NMA – homework from week **2**

1. Fixed Point Iterations (*FPI*): Consider a linear system $x = Ux + v$, where

$$U = \begin{bmatrix} 0 & -0.49 \\ 1 & 0 \end{bmatrix} \qquad v = \begin{bmatrix} 5 \\ 10 \end{bmatrix}.$$

(a) Will *FPI* converge for this system? Give reasons for your answer.

(b) Use $x^{(0)} = v$ and compute $x^{(1)}$ and $x^{(2)}$ using *FPI*.

(c) Compute $\|x^{(2)} - x^{(1)}\|_\infty$, i.e. the row norm of the difference between the vectors $x^{(2)}$ and $x^{(1)}$.

2. Jacobi (*J*) method: Consider a linear system $Ax = b$, where

$$A = \begin{bmatrix} 1 & -10 & -2 \\ -1 & 5 & 0 \\ 2 & 0 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}.$$

(a) Will J method converge for the given system? Give reasons for your answer.

(b) Choose $x^{(0)} = b$ and compute $x^{(1)}$ and $x^{(2)}$ using J method.

3. Gauss-Seidel (*GS*) method: Consider a linear system $Fx = g$, where

$$F = \begin{bmatrix} 3 & 1 & 0 \\ 1 & p & 1 \\ 0 & 1 & 3 \end{bmatrix} \qquad g = \begin{bmatrix} 3p \\ -1 \\ 4 \end{bmatrix}, \qquad p \in R \text{ is a parameter.}$$

(a) Find all values of p such that the matrix F is strictly diagonally dominant.

(b) Find all values of p such that the matrix F is symmetric positive definite.

(c) Choose $p = 1$, $x^{(0)} = [0, 0, 0]^T$ and compute $x^{(1)}$ and $x^{(2)}$ using *GS* method.

(d) Will *GS* method converge for $p = 1$? Give reasons for your answer.