

## Example

Solve the equation

$$2x + \sin x = 3 \quad (\text{exact solution to 4 decimal places is } 1.0631)$$

using both Newton's method and Fixed point iterations.

**Newton's method:**

$$\text{equation solved: } f(x) = 0, \quad \text{Newton's method: } x_{k+1} = x_k - \frac{f(x)}{f'(x)}$$

$$\underbrace{2x + \sin x - 3}_{f(x)} = 0, \quad f'(x) = 2 + \cos x, \quad x_{k+1} = x_k - \frac{2x_k + \sin x_k - 3}{2 + \cos x_k}$$

Results for different choices of  $x_0$ :

$x_0$	0	20	-20
$x_1$	1.0000	4.2560	-1.7643
$x_2$	1.0624	1.2967	2.3902
$x_3$	1.0631	1.0518	0.4496
$x_4$		1.0631	1.0240
$x_5$			1.0628
$x_6$			1.0631

**FPI:**

$$\text{equation solved: } x = g(x), \quad \text{FPI: } x_{k+1} = g(x_k)$$

$$x = \underbrace{0.5(3 - \sin x)}_{g(x)}, \quad x_{k+1} = 0.5(3 - \sin x_k)$$

Results for different choices of  $x_0$ :

$x_0$	0	20	-20
$x_1$	1.5000	1.0435	1.9565
$x_2$	1.0013	1.0679	1.0367
$x_3$	1.0789	1.0619	1.0696
$x_4$	1.0593	1.0634	1.0240
$x_5$	1.0640	1.0630	1.0615
$x_6$	1.0628	1.0631	1.0630
$x_7$	1.0631		1.0631