Numerical solution of differential equations

$$\boxed{\text{differential equation}} \xrightarrow{\text{discretization}} \boxed{\text{(system of) algebraic eq.}}$$

$$\boxed{\text{exact solution}} \xleftarrow{\text{convergence}} \boxed{\text{numerical solution}}$$
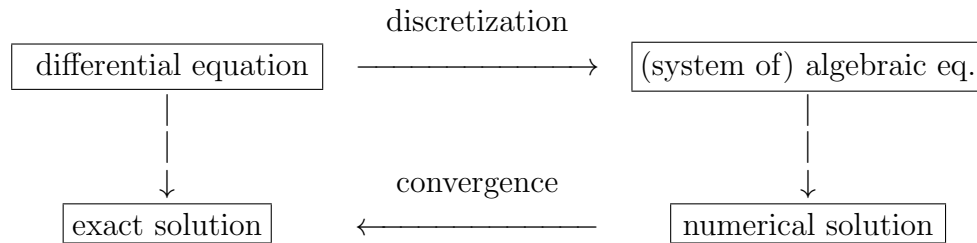
Example

Poisson equation $-y'' = 12x^2$ with boundary conditions $y(-3) = y(3) = 0$

(can represent stationary heat or diffusion problem, or many others).

Discretization using the Finite Difference method with nodes $X = (-2, -1, 0, 1, 2)$ leads to

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 48 \\ 12 \\ 0 \\ 12 \\ 48 \end{bmatrix}$$

where y_i are approximate values of the exact solution $y(x)$ at the nodes.

Comparison of the exact solution ($y(x) = 81 - x^4$, blue) and the numerical one (black dots):
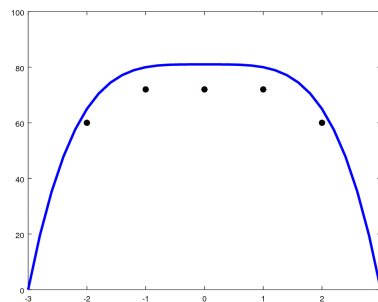
Our roadmap

1. Methods for **solving a system of algebraic equations** – both linear and nonlinear

2. **Discretization** of ODR, PDR, **convergence** of the numerical solution to the exact solution

3. **Interpolation, approximation** - minor topic (1 week)