

NMA – homework from week 3

1. Newton's method: Consider the nonlinear system

$$\frac{x^2}{4} + y^2 = 1, \quad y = 2 \cos(\pi x).$$

- (a) Find graphically approximate position of all roots (solutions of the system).
- (b) Choose $x^{(0)} = [-0.5, -1]^T$ and compute $x^{(1)}$ using the Newton's method.
- (c) Compute $\|x^{(0)} - x^{(1)}\|_1$, i.e. the column norm of the difference of the vectors $x^{(0)}$ and $x^{(1)}$.

2. Fixed Point Iterations (FPI): Consider the nonlinear system

$$x = \cos \frac{x - y}{3}, \quad y = \sin \frac{x - y}{4}.$$

- (a) Prove that there exists unique solution of the equations (unique fixed point) and that *FPI* converge to this fixed point for any starting point $x^{(0)}$ in R^2 .
- (b) Choose $x^{(0)} = [2, 2]^T$ and compute $x^{(1)}$ and $x^{(2)}$ using *FPI*.