = \log_6 12 - \log_6 \frac{1}{12} - 2$ d) $x_4 = \ln 4 + \ln \frac{1}{3} - 2\left(\ln 2 + \ln \frac{1}{4}\right)$

Solutions:

68 a) $\log_3 9 = 2$; b) $\log_2 \frac{1}{8} = -3$; c) $\log_9 3 = \frac{1}{2}$.

69 a) -2; b) 3; c) $\frac{1}{2}$; d) -8; e) $\frac{1}{6}$; f) 0; g) -1; h) 2.

70 a) 81; b) 4; c) 5; d) $\frac{\sqrt[3]{4}}{2}$; e) 1; f) $\frac{1}{8}$; g) 17; h) $\sqrt[5]{0{,}001}$.

71 a) $a = 3$; b) $a = \frac{1}{2}$; c) $a = \frac{1}{27}$; d) $a = \frac{1}{5}$; e) $a = 256$; f) $a = \sqrt{2}$; g) $a = \sqrt{2}$; h) $a = 10$.

72 a) $x_1 = \log 8$, $0 < x_1 < 1$; b) $x_2 = \log_6 4$, $0 < x_2 < 1$; c) $x_3 = \log_{\frac{1}{5}} 8$, $x_3 < 0$; d) $x_4 = \ln \frac{16}{3}$, $x_4 > 1$.

76 Určete reálná čísla a, b tak, aby pro funkci h_1: $y = a\log_2 x + b$ platilo $h_1(4) = 5$, $h_1(\frac{1}{4}) = -7$.

a,b = ? in order to hold: h1 at x=4 ...

77 Určete reálná čísla a, b tak, aby graf funkce h_2: $y = \log_{\frac{1}{3}}(x+a)+b$ procházel body $K[2;1]$, $L[8;0]$.

a,b = ? in order to graph h2 pass through points K and L.

78 Určete definiční obory funkcí f_1 až f_{10}:

f_1: $y = \log_3 x$

f_2: $y = \log_3(x+6)$

f_3: $y = \log(x^2-4)$

f_4: $y = \dfrac{1}{\log x - 1}$

f_5: $y = \dfrac{1}{\log_2(x+7)-1}$

f_6: $y = \log(|x+1|-7)$

f_7: $y = \log(-|x+3|+|x|+1)$

f_8: $y = \sqrt{\log(x+3)}$

f_9: $y = \log\dfrac{x}{2x-1}$

f_{10}: $y = \sqrt{\log_{0,2}\dfrac{x+5}{x}}$

What is the domain of functions?

Solutions:

4.13 Logaritmická funkce

76 $a = 3 \wedge b = -1 \Rightarrow y = 3\log_2 x - 1$. **77** $a = 1 \wedge b = 2 \Rightarrow y = \log_{\frac{1}{3}}(x+1)+2$.

78 $\mathrm{D}(f_1) = (0;\infty)$
$\mathrm{D}(f_2) = (-6;\infty)$
$\mathrm{D}(f_3) = (-\infty;-2)\cup(2;\infty)$
$\mathrm{D}(f_4) = (0;\infty) - \{10\}$
$\mathrm{D}(f_5) = (-7;\infty) - \{-5\}$
$\mathrm{D}(f_6) = (-\infty;-8)\cup(6;\infty)$
$\mathrm{D}(f_7) = (-\infty;-1)$
$\mathrm{D}(f_8) = \langle -2;\infty)$
$\mathrm{D}(f_9) = (-\infty;0)\cup(\frac{1}{2};\infty)$
$\mathrm{D}(f_{10}) = (-\infty;-5)$

Ex6: Determine values of logarithmic function

$\log(1)$ $[0]$

$\ln(0)$ $[-]$

$\log_5(5)$ $[1]$

$\ln\left(\frac{1}{e^2}\right)$ $[-2]$

Ex7: Simplify and determine expression V:

$$\ln V = \ln 4 - \ln 3 + \ln \pi + \ln r \qquad \left[V = \tfrac{4}{3}\pi r^3\right]$$

$$\log_2 V = 3\log_2 x + (n+3)\log_2 y - 3 \qquad \left[V = \frac{x^3 y^{n+3}}{8}\right]$$

$$\log_a V = \tfrac{3}{4}\log_a(x+2) - 2\log_a y \qquad \left[V = \sqrt[4]{(x+2)^3}/y^2\right]$$

$$\log_5 V = 2\log_5(x-2) + 3\log_5(x+2) - 2\log_5(x^2-4) \qquad [V = x+2]$$

Ex8: Solve given equations in real numbers:

- $3^x = 81 \qquad [x = 4]$
- $\left(\frac{1}{4}\right)^x = 16 \qquad [x = -2]$
- $\sqrt{128} = 8^x \qquad [x = 7/6]$
- $5^{x^2-2} \cdot 5^{3x+4} = 1 \qquad [x_1 = -1,\ x_2 = -2]$
- $x^2e^x + 3xe^x - 4e^x = 0 \qquad [x_1 = 1,\ x_2 = -4]$
- $e^{\frac{1}{x}} + xe^{\frac{1}{x}}\left(-\frac{1}{x^2}\right) = 0 \qquad [x = 1]$
- $\ln(\sqrt{x}) = -2 \qquad \left[x = \frac{1}{e^4}\right]$
- $\ln(x+1) = 0 \qquad [x = 0]$
- $2\ln x - 1 = 0 \qquad [x = \sqrt{e}]$
- $2x + 3x\ln x = 0 \qquad \left[x = \frac{1}{\sqrt[3]{e^2}}\right]$

5 Exponenciální a logaritmické rovnice a nerovnice

5.1 Exponenciální rovnice

1 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $2^{3x-1} \cdot 4 = 8^{x+1} \cdot \left(\frac{1}{2}\right)^x$

b) $\sqrt[4]{4^x} \cdot \sqrt[3]{2^{x-3}} = \sqrt[6]{16}$

c) $\frac{1}{3^x} = \frac{1}{\sqrt{3}} \cdot \sqrt[6]{27^{3-3x}} \cdot \left(\frac{1}{9}\right)^{x+3}$

d) $0.25 \cdot \left(\frac{1}{4}\right)^{2x} = 1$

e) $2 \cdot 0{,}5^{x^2+\frac{8}{3}x} = \frac{8}{\sqrt[3]{4}}$

f) $5^x \cdot 2^x = 100^{x-1}$

g) $\frac{81}{16} = \left(\frac{2}{3}\right)^x \cdot \left(\frac{9}{4}\right)^{x+1}$

h) $\frac{3^x}{2 \cdot 3^{\sqrt{3}}} = 4{,}5$

2 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $3^x + 3^{x+1} = 108$

b) $2^{x+1} + 2^{x-1} + 2^{x+3} = \frac{21}{8}$

c) $7 \cdot 4^{-x+2} = 3 \cdot 4^{-x+3} - 5$

d) $\frac{4}{5} \cdot 5^0 + 5^{-1} - 25^x + 20 \cdot 25^{x-1} = 0$

e) $3^x \cdot \left(\frac{1}{2}\right)^x + 3^{x+1} \cdot \left(\frac{1}{2}\right)^{x+1} = \frac{5}{3}$

f) $2^{2x} \cdot 5^x - 2^{2x-1} \cdot 5^{x+1} = -600$

3 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $4^{2x} - 6 \cdot 4^x + 8 = 0$

b) $\frac{1}{4} \cdot 2^x + \frac{1}{2} \cdot 4^x = 9$

c) $9 \cdot 3^x + 3^{-x} = 10$

d) $9^{x-0,5} + 9^{0,5-x} = \frac{10}{3}$

e) $2\left(\frac{1}{4}\right)^x - 3\left(\frac{1}{2}\right)^x = \left[1 + \left(\frac{1}{2}\right)^x\right]\left(\frac{1}{4}\right)^{-1}$

f) $5^{2x} \cdot (5^{2x} - 5) = 3 \cdot (5^{2x+1} + 5^{2x}) + 50$

4 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1}$

b) $2^{x-1} - 2^{x-2} = 5^{x-3} + 2^{x-3}$

c) $2 \cdot 4^x + 5^{x-\frac{1}{2}} = 5^{x+\frac{1}{2}} - 2^{2x-1}$

d) $3^x + \frac{9^x}{3} = 3^{x+1} + \frac{9^x}{9}$

5 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $4^x + 6^x = 2 \cdot 9^x$

b) $16^x = 8 \cdot 4^x + 2 \cdot 8^x$

6 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $3^x = 10$

b) $5^{x+1} = 4$

c) $2^x \cdot 3^{x-1} = 6$

d) $5^x \cdot 7^{2x} = 16^{x-1}$

7 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $6 \cdot 7^{x+3} - 7^{x+2} = 82$

b) $3^{x+1} + 2 \cdot 3^{-x} = 7$

c) $2^x - 3^x = 2^{x-1} + 5 \cdot 3^{x-1}$

d) $7 \cdot 6^x - 2 \cdot 4^x = 6 \cdot 9^x$

8 Určete všechna čísla $x, y \in \mathbb{R}$ tak, aby byla řešením dané soustavy:

a) $2^x + 5 \cdot 3^y = 53$
$7 \cdot 2^x - 3^y = 47$

b) $2^{x+1} + 3^y = 31$
$2^x - 3^{y-2} = -1$

c) $2 \cdot 2^{x-y} + 2^{x+y-1} = 20$
$10 \cdot 2^{x-y-1} - 2^{x+y} = -22$

d) $16^{x+y} = 8$
$16^x + 16^y = 6$

e) $5^{\frac{x+y}{2}} : 5^{\frac{x+y}{4}} = 625$
$6^{\frac{x-y}{2}} : 6^{\frac{x-y}{4}} = 216$

f) $x^{y+1} = 27$
$x^{-y+1} = 3^{-1}$

5.2 Logaritmické rovnice

9 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_2(x+1) = 3$

b) $4\log_3(2x-1) = 12$

c) $\log_{\frac{1}{2}}(2-x) = -2$

d) $\log_4(5x-4) = 2$

e) $\log_2 \log_3 \log_{\frac{1}{2}} x = 0$

f) $\log_{\frac{1}{2}} \log_3(1 + 20\log_2 x) = -2$

g) $\log_2[14 + 2\log_7(1 + 2\log_{\frac{1}{2}} x)] = 4$

h) $\log_9\{3\log_2[1 + \log_3(1 - 2\log_3 x)]\} = 0{,}5$

10 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_5(x^2 + 2x) = \log_5(-3x)$

b) $\log x^2 = \log(4 - x^2)$

c) $\log_{0,1}(x^2 - 5x) = \log_{0,1}(5x + 11)$

d) $\log_2(x^2 - x) = \log_2 x$

11 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log x = 2\log 5 + \log 4$

b) $\frac{\log_3 x}{1 + \log_3 2} = 2$

c) $\log_6(x+1) + \log_6 x = 1$

d) $\log_2(x+7) - \log_2 x = 3$

e) $\log(x+3) = \log x + \log 3$

f) $\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1$

g) $\log x^5 - \log x^4 + \log x^3 = 12$

h) $\log \sqrt{x} + \log \frac{1}{x^2} - \log x^3 + \frac{11}{2} = \frac{\log x^2}{1 + \log 10}$

i) $3\log 2x^2 + 2\log 3x^3 = 5\log x + 2\log 6x^3$

j) $0{,}5(3\log 5 - 1 - \log x) = 2 - \log 5$

k) $\log_4(3x+2) - 2\log_4 x = 2 - \log_4 8$

l) $1 + \log_3(5-x) - \log_3(2x-1) = \log_3(2x-1)$

12 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_2 \frac{3-x}{x+3} = -2$

b) $\log_3 \frac{6x-2}{x-3} = 2$

c) $\log_{\frac{1}{2}} \frac{x}{x+14} = \frac{\log 125}{\log 5}$

d) $\log_7 \frac{x^2+1}{x^2-1} = 1$

13 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\frac{\log_3(6x-2)}{\log_3(x-3)} = 2$

b) $\frac{\log_5(x-\frac{1}{4})}{\log_5(x+\frac{7}{2})} = -1$

c) $\frac{2\log 3x}{\log(2-7x)} = 1$

d) $\frac{\log x}{\log(x-2)} = \frac{\log 9}{\log 3}$

14 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_2^2 x + 2\log_2 x - 3 = 0$

b) $4\log_9 x(\log_9 x - 1) = 2 + 3\log_9 x$

c) $\log_{\frac{1}{2}}^2(x+1) + 5\log_{\frac{1}{2}}(x+1) = 6$

d) $4\log_3(2x+1) + \log_3\sqrt{2x+1} = \frac{3}{2}\log_3^2(2x+1) - 6$

15 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\dfrac{\log x + 3}{3 - \log x} = 5$

b) $\dfrac{\log x + 1}{2 + \log x} + \dfrac{2\log x - 1}{\log x} = 3$

16 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_{\frac{1}{7}} x + \dfrac{1}{\log_{\frac{1}{7}} x} = -2$

b) $\log x^3 + 2 = \dfrac{10}{\log x^2}$

c) $\log_3^2(x+1) + \dfrac{1}{\log_3^2(x+1)} = \dfrac{17}{4}$

d) $\log_4^2 x^3 - \dfrac{4}{\log_4^2 x^2} = 8$

17 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log 100x + \log 10x = 7$

b) $\log 100x + \log^2 10x = 7$

c) $\log 100x + \log 10x^2 = 7$

d) $\log 100x^2 + \log^2 10x^2 = 7$

18 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\dfrac{3}{2}\log\dfrac{x^2}{10} + \log\dfrac{100}{x^3} - \log\dfrac{\sqrt{10}}{x} = -2$

b) $\dfrac{\log_2 x - 2}{\log_2 \frac{x}{4}} - 2\log_2\sqrt{x} = \log_2^2 x + 1$

c) $\dfrac{\log_2 2x}{\log_2 8x} + \log_2\dfrac{x}{8} = \dfrac{2}{\log_2 4 - \log_2 0{,}5}$

d) $\log_{\frac{1}{2}} x \cdot \log_{\frac{1}{2}} 4x = \dfrac{\log_{\frac{1}{2}} 16x}{\log_{\frac{1}{2}} 8} + \log_{\frac{1}{2}} 4$

e) $\dfrac{\log_3^2 9x}{\log_3 81x^2} = \dfrac{3}{2}$

f) $\dfrac{\log\frac{x}{10}}{\log^2\sqrt{x}} + 1 = \log x$

19 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $x^{\log x} = 100x$

b) $x^{\log x + 2} = 100x$

c) $1\,000x^2 = x^{\log x}$

d) $27x^2 = x^{\log_3 x}$

e) $x^{\log_7 x^2} = 49x^3$

f) $(\sqrt{x})^{\log_2 x + 1} = 2$

20 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $\log_2 x + \log_4 8 = \log_8 16$

b) $\log_9 x + \log_3 x = 6$

c) $\log_2 x - \log_4 x + \log_{16} x = \dfrac{3}{4}$

d) $\log_x 2 + \log_{4x} 8 = 2\log_{4x} 16$

21 Určete všechna čísla $x, y \in \mathbb{R}$ tak, aby byla řešením dané soustavy:

a) $x + y = 7$
$\log x + \log y = 1$

b) $x^2 + y^2 = 20$
$\log_2 x + \log_2 y = 3$

c) $\log_2 x + \log_4 y = 2$
$\log_4 x + \log_2 y = 2{,}5$

d) $\log_3(x+1) = 3^{\log_3 1}$
$\log_{\frac{1}{3}}(x+y) = 1 + \log_{\frac{1}{3}} 6$

22 Určete všechna čísla $x, y \in \mathbb{R}$ tak, aby byla řešením dané soustavy:

a) $\log x + \log y = 1$
$2\log x - \log y = 5$

b) $\log_2 x + 2\log_2 y = 10$
$2\log_2 x^2 - \log_2 y^2 = 10$

c) $\log_{\frac{1}{2}} x + \log_2 y = 3$
$\dfrac{6\log_{\frac{1}{2}} x}{\log_2 y} - \dfrac{2\log_2 y}{\log_{\frac{1}{2}} x} = -1$

d) $xy = 100$
$\log^2 x + \log^2 y = 10$

e) $2^{x+y} \cdot 4^{2x-y} = 8$
$\log_3(x+y) = 2$

f) $5^{\log x} + 3^{\log y} = 4$
$5^{2\log x} - 3^{2\log y} = -8$

23 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $x + \log_2(8 + 2^x) = 7$

b) $x + \log_2(1 - 3\cdot 2^x) = x\log_2 4$

c) $\log_2 3 + \log_2 4^{x+\sqrt{x}} = \log_2(2^{x+\sqrt{x}+1} + 4) + 2$

d) $\log_3 4\cdot 2^x + \log_3 3^{x+2} = \log_3 36^x$

e) $\log_5 10\cdot 25^x - \log_5(5^x + 25) = x + 1$

24 Řešte rovnice s neznámou $x \in \mathbb{R}$:

a) $3^{\log_5 x} + 45 = 2\cdot 3^{\log_5 x + 1}$

b) $5\cdot(4^{\log_3 x} - 4^0) = 4^{\log_3 x + 1} - 4^{\log_3 x - 1}$

c) $3\cdot 16^{\log x} - 5\cdot 4^{\log x} - 2 = 0$

d) $2^{\log x} + 3^{\log x - 1} = 2^{\log x + 1} - 3^{\log x - 2}$

5.3 Exponenciální nerovnice

25 Řešte nerovnice s neznámou $x \in \mathbb{R}$:

a) $3^{x+5} < 1$

b) $2^{3x-4} \geqq 1$

c) $0{,}1^{2x} \leqq 1$

d) $\left(\dfrac{1}{2}\right)^{x+3} > 1$

e) $3^{x-5} < 0$

f) $\left(\dfrac{1}{5}\right)^{3x+2} \geqq 0$

26 Řešte nerovnice s neznámou $x \in \mathbb{R}$:

a) $3^{x-1} > 9$

b) $0{,}125^{2x+3} \leqq 16$

c) $3^{2x} \leqq 7$

d) $0{,}2^x > 3$

e) $4^x \cdot 2^x \leqq 100$

f) $\left(\dfrac{2}{7}\right)^x \cdot 3^{x+4} > -1$

27 Řešte nerovnice s neznámou $x \in \mathbb{R}$:

a) $3^{x+4} \geqq 3^{2x-1}$

b) $4^{x+5} < 16^{x+1}$

c) $0{,}2^{3x-2} \geqq 0{,}2^{x+5}$

d) $\left(\dfrac{1}{6}\right)^{x^2} < \left(\dfrac{1}{6}\right)^{2x+8}$

e) $2^x \cdot 2^{x+1} < 2^{x+3}$

f) $\left(\dfrac{1}{2}\right)^x \cdot \dfrac{1}{2} < \dfrac{1}{8}$

28 Řešte nerovnice s neznámou $x \in \mathbb{R}$:

a) $3^{x+1} + 3^{x+2} < 36$

b) $\left(\dfrac{1}{2}\right)^{x-1} + \left(\dfrac{1}{2}\right)^{x-2} \leqq 3$

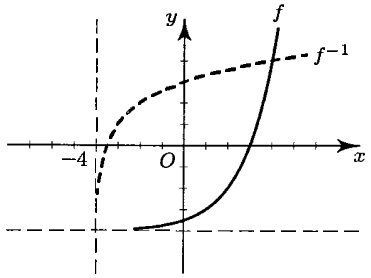
c) $2^x - 3^x > 2^{x+2} - 3^{x+1}$

d) $25^x - 9\cdot 5^x + 20 < 0$

e) $\left(\dfrac{1}{2}\right)^{-x} - \left(\dfrac{1}{2}\right)^{x-1} \geqq 1$

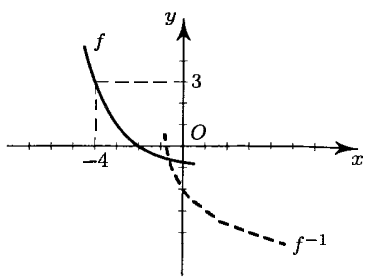
f) $2^x - 5\cdot 4^{x-2} < 1 - 2^{x-1}$

m)



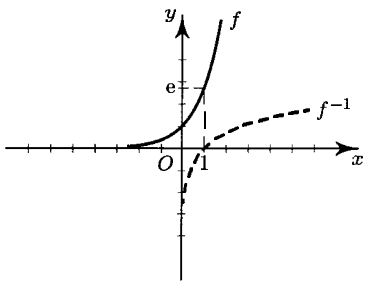
f^{-1} $y = \log_2(x+4)+1$

n)



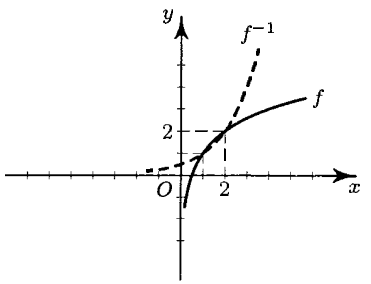
f^{-1} $y = \log_{\frac{1}{2}}(x+1)-2$

o)



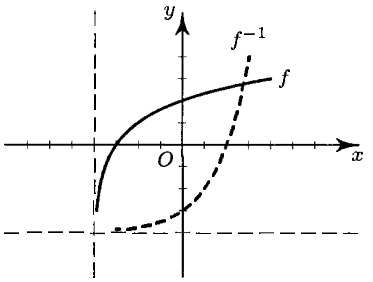
f^{-1} $y = \ln x$

p)



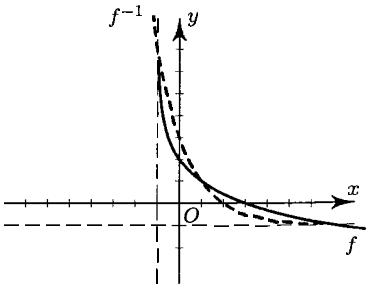
f^{-1} $y = 2^{x-1}$

q)



f^{-1} $y = 2^x - 4$

r)



f^{-1} $y = (\frac{1}{2})^{x-2} - 1$

K řešení úlohy 90

5 Exponenciální a logaritmické rovnice a nerovnice

5.1 Exponenciální rovnice

1 Převedte na stejny zaklad
a) {2}, b) {2}, c) {−2}, d) $\{-\frac{1}{2}\}$, e) $\{-2, -\frac{2}{3}\}$, f) {2}, g) {2}, h) $\{2+\sqrt{3}\}$

2 Vytykejte a) $3^x + 3^x\ 3^1 = 108 \Rightarrow 3^x\ (1+3^1) = 108 \Rightarrow 3^x = 27 \Rightarrow x = 3$,
b) {−2}, c) {2}, d) $\{\frac{1}{2}\}$, e) {−1}, f) {2}

3 Substituce na kvadratickou rovnici
a) $\{\frac{1}{2}, 1\}$, b) {2}, c) {−2, 0}, d) {0, 1}, e) {−2}, f) {1}

4 a) Vytkněte $3^x(1+3) = 4^x(7-4)$, potom dělte $\frac{3^x}{4^x} = \frac{3}{4} \Rightarrow (\frac{3}{4})^x = \frac{3}{4} \Rightarrow x = 1$,

b) {3}, c) $\{\frac{3}{2}\}$, d) {2}

5 a) Upravte $2^x\ 2^x + 2^x\ 3^x = 2\ 3^x\ 3^x$, potom dělte součinem $2^x\ 3^x$ a dostanete $(\frac{2}{3})^x + 1 = 2\ (\frac{3}{2})^x$ Po substituci kvadraticka rovnice $x = 0$,
b) upravit, dělit součinem $4^x\ 2^x$, $x = 2$

6 Logaritmujte
a) $x = \log_3 10 = \frac{\log 10}{\log 3} = 2{,}10$, b) $x = \log_5 \frac{4}{5} = \frac{\log \frac{4}{5}}{\log 5} = -0{,}14$,
c) $x = \log_6 18 = \frac{\log 18}{\log 6} = 1{,}61$, d) $x = \log_{\frac{16}{245}} 16 = \frac{\log 16}{\log 16 - \log 245} = -1{,}02$

7 a) $\{\log_7 \frac{2}{49}\}$, b) $\{\log_3 2, -1\}$ c) $\{\log_{\frac{2}{3}} \frac{16}{3}\}$, d) $\{\log_{\frac{2}{3}} 2, -1\}$

8 a) Substitute $2^x = a$, $3^y = b$, [3, 2], b) substitute $2^x = a$, $3^y = b$, [1, 3],
c) substitute $2^{x-y} = a$, $2^{x+y} = b$, [3, 2],
d) substitute $16^x = a$, $16^y = b$, 2 řešení: $[\frac{1}{2}, \frac{1}{4}]$, $[\frac{1}{4}, \frac{1}{2}]$,
e) v první rovnici převedte obě strany na zaklad 5, ve druhe na zaklad 6 [14, 2],
f) substitute $x^y = a$, [3, 2]

5.2 Logaritmické rovnice

9 Užijte definici logaritmu čísla
a) {7}, b) {14}, c) {−2}, d) {4}, e) $\{\frac{1}{8}\}$, f) {16}, g) $\{\frac{1}{8}\}$, h) $\{\frac{1}{3}\}$

10 Rovnají-li se logaritmy o stejnem zakladu, potom se rovnají i logaritmovane vyrazy
a) {−5}, b) $\{\pm\sqrt{2}\}$, c) {−1, 11}, d) {2}

11 Užijte pravidla pro počitani s logaritmy
a) {100}, b) {36}, c) {2}, d) {1}, e) $\{\frac{3}{2}\}$, f) {2},
g) {1 000}, h) {10}, i) $\{\frac{1}{2}\}$, j) $\{\frac{1}{32}\}$, k) {2}, l) {2}

12 Užijte definici logaritmu, viz uloha 9 a) $\{\frac{9}{5}\}$, b) $\{\frac{25}{3}\}$, c) {2} d) $\{\pm\frac{2\sqrt{3}}{3}\}$

13 V rovnici odstraňte zlomek, pak užijte vztah $k\ \log x = \log x^k$, dale viz úlohu 10
a) {11}, b) $\{\frac{1}{2}\}$, c) $\{\frac{2}{9}\}$, d) {4}

14 Substituce $\log_a x = t$, dostanete kvadratickou rovnici
a) $\{2, \frac{1}{8}\}$, b) $\{81, \frac{\sqrt{3}}{3}\}$, c) $\{-\frac{1}{2}, 63\}$, d) $\{-\frac{1}{3}, 40\}$

15 Substituce $\log x = t$, dostanete rovnici s neznamou ve jmenovateli a) {100} b) $\{\frac{1}{10}\}$

16 a), c) Substituce, uprava, kvadratická rovnice
b), d) Pravidlo $\log x^k = k\ \log x$, potom substituce, uprava, kvadraticka rovnice
a) {7}, b) $\{10; \frac{\sqrt[3]{10}}{100}\}$, c) $\{-\frac{8}{9}, 8, -1+\sqrt{3}, -1-\frac{\sqrt{3}}{3}\}$, d) $\{4, \frac{1}{4}\}$

17 Dejte si pozor, kde je napsano na druhou Užijte pravidlo pro logaritmus součinu
a) {100}, b) $\{10^{-4}, 10\}$, c) $\{10\sqrt[3]{10}\}$, d) $\{\pm 0{,}01, \pm\sqrt{10}\}$

18 Pravidla pro počitaní s logaritmy Složitější ulohy
a) $\{\frac{1}{100}\}$, b) $\{\frac{1}{2}, 1\}$, c) $\{8, \frac{\sqrt[3]{4}}{16}\}$, d) $\{\frac{1}{2}, \frac{\sqrt[3]{2}}{2}\}$, e) {3}, f) {0,01, 10, 100}

19 Logaritmujte a) $x^{\log x} = 100x \Rightarrow \log x^{\log x} = \log 100x \Rightarrow \log x\ \log x = \log 100 + \log x$, potom substituce $\log x = a$
a) {0,1, 100}, b) {0,01, 10}, c) {0,1, 1 000}, d) $\{\frac{1}{3}, 27\}$, e) $\{\frac{\sqrt{7}}{7}, 49\}$, f) $\{\frac{1}{4}\ 2\}$

20 Užijte vzorec pro změnu zakladu logaritmu $\log_a x = \frac{\log_b x}{\log_b a}$
a) $\{\frac{\sqrt[6]{32}}{2}\}$, b) {81}, c) {2}, d) $\{\sqrt{2}\}$

21 a) $x = 7 - y$, dosadit o druhe rovnice [5, 2], [2, 5]
b) Ze druhe rovnice plyne $x\ \ y = 2^3 \Rightarrow x = \frac{8}{y}$, dosadit do prvni rovnice [4, 2], [2 4]
c) Převedte všechny logaritmy na zaklad 2, potom $\log_2 x = a$, $\log_2 y = b$ [2, 4]
d) Nejdříve upravte prave strany obou rovnic, potom užijte definice logaritmu [2 0]

22 a) Substituce $\log x = a$, $\log y = b$ [100 0,1]
b) Nejdříve ve 2 rovnici pravidlo pro logaritmus mocniny, potom substituce [16, 8]
c) Substituce $\log_{\frac{1}{2}} x = a$, $\log_2 y = b$ [64, 512], $[\frac{1}{2}, 4]$
d) $x = \frac{100}{y}$ dosadit do druhe rovnice [1 000, 0,1], [0,1, 1 000]
e) V prvni rovnici obě strany na stejny zaklad, ve druhe užijte definici logaritmu [2, 7]
f) Substituce $5^{\log x} = a$, $3^{\log y} = b$ [1, 10]