Matrix powers – transformation of the unit circle

Example 1 – a general matrix with 2 different real eigenvalues

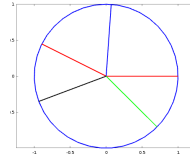$$A = \begin{bmatrix} 0.9 & 0.6 \\ 0 & 0.6 \end{bmatrix}$$

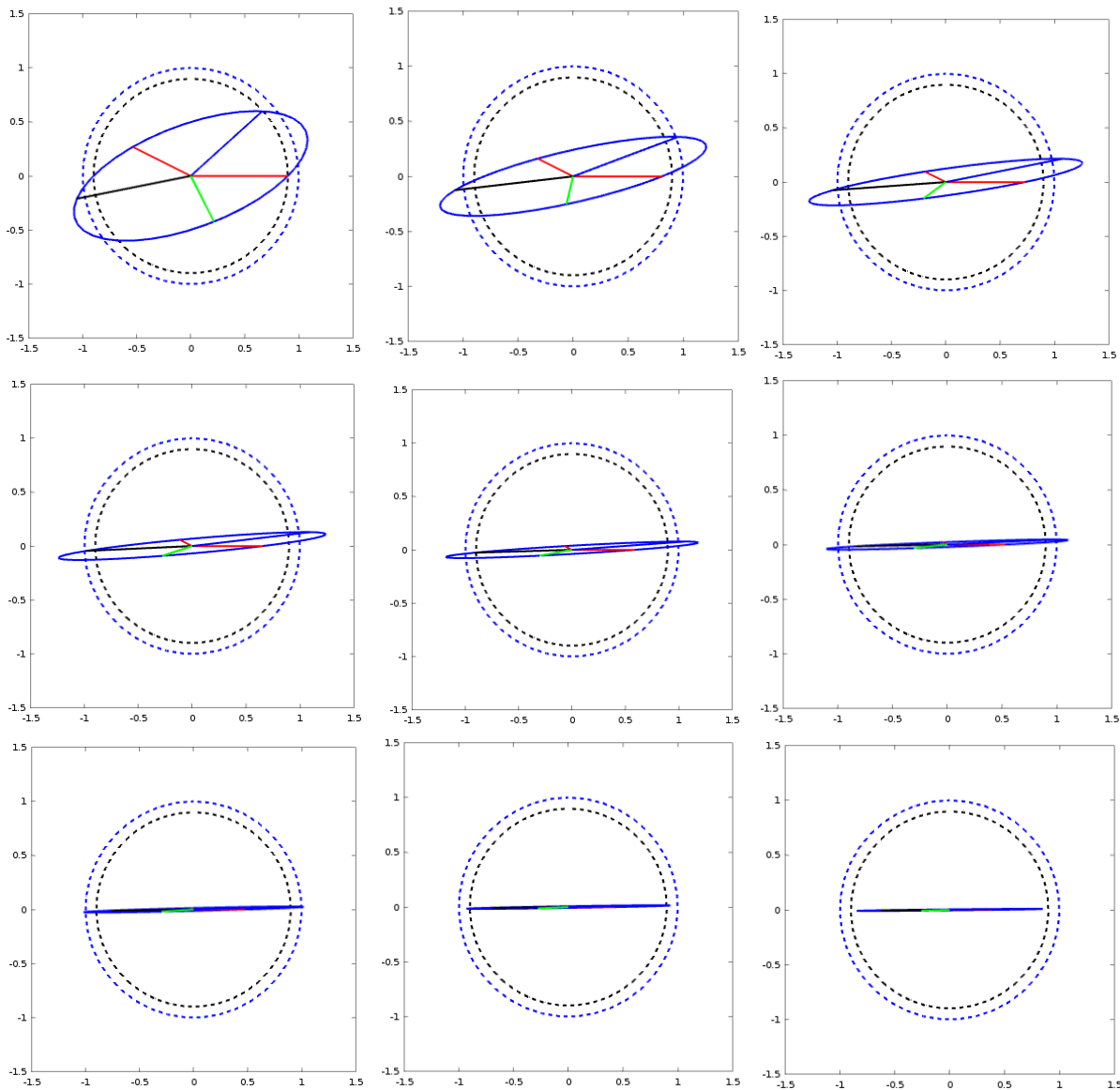$\|A\|_2 = 1.1432$

$\|A\|_F = 1.2369$

$\|A\|_1 = 1.2$

$\|A\|_\infty = 1.5$

$\rho(A) = 0.9$

Right: Unit circle (blue), unit eigenvectors (red), some other unit vectores (different colours).

Bellow: Images of the unit circle and of the vectors using powers A, A^2, ..., A^{10}
(black dashed circle has radius $\rho(A)$, unit circle is blue dashed).

Example 2 – symmetric matrix
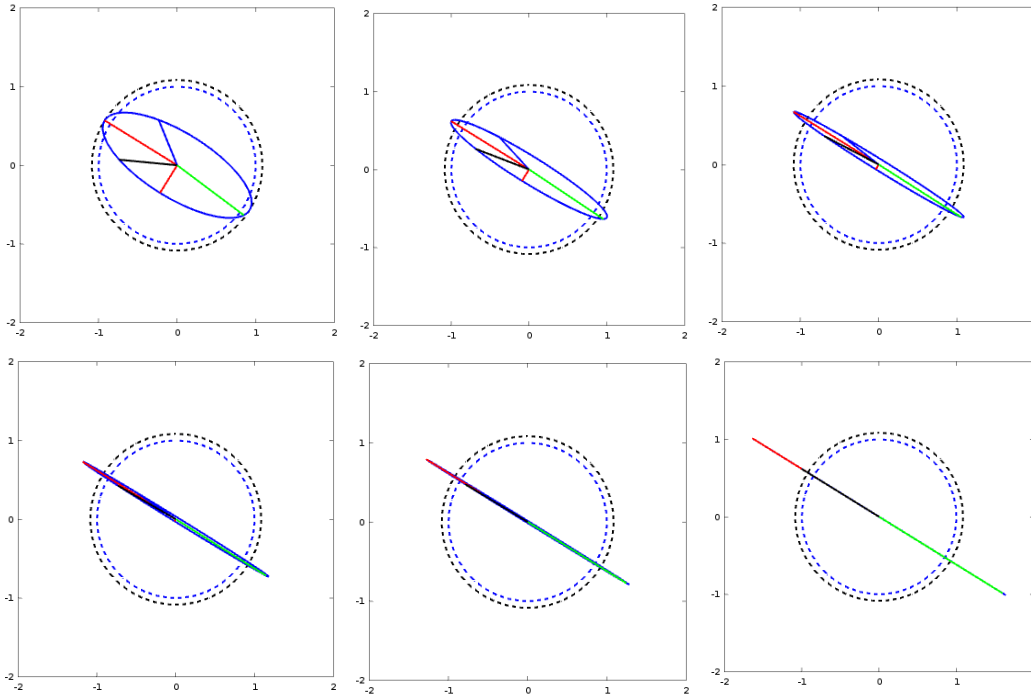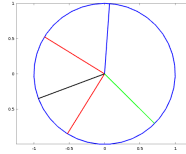eigenvalues are always real, eigenvectors are orthogonal and parallel to axes of the elipse

$A = 0.3 \cdot \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$\rho(A) = 1.0854$

$\|A\|_2 = 1.0854$
$\|A\|_F = 1.1619$
$\|A\|_1 = \|A\|_\infty = 1.2$

Example 3 – matrix with complex eigenvalues

$A = 0.8 \cdot \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$

$\rho(A) = 1.3856$

$\|A\|_2 = 1.8422$
$\|A\|_F = 2.1166$
$\|A\|_1 = \|A\|_\infty = 2.4$