Computer graphics

Lesson 5

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Surfaces

- given by set of points → points mesh | set of definition curves
- **types**:
  1. Approximation surface – mesh of control points, control mesh
  2. Interpolation surface – mesh of definition points, definition mesh
- joining of individual surface elements (= patches) with given continuity is ***patching***
- consider easiest free-form surfaces: ***ruled surface*** (interpolation), ***surface of hyperbolic paraboloid*** (interpolation), ***Coons bilinear surface*** (interpolation), ***Bézier surface*** (approximation)

- curvatures, tangent planes etc. → page 24-28

Vector equation of a surface

- square area of parametrization $\langle 0; 1\rangle^2$
- input data in matrix form – map of surface
- forms of equation:

$$\mathbf{P}(u,v) = (x(u,v), y(u,v), z(u,v)),\ \ (u,v) \in [0,1]^2$$

1. matrix form (preferred)
2. linear combination of basis functions → multiplication of matrices

$$\mathbf{P}(u,v) = H_0(u)H_0(v)\mathbf{M}_{0,0} + H_0(u)H_1(v)\mathbf{M}_{0,1} + \ldots + H_m(u)H_n(v)\mathbf{M}_{m,n},\ \ (u,v) \in [0,1]^2$$

3. expression by summation

$$\mathbf{P}(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} H_i(u)H_j(v)\mathbf{M}_{i,j},\ \ (u,v) = [0,1]^2$$

$$\begin{aligned}\mathbf{P}(u,v) &= \mathbf{H}(u)\cdot\mathbf{M}\cdot\mathbf{H}(v) = \\ &= (H_0(u), H_1(u), \ldots, H_m(u)) \cdot \begin{pmatrix} \mathbf{M}_{0,0} & \mathbf{M}_{0,1} & \ldots & \mathbf{M}_{0,n} \\ \mathbf{M}_{1,0} & \mathbf{M}_{1,1} & \ldots & \mathbf{M}_{1,n} \\ \vdots & \vdots & & \vdots \\ \mathbf{M}_{m,0} & \mathbf{M}_{m,1} & \ldots & \mathbf{M}_{m,n} \end{pmatrix} \cdot \begin{pmatrix} H_0(v) \\ H_1(v) \\ \vdots \\ H_n(v) \end{pmatrix} \\ & (u,v) \in [0,1]^2. \end{aligned}$$

Tangent vector of a surface

Definition 1.25 – Tangent vectors of parametric curves. The first partial derivative

$$\mathbf{P}^u(u,v) = \frac{\partial \mathbf{P}(u,v)}{\partial u} = (x^u(u,v),\ y^u(u,v),\ z^u(u,v)),\ (u,v) \in I \qquad (1.44)$$

is a vector function. For $(\alpha, \beta) \in I$, this vector function determines a *tangent vector of parametric u-curve* $\mathbf{P}^u(u,v)$ at point $\mathbf{P}(\alpha,\beta)$

$$\mathbf{P}^u(\alpha,\beta) = \left.\frac{\partial \mathbf{P}(u,v)}{\partial u}\right|_{u=\alpha, v=\beta} = (x^u(\alpha,\beta),\ y^u(\alpha,\beta),\ z^u(\alpha,\beta)). \qquad (1.45)$$

→ Textbook pg. 28

- similar for *v*-curve
- tangent plane is determined by tangent vectors of parametric *u*-curve and *v*-curve in the given point on surface

Twist vector of a surface

Definition 1.28 – Twist vector. The second mixed partial derivative of vector function

$$\begin{aligned}\mathbf{P}^{uv}(u,v) &= \frac{\partial^2 \mathbf{P}(u,v)}{\partial u \partial v} = \frac{\partial^2 \mathbf{P}(u,v)}{\partial v \partial u} = \\ &= (x^{uv}(u,v),\ y^{uv}(u,v),\ z^{uv}(u,v)) = (x^{vu}(u,v),\ y^{vu}(u,v),\ z^{vu}(u,v)),\\ &\quad (u,v) \in I \end{aligned} \tag{1.49}$$

is a vector function. For $(\alpha, \beta) \in I$, this vector function determines a *twist vector* of the surface at point $\mathbf{P}(\alpha, \beta)$. □

Twist vectors correspond to elevation of the surface from its tangent plane.

→ Textbook pg. 28

Ruled surface

- interpolates 2 given opposite boundaries (curves given by vector function of one variable from linear interpolation between given boundaries – connection of one and one point on each curve by straight line segment)
- corners: $P_{0,0} = P(0,0)$, $P_{0,1} = P(0,1)$, $P_{1,0} = P(1,0)$, $P_{1,1} = P(1,1)$
- boundaries: $P_0(u) = P(u,0)$, $P_1(u) = P(u,1)$,

$$P_0(v) = P(0,v),\ P_1(v) = P(1,v)$$

ruled surface given by boundaries in u-direction is

$$\mathbf{P}(u,v) = (1-v)\mathbf{P}_0(u) + v\mathbf{P}_1(u),\ \ (u,v) \in [0,1]^2$$

v-direction is

$$\mathbf{P}(u,v) = (1-u)\mathbf{P}_0(v) + u\mathbf{P}_1(v),\ \ (u,v) \in [0,1]^2$$

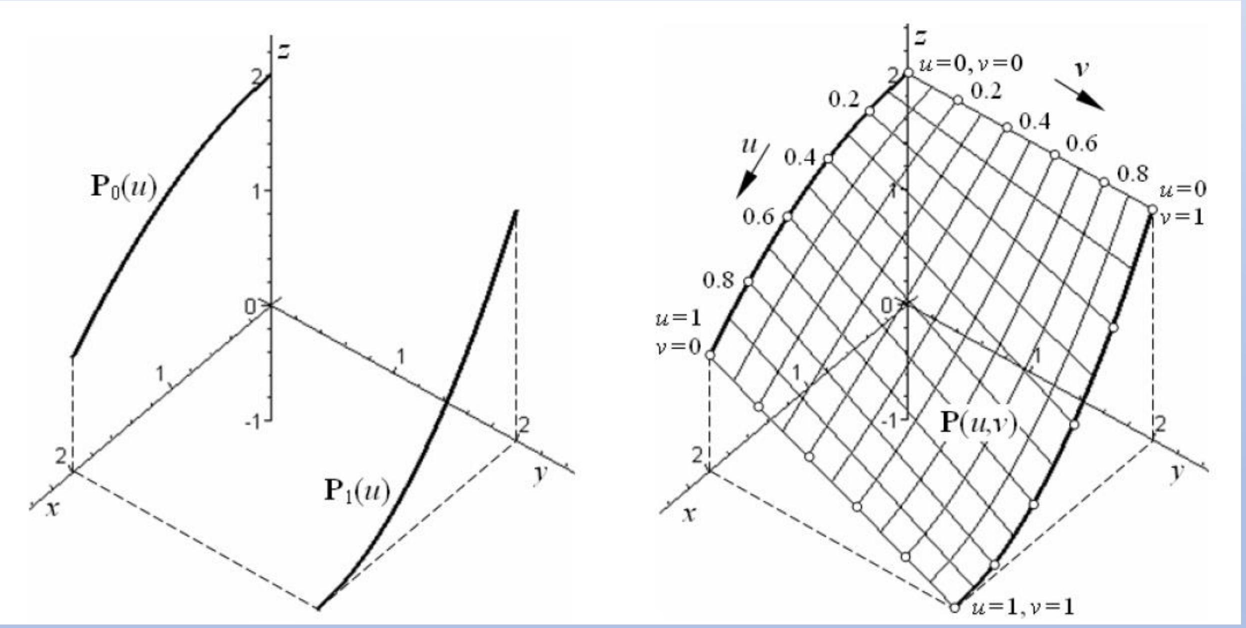
Ruled surface

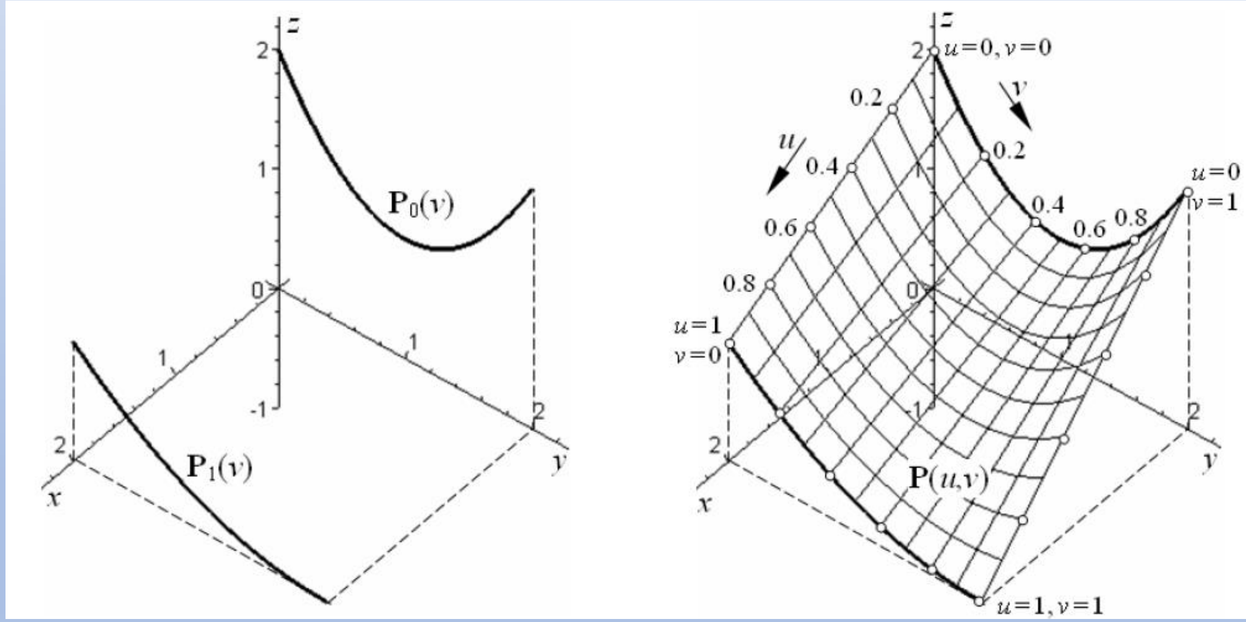
- properties:
  - interpolates given boundary curves
  - the boundaries in *v*-direction of ruled surface given by boundaries in *u*-direction are straight line segments
  - the boundaries in *u*-direction of ruled surface given by boundaries in *v*-direction are straight line segments

Ruled surface

- derivation of ruled surface equation

- **Example 3.1** **Example 3.2**





Surface of hyperbolic paraboloid

- interpolates 4 given points (= corners)
- bilinear interpolation – linear interpolation between 4 points

- vector equation of surface of HP given by corners $P_{0,0}$, $P_{0,1}$, $P_{1,0}$, $P_{1,1}$:

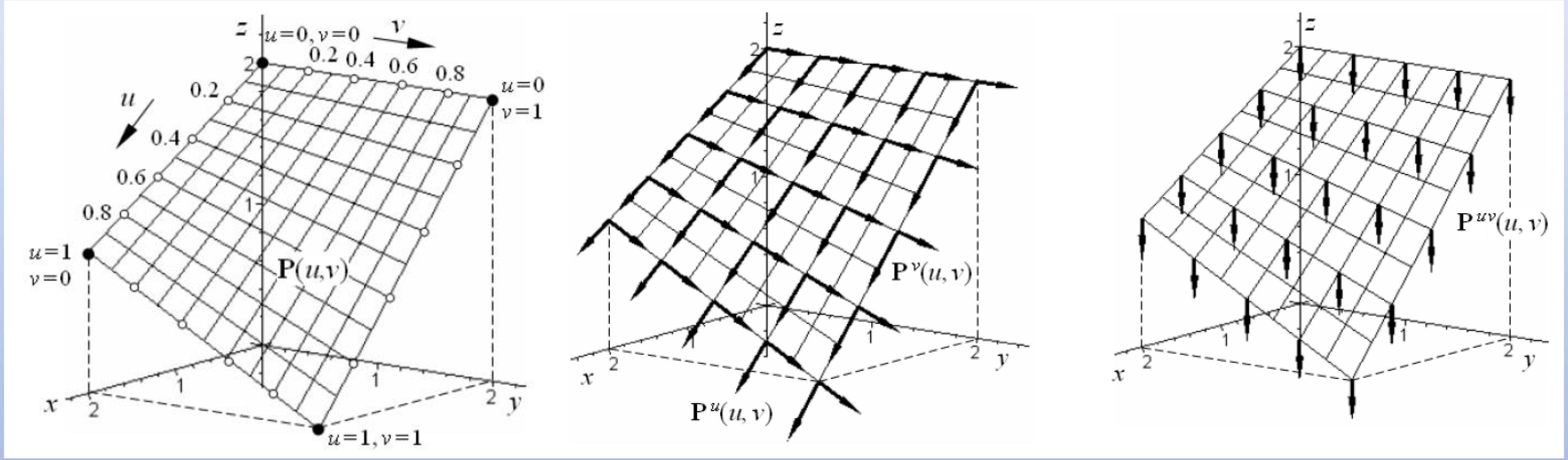
$$\mathbf{P}(u,v) = (1-u)(1-v)\mathbf{P}_{0,0} + (1-u)v\mathbf{P}_{0,1} + u(1-v)\mathbf{P}_{1,0} + uv\mathbf{P}_{1,1},\ (u,v) \in [0,1]^2$$

Surface of hyperbolic paraboloid

- **properties:**
  - interpolates given corners of the patch
  - parametric curves in both directions (also boundaries) are straight line segments
  - surface of HP is identical with ruled surface given by straight line segments as boundaries

Surface of hyperbolic paraboloid

- **Example 3.4**



Resulted surface

Tangent vectors of parametric curves

Twist vectors