Computer graphics
Lesson 4

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Interpolation cubic curve

- given 4 definition points A, B, C and D
- create C^2 continuous segmented curve
- segments from Bézier cubic curve and create claped curve

- lets have $A = (4;1), \mathrm{B} = (2;6), \mathrm{C} = (6;6), \mathrm{D} = (10;1)$

  $\rightarrow$ to interpolate we need 3 segments $P_1(t), P_2(t), P_3(t)$

  $\rightarrow$ conditions from Bézier cubic curves segments:

  C^0 continuity: $P_1(1) = P_2(0)$ & $P_2(1) = P_3(0)$

  C^1 continuity: $P'_1(1) = P'_2(0)$ & $P'_2(1) = P'_3(0)$

  C^2 continuity: $P''_1(1) = P''_2(0)$ & $P''_2(1) = P''_3(0)$

Interpolation cubic curve

C^0 continuity: $V_3 = Q_0$, $Q_3 = W_0$, where $V_3 = B$, $Q_3 = C$

C^1 continuity geometrically:

$$V_4 = V_3 + (V_3 - V_2)$$

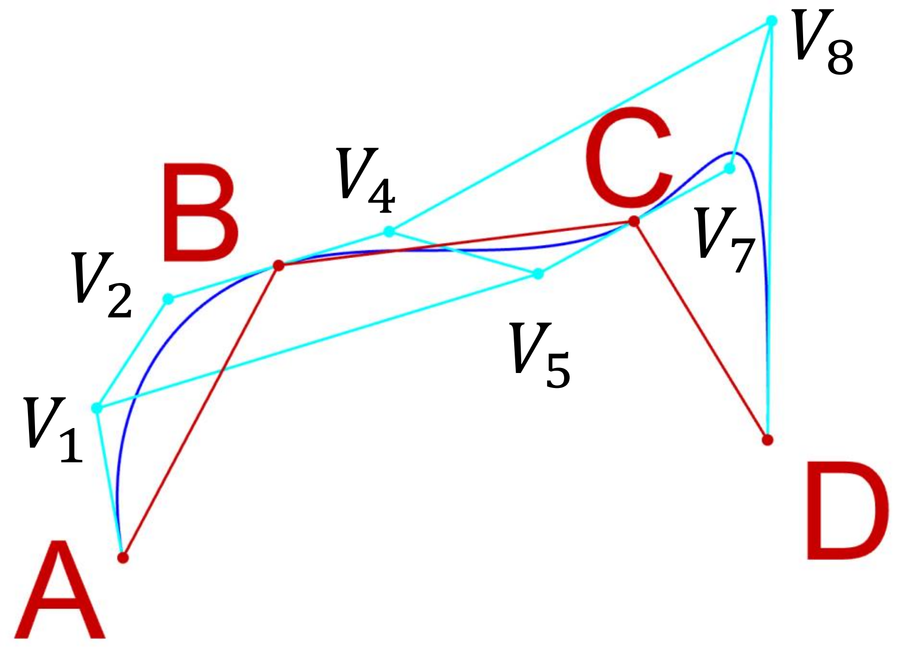
C^2 continuity geometrically:

$$V_5 = V_1 + 4 \cdot (V_3 - V_2)$$

$\rightarrow$ solve equations:

$$V_4 = V_3 + (V_3 - V_2)$$
$$V_7 = V_6 + (V_6 - V_5)$$
$$V_5 = V_1 + 4 \cdot (V_3 - V_2)$$
$$V_8 = V_4 + 4 \cdot (V_6 - V_5)$$

$\rightarrow$ 4 equations for 6 unknown



Interpolation cubic curve

→ 4 equations for 6 unknown → 2 conditions, choose $V_1 = (1;3), V_8 = (7;3)$

→ calculate V_2, V_4, V_5, V_7 using only A, B, C, D, V_1 and V_8

Interpolation cubic curve

$$\rightarrow \quad V_2 = \frac{14}{15}B - \frac{4}{15}C + \frac{4}{15}V_1 + \frac{1}{15}V_8$$

$$V_4 = \frac{16}{15}B + \frac{4}{15}C - \frac{4}{15}V_1 - \frac{1}{15}V_8$$

$$V_5 = \frac{4}{15}B + \frac{16}{15}C - \frac{1}{15}V_1 - \frac{4}{15}V_8$$

$$V_7 = -\frac{4}{15}B + \frac{14}{15}C + \frac{1}{15}V_1 + \frac{4}{15}V_8$$

→ substitute coordinates

Interpolation cubic curve

$\rightarrow$ $V_2 = (1; 5)$

$V_4 = (3; 7)$

$V_5 = (5; 7)$

$V_7 = (7; 5)$

$\rightarrow$ sketch the curve and create the clamped curve from it