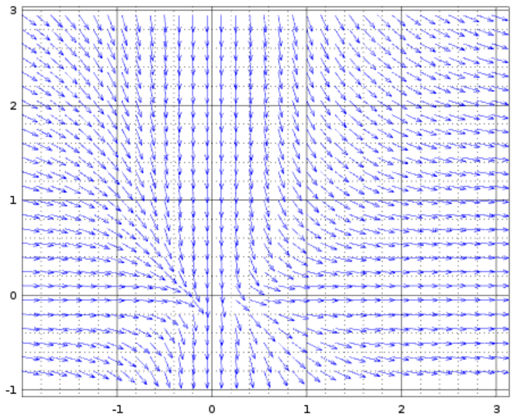
Importance of existence and uniquenes of exact solution

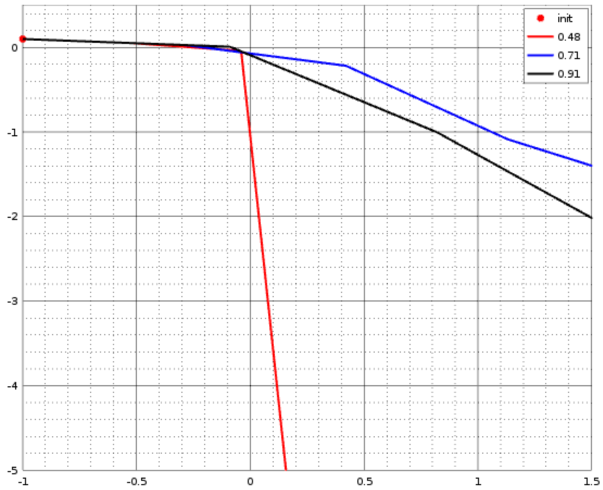
Example 1:

Use Euler method for equation $y' = -\frac{|y|}{x^2}$ with initial condition $y(-1) = 0.1$ and step sizes $h = 0.48$, $h = 0.71$ and $h = 0.91$.

The domain of existence and uniqueness of exact solution: $(-\infty, 0) \times R$ $\quad (-\frac{|y|}{x^2}$ is Lipschitz.)

Numerical solution outside the domain is wrong:



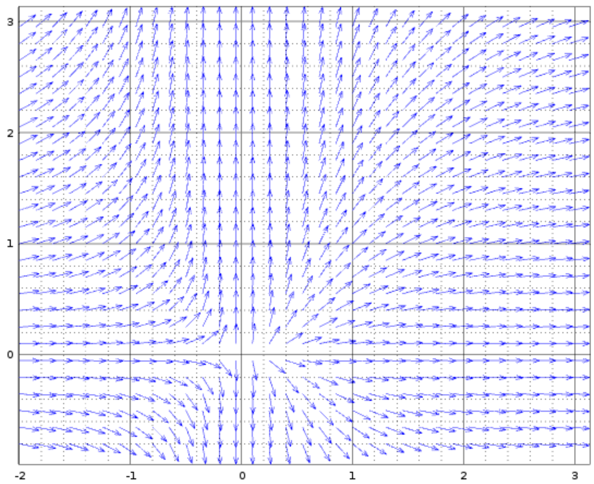


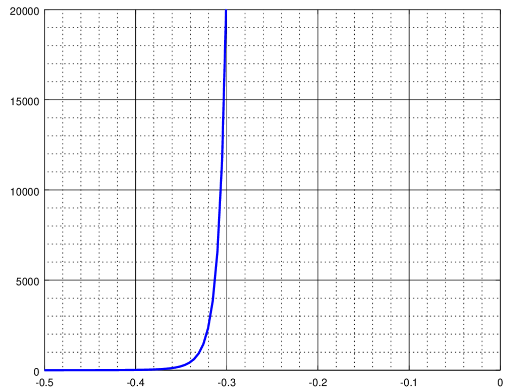
Example 2:

Use Euler method for equation $y' = \frac{y}{x^2}$ with initial condition $y(-1.7) = 0.1$ and step sizes $h = 0.48$, $h = 0.71$ and $h = 0.91$.

The interval of existence and uniqueness of exact solution: $(-\infty, 0)$

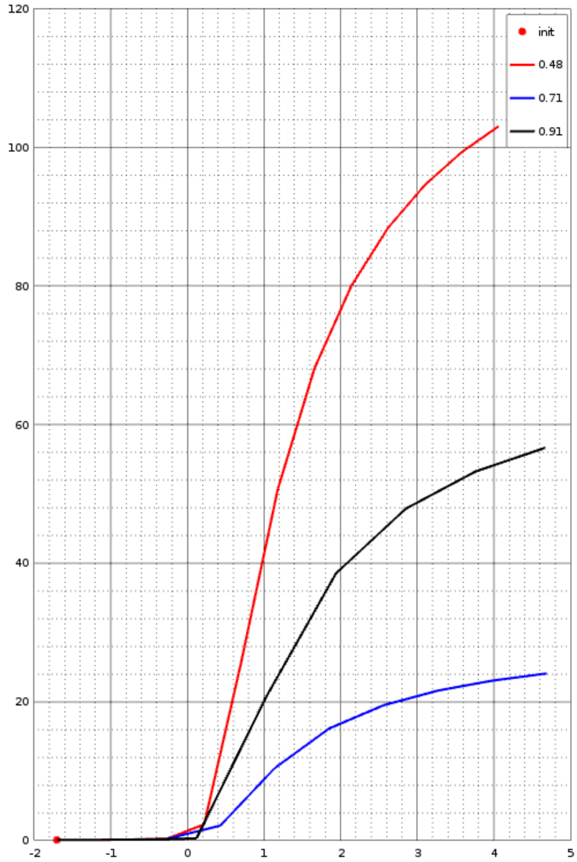
Exact solution: $y = c\, \mathrm{e}^{-\frac{1}{3x^3}}$, where $c \approx 0.09344$

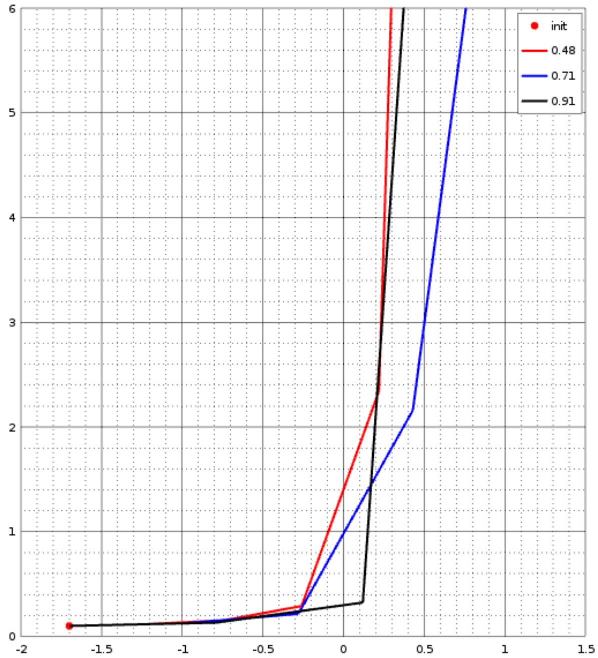




Left: Direction field $\qquad$ Right: part of the exact solution in interval $(-0.5, -0.3)$

Numerical solution outside the interval $(-\infty, 0)$ is wrong:





On the right, the area close to origin is zoomed in.