

NMA – homework from week 1

1. Consider a matrix A and a vector v :

$$A = \begin{bmatrix} 4 & 2 & 5 \\ 0 & 3 & 1 \\ 0 & -13 & -1 \end{bmatrix} \quad v = \begin{bmatrix} -6 \\ 4 \\ -1 \end{bmatrix}$$

- Compute row, column and Frobenius norms of the matrix A and the vector v .
- Compute spectral radius $\rho(A)$ of the matrix A and compare it with the norms computed before.
- What can you now say about the value of the spectral norm of the matrix A ? (Hint: you know its Frobenius norm and its spectral radius.)
Check your forecast by computing the spectral norm (using Matlab).

2. Consider a matrix A :

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

- Compute row, column and Frobenius norms of the matrix A .
- Name some important properties of the matrix A and prove them.
- Which of the five values $\{1, 0, -1, 3 - 2i, 6\}$ cannot be eigenvalues of the matrix A and why? Answer this question before you compute the actual eigenvalues; use reasoning based on your previous answers to a) and b) only.
Then compute all eigenvalues λ_i of the matrix A .
- Compute spectral radius $\rho(A)$ of the matrix A and compare it with other norms computed before. What can you say about spectral norm of this matrix, if you know its spectral radius?